I) THE ACCRETION WEDGE.

I1) The model.

Consider a 2-D prismatic body having a triangular vertical section (Fig.I1), in the presence of gravity. The wedge rests on a rigid basement having the slope equal to θ_0. Both the compressional force acting on the left side of the wedge and the friction to the basement cause thickening of the incompressible material and the development of a topographical slope equal to α. It is assumed that the material is into a state of plastic yielding according to the VON MISES-HENCKY criterion. It follows to obtain a condition relating the slopes of the topography and that of the basement to the geometry of the wedge, its yield strength and the friction coefficient to the basement.

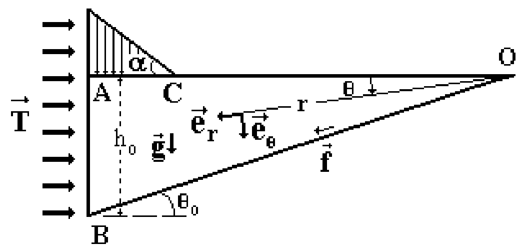

Fig.I1. The 2-D accretion wedge.

I2) Equations of equilibrium. Yield condition. Stress field.

Taking into account that the 2-D case is discussed, the stress in polar co-ordinates is

$$\sigma = \begin{pmatrix} \sigma_{rr} & \sigma_{r\theta} & 0 \\ \sigma_{r\theta} & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \nu(\sigma_{rr}+\sigma_{\theta\theta}) \end{pmatrix} . \qquad \text{(i1)}$$

Because the material is assumed to be incompressible, the POISSON coefficient is $\nu = 1/2$.
Using polar co-ordinates, the equilibrium equation (d40)-(d41) in the presence of gravity are

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{r\theta}}{r\partial\theta} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} + \rho g \sin\theta = 0 \qquad \text{(i2)}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{\theta\theta}}{r\partial\theta} + 2\frac{\sigma_{r\theta}}{r} + \rho g \cos\theta = 0 \qquad \text{(i3)}$$

Using (i1), the yield condition (r54) is

$$\sigma_1 - \sigma_3 = 2k \qquad \text{(i4)}$$

Taking into account again that the compressive stress is assumed to have positive sign, eqs.(d11)-(d13) give

$$\begin{cases} \sigma_{rr} = -f(r,\theta) - k\cos 2\psi \\ \sigma_{\theta\theta} = -f(r,\theta) + k\cos 2\psi \\ \sigma_{r\theta} = k\sin 2\psi \end{cases} , \qquad \text{(i5)}$$

where the trace of the stress has been denoted by $f(r,\theta) = \frac{1}{2}\mathrm{tr}(\sigma)$ and $\psi = \psi(r,\theta)$ is the angle between the radius and the local direction of the maximum compressive stress (variable inside the wedge). However, it will be assumed that $\psi = \psi(\theta)$ only. After some manipulations, substituting (i5) into (i2)-(i3) gives

$$\begin{cases} \dfrac{\partial f}{\partial r} = \dfrac{2k}{r}\cos 2\psi \dfrac{d\psi}{d\theta} - \dfrac{2k}{r}\cos 2\psi + \rho g \sin\theta \\ \dfrac{\partial f}{\partial \theta} = -2k\sin 2\psi \dfrac{d\psi}{d\theta} + 2k\sin 2\psi + \rho g r\cos\theta \end{cases} \qquad \text{(i6)}$$

Eqs.(i6a) and (i6b) are differentiated with respect to θ, r respectively, the results of the differentiation being equal each other. After some elementary manipulations, it follows that

$$\frac{d}{d\theta}\left(\cos 2\psi \frac{d\psi}{d\theta} - \cos 2\psi\right) = 0 \quad , \tag{i7}$$

i.e.

$$\frac{d\psi}{d\theta} = 1 + \frac{C}{\cos 2\psi} \quad , \tag{i8}$$

where C is a constant of integration. Substituting (i8) into (i6a) gives

$$\frac{\partial f}{\partial r} = \frac{2Ck}{r} + \rho g \sin\theta \quad , \tag{i9}$$

i.e.

$$f = 2Ck \ln r + \rho g r \sin\theta + g(\theta) \quad , \tag{i10}$$

To find the unknown function $g = g(\theta)$, eq.(i10) is substituted into (i6b) to obtain

$$\frac{dg}{d\theta} = -2Ck \tan 2\psi \quad . \tag{i11}$$

Using (i8), it follows that

$$\frac{dg}{d\psi} = -2Ck \frac{\sin 2\psi}{\cos 2\psi + C} \quad , \tag{i12}$$

i.e.

$$g = Ck \ln(C + \cos 2\psi) + A \quad , \tag{i13}$$

where A is another constant of integration. Hence the final stress inside the wedge is

$$\begin{cases} \sigma_{rr} = -2Ck \ln r - Ck \ln(C + co2\psi) - \rho g r \sin\theta - A - k\cos 2\psi \\ \sigma_{\theta\theta} = -2Ck \ln r - Ck \ln(C + co2\psi) - \rho g r \sin\theta - A + k\cos 2\psi \\ \sigma_{r\theta} = k \sin 2\psi \end{cases} \quad . \tag{i14}$$

I3) Boundary conditions. Final results.

Consider the segment AC placed on the side OA of the wedge, having $\theta = 0$ and $OC \le r \le OA$, where the point C is very closed to the point A. The outward pointing normal vector is $\vec{n} = -\vec{e}_\theta$. Here, is acting the lithostatic pressure due to the topography. Hence

$$\sigma\left(-\vec{e}_\theta\right) = \rho g r \tan\alpha \, \vec{e}_\theta \quad , \tag{i15}$$

i.e.

$$\begin{cases} \sigma_{r\theta}(\theta = 0) = 0 \\ \sigma_{\theta\theta}(\theta = 0) = -\rho g r \tan\alpha \end{cases} \quad . \tag{i16}$$

Because the angle θ_0 has very small values, it will be assumed for all angles θ that

$$\sigma_{\theta\theta} = -\rho g r \tan\alpha \quad , \tag{i17}$$

or

$$\frac{\partial \sigma_{\theta\theta}}{\partial r} = -\rho g \tan\alpha \quad . \tag{i18}$$

However, all the next derivations are supposed to be valid at the rear of the wedge, where the topography is generated due to the horizontal compression, i.e. the radius r is a mean value of the lengths OA and OC, the point C being closed to A. By differentiating (i14b), eq.(i18) leads to

$$\frac{2Ck}{r} = \rho g \tan\alpha \quad , \tag{i19}$$

where

$$r \cong \frac{h_0}{\theta_0} \qquad . \qquad \text{(i20)}$$

Consider now the side OB of the wedge, having $\theta = \theta_0$ and the outward pointing normal vector $\vec{n} = \vec{e}_\theta$. Here, is acting the friction force due to the basement, assumed to have the magnitude equal to λk, where λ is a friction coefficient. Hence

$$\sigma \vec{e}_\theta = \lambda k \vec{e}_r \quad , \qquad \text{(i21)}$$

i.e.

$$\sigma_{r\theta}(\theta = \theta_0) = \lambda k \quad . \qquad \text{(i22)}$$

The next partial derivative follows to be evaluated in two ways. In the first approach, eqs.(i22), (i16a) and (i20) are used to give

$$\frac{\partial \sigma_{r\theta}}{r\partial\theta} \approx \frac{1}{r} \frac{\sigma_{r\theta}(\theta = \theta_0) - \sigma_{r\theta}(\theta = 0)}{\theta_0 - 0} = \frac{\lambda k}{h_0} \qquad . \qquad \text{(i23)}$$

The dominant stress is into the wedge is the horizontal compression. Hence $\psi \cong \theta$, both angles having small values. It follows $\cos 2\psi \cong 1$. Eqs.(i5c), (i8), (i19) and (i20) give

$$\frac{\partial \sigma_{r\theta}}{r\partial\theta} = \frac{2k}{r} \cos 2\psi \frac{d\psi}{d\theta} = \frac{2k}{r}(C + \cos 2\psi) \approx \frac{2Ck}{r} + \frac{2k}{r} = \rho g \tan\alpha + \frac{2k\theta_0}{h_0} \qquad \text{(i24)}$$

From (i23) and (i24) it follows

$$\rho g h_0 \tan\alpha + 2k\theta_0 \cong \lambda k \quad , \qquad \text{(i25)}$$

showing that the friction force (resistance to sliding of the wedge onto the basement) is balanced by tqo forces. The first one is due to the topography and the second force is related to the compressive stress and to the slope of the basement. Further details related to the application of eq.(i25) in real cases are presented by Ranalli (1987).