CWP-175
May 1995

Moveout analysis for transversely isotropic media with a tilted symmetry axis

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303/273-3557

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ABSTRACT

The transversely isotropic model with a tilted axis of symmetry may be typical, for instance, for sediments near the flanks of salt domes. Here, I use the exact normal-moveout (NMO) equation of Tsvankin (1995a) to study reflection moveout in the symmetry plane of this medium that contains the symmetry axis. While for vertical and horizontal transverse isotropy zero-offset reflections exist for the full range of dips up to 90 degrees, it is no longer the case for intermediate axis orientations. For typical homogeneous models with a symmetry axis tilted towards the reflector, wavefront distortions make it impossible to generate specular zero-offset reflected rays from steep interfaces. These "missing" dipping planes can be imaged only in vertically inhomogeneous media by using *turning* waves. In another situation, typical for a symmetry axis tilted away from the reflector, specular zero-offset reflections from overhang structures can be recorded in the absence of velocity gradient. Implications of these unusual phenomena in salt and sub-salt imaging deserve a special study.

In elliptically anisotropic media with any orientation of the elliptical axes the dependence of the P–wave NMO velocity on the ray parameter p (the "dip-moveout signature") was proved to be identical to the isotropic equation used in conventional constant-velocity dip-moveout (DMO) algorithms. Not just DMO, but all other isotropic time-processing methods including time migration are entirely valid for elliptical anisotropy. In nonelliptical TI media the tilt of the symmetry axis has a drastic influence on moveout from horizontal reflectors, as well as on the dip-dependence of NMO velocity. The DMO signature retain the same character as for vertical transverse isotropy only for near-vertical and near-horizontal orientation of the symmetry axis. For this relatively narrow range of tilt angles the P–wave NMO velocity is tightly controlled by the parameter η introduced by Alkhalifah and Tsvankin (1995) and, for typical $\eta > 0$, increases with p much faster than in isotropic media. The behavior of NMO velocity rapidly changes if the symmetry axis is tilted away from vertical, with a tilt of ± 20 degrees being sufficient to practically eliminate the influence of the anisotropy on the DMO signature. For larger tilt angles and typical positive values of the difference between the anisotropic parameters ϵ and δ, the NMO velocity increases with p slower than in homogeneous isotropic media – a dependence usually caused by vertical velocity gradient. Dip-moveout processing for a wide range of tilt angles

requires application of anisotropic DMO algorithms, such as the Hale-type method by Anderson and Tsvankin (1995).

The strong influence of tilt on the P–wave moveout can be used to constrain the tilt angle using P–wave moveout data in the plane that includes the symmetry axis. However, if the azimuth of the axis is unknown, the inversion for the axis orientation cannot be performed without a 3–D analysis of reflection traveltimes on lines with different azimuthal directions.

INTRODUCTION

The behavior of reflection moveouts has a strong influence on most seismic processing steps, such as velocity analysis, normal-moveout (NMO) correction, dip-moveout (DMO) removal and migration. In seismic exploration, recording is conventionally carried out on common-midpoint (CMP) spreads with the length close to the depth of the target horizon. For these moderate offset-to-depth ratios, P–wave reflection moveout is usually close to hyperbolic and is adequately described by the normal-moveout velocity calculated in the zero-spread limit (Taner and Koehler, 1969; Tsvankin and Thomsen, 1994).

If the medium is anisotropic, normal-moveout velocity is generally different from the vertical velocity. The influence of anisotropic parameters on NMO velocity represents a fundamental problem in the velocity analysis for anisotropic media. This problem has attracted considerable attention in the literature, but most existing work is restricted to a relatively simple (although common) model – transversely isotropic media with a vertical symmetry axis (VTI). Beginning with the work by Lyakhovitsky and Nevsky (1971), several authors gave analytic expressions (in different notation) for NMO velocities of pure modes (P-P, SV-SV, SH-SH) from a horizontal reflector (I will omit the qualifiers in "quasi–P-wave" and "quasi–SV-wave"). For P–waves, NMO velocity in a VTI layer depends just on the vertical velocity and parameter δ introduced by Thomsen (1986):

$$V_{\rm nmo}\,[P\text{—wave}] = V_{P0}\,\sqrt{1+2\delta}\,. \tag{1}$$

Hake et al. (1984) showed that NMO velocity in layered VTI media is equal to the root-mean-square (rms) of the NMO velocities in the individual layers; they also presented the corresponding equations for the quartic moveout term of pure modes. Normal-moveout velocity in homogeneous VTI models formed by interbedding of isotropic layers was studied numerically by Levin (1979). Banik (1984) demonstrated on North Sea data that the deviation of the P–wave moveout velocity from the rms vertical velocity in anisotropic media may lead to substantial errors in time-to-depth conversion. Normal-moveout velocity for converted P–SV–waves was expressed through the moveout velocities of P–P and SV–SV–waves by Seriff and Sriram (1991). Tsvankin and Thomsen (1994) gave an analytic description of reflection moveout in layered VTI media valid for both the short-spread moveout and the nonhyperbolic (long-spread) portion of the moveout curve; their treatment can be extended in a straightforward way to symmetry planes of any anisotropic medium.

Anisotropy distorts not only the moveout in horizontally-layered media, but also the NMO velocity for dipping reflectors. In isotropic, homogeneous media the dip-dependence of NMO velocity in the dip plane of the reflector is given by (Levin, 1971)

$$V_{\rm nmo}(\phi) = \frac{V_{\rm nmo}(0)}{\cos\phi}, \tag{2}$$

where ϕ is the dip angle. Deviations from the cosine-of-dip dependence are of primary importance in DMO processing, as well as in the inversion for the anisotropic parameters. Analytic expressions for the dip-dependent NMO velocity in elliptically anisotropic media were presented by Byun (1982) and Uren et al. (1990); however, elliptical models are no more than a subset of transversely isotropic media that is not typical for subsurface formations (Thomsen, 1986). Levin (1990) carried out a numerical study of the P-wave NMO velocities for dipping reflectors beneath homogeneous TI media and showed that there is no apparent correlation between the magnitude of velocity anisotropy and errors in the cosine-of-dip dependence. The same conclusion was drawn by Larner (1993) in his modeling study of the P-wave NMO velocities in VTI media with vertical-velocity gradient.

Tsvankin (1995a) derived an equation for NMO velocity from dipping reflectors valid for pure modes in any symmetry plane in anisotropic media. He also transformed this exact equation into a much simpler expression for weakly anisotropic VTI media and gave an analytic explanation for the results of Levin (1990) and Larner (1993). Deviations from the cosine-of-dip dependence [equation (2)] in VTI media turned out to be primarily controlled by the *difference* between the Thomsen's anisotropy parameters ϵ and δ. Alkhalifah and Tsvankin (1995) used the moveout equation of Tsvankin (1995a) to study NMO velocity for dipping reflectors as a function of the ray parameter – the quantity that is easier to obtain from reflection data than the dip angle. They showed that for P–waves in VTI media, the dip-dependent NMO velocity is described by just two parameters – the zero-dip NMO velocity $V_{\rm nmo}(0)$ [equation (1)] and the anisotropic parameter denoted as η:

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta}. \tag{3}$$

Alkhalifah and Tsvankin (1995) proved that in VTI media the parameters $V_{\rm nmo}(0)$ and η are sufficient to perform all time-related processing steps, such as NMO correction, dip-moveout (DMO) removal, prestack and poststack time migration. Furthermore, both parameters can be reliably recovered from P–wave surface data using NMO velocities and ray parameters measured for two different dips. Alkhalifah and Tsvankin (1995) also generalized the NMO equation of Tsvankin (1995a) to models consisting of a stack of anisotropic horizontal layers above a dipping reflector. A more detailed overview of the moveout analysis in VTI media can be found in Tsvankin (1995b).

The papers mentioned above are focused on the behavior of NMO velocity in transversely isotropic media with a vertical symmetry axis. The only non-VTI (azimuthally anisotropic) model examined by Tsvankin (1995a) is the one with an in-plane symmetry axis normal to the reflector; in this case, in agreement with the numerical results of Levin (1990), the isotropic cosine-of-dip dependence remains entirely valid. Another type of azimuthal anisotropy discussed in the literature is TI media with a horizontal symmetry axis (Thomsen, 1988; Sena, 1991). Reflection moveout for horizontal transverse isotropy is studied in detail in a companion paper (Tsvankin, I., Inversion of moveout velocities for horizontal transverse isotropy: this volume).

Here, the NMO equation of Tsvankin (1995a) is used to examine moveout velocity in transversely isotropic media with a tilted in-plane axis of symmetry. I demonstrate that the tilt of the symmetry axis may lead to the disappearance of specular zero-offset reflections from steep interfaces. For the symmetry axis tilted at an arbitrary angle, the NMO velocity is studied both analytically and numerically with the emphasis on implications for DMO processing and anisotropic inversion. The analysis shows that the zero-dip NMO velocity $V_{\rm nmo}(0)$ and parameter η are sufficient to describe P–wave NMO velocity in a limited range of tilt angles that includes near-vertical and near-horizontal orientations of the symmetry axis.

NMO VELOCITY AND PARAMETRIZATION FOR TI MEDIA

Normal moveout velocity of pure modes in symmetry planes of any anisotropic homogeneous medium is given by (Tsvankin, 1995a)

$$V_{\rm nmo}(\phi) = \frac{V(\phi)}{\cos\phi} \frac{\sqrt{1 + \frac{1}{V(\phi)} \frac{d^2V}{d\theta^2}}}{1 - \frac{\tan\phi}{V(\phi)} \frac{dV}{d\theta}}, \qquad (4)$$

where V is the phase velocity as a function of the phase angle θ with vertical, and ϕ is the dip angle of the reflector; the derivatives of phase velocity should be evaluated at the angle ϕ. Equation (4) is valid if the incidence plane coincides with the dip plane of the reflector. This assumption makes the problem two-dimensional by eliminating out-of-plane components of phase- and group-velocity vectors of the reflected waves.

If the medium is transversely isotropic (hexagonal), equation (4) can be applied in the vertical plane that contains the symmetry axis and, if the symmetry axis is horizontal, in the isotropy plane. For an arbitrary mutual orientation of the symmetry axis and the incidence plane in TI media, equation (4) can be used only under the assumption of weak azimuthal anisotropy. Here, our goal is to study the dependence of NMO velocity on the anisotropy parameters and reflector dip for TI media with a symmetry axis confined to the incidence plane. To cover all possible mutual orientations of the reflector normal and the symmetry axis, the tilt angle ν spans the range

$-90^\circ < \nu < 90^\circ$, while the dip ϕ is restricted to positive angles $0^\circ < \phi < 90^\circ$. Thus, positive values of ν mean that the axis is tilted towards the reflector, while $\nu < 0$ corresponds to the axis tilted away from the reflector.

The transversely isotropic model will be described by the Thomsen (1986) parameters defined in the coordinate system associated with the symmetry axis.

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}}, \tag{5}$$

$$V_{S0} \equiv \sqrt{\frac{c_{55}}{\rho}}, \tag{6}$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}, \tag{7}$$

$$\delta \equiv \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, \tag{8}$$

$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}}, \tag{9}$$

where ρ is the density, and the elastic constants c_{ij} correspond to the coordinate frame with the x_3 axis pointing in the symmetry direction. V_{P0} and V_{S0} are the P–wave and S–wave velocities, respectively, along the symmetry axis; the dimensionless anisotropic coefficients ϵ, δ, and γ go to zero in isotropic media. A detailed description of notation for transversely isotropic media is given by Tsvankin (1995b).

$P - SV$ propagation for vertical transverse isotropy is determined by four coefficients – V_{P0}, V_{S0}, ϵ, and δ. P–wave velocities and traveltimes in VTI media depend largely on V_{P0}, ϵ, and δ, while the influence of the shear-wave velocity V_{S0} is practically negligible (Tsvankin and Thomsen, 1994; Tsvankin, 1995b). Hence, for TI media with a tilted axis of symmetry, P–wave normal-moveout velocity is determined by V_{P0}, ϵ, δ, and the tilt angle ν.

EXISTENCE OF DIPPING EVENTS IN SYMMETRY PLANES OF ANISOTROPIC MEDIA

Before studying the dip-dependence of NMO velocity, we have to find out whether it is possible to record dipping events for the full range of dips in anisotropic models. The wavefront excited by a point source in homogeneous, isotropic media is spherical with the rays normal to the wavefront. This means that for any dip from 0 to 90 degrees there exists a section of the wavefront parallel to the reflector. This section generates the normal-incidence (zero-offset) reflection that will be recorded at the source location; likewise, there always exists a specular reflection for non-zero source-receiver offsets. For a vertical reflector that extends all the way to the surface, the raypaths of the reflected waves are horizontal, and the traveltimes in the CMP geometry are independent of offset, which implies that the normal-moveout velocity becomes infinite. For any other dip in the range from 0 to 90 degrees NMO velocity has a finite value. This is corroborated by the familiar cosine-of-dip dependence of NMO velocity in isotropic media [equation (2)] that can be obtained from equation (4) by setting $V(\theta) = const.$

Angular velocity variations in anisotropic media distort the shape of the wavefront and the angular distribution of the wavefront normals. However, for the special cases of vertical (VTI) and horizontal (HTI) orientations of the symmetry axis in TI media, we still have no "gaps" in the dip coverage of reflection data. Since the phase- and group-velocity vectors coincide with each other in the vertical and horizontal direction, the wavefront contains the full range of phase angles, despite all wavefront distortions at oblique angles of incidence (SV-wave cusps are not discussed here). Also, as for isotropic media, NMO velocity in VTI and HTI models becomes infinite for a vertical reflector. Indeed, the denominator in expression (4) for normal-moveout velocity can be represented as

$$D = \cos\phi \left(1 - \frac{\tan\phi}{V(\phi)}\frac{dV}{d\theta}\right) = \cos\phi - \frac{\sin\phi}{V(\phi)}\frac{dV}{d\theta}\,. \tag{10}$$

Since for both vertical and horizontal symmetry axes the derivative of phase velocity at $\phi = 90°$ goes to zero, the denominator D vanishes for a vertical reflector.

The situation becomes much more complicated if the symmetry axis is tilted (within the incidence plane) at an arbitrary angle. It is still true that the wavefront from a point source in a homogeneous anisotropic halfspace contains the full 90-degreee range of *group* (*ray*) angles in any quadrant, but not necessarily the full range of *phase* angles that determine the direction of the wavefront normal. As a result, for some anisotropic models there are no wavefront normals perpendicular to dipping reflectors within a certain range of dips and, therefore, no corresponding zero-offset and small-offset reflections. Of course, this argument is based on the geometrical-seismics approximation; even in the absence of the specular reflection, the seismogram at the source location will contain some reflected energy that does not travel along the geometrical raypath. However, unless the reflector is close to the surface, we cannot expect this non-specular reflection energy to be significant.

Figure 1 shows a typical P–wavefront for a symmetry axis tilted towards the reflector. Due to the increase in the phase and group velocity from the symmetry axis towards horizontal, the maximum angle between the wavefront normal and vertical in the lower right quadrant is limited to $\phi_{\max}$ – the value corresponding to the horizontal ray. There are no wavefront normals in the angular range $\phi_{\max} < \phi < 90°$ and, therefore, no specular reflections for dips larger than $\phi_{\max}$.

To find the maximum dip that would generate a specular reflection, let us express the group-velocity vector in the incidence $[x, z]$ plane through the phase velocity and phase angle (Berryman, 1979):

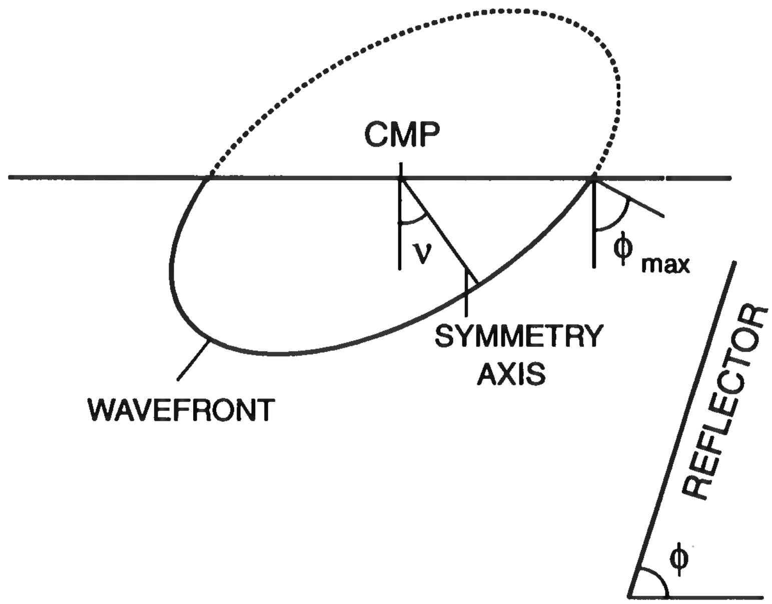

FIG. 1. The wavefront for a transversely isotropic medium with a symmetry axis tilted towards the reflector. The increase in phase and group velocity away from the symmetry axis in this model reduces the angular range of the wavefront normals. The maximum phase (wavefront) angle in the right lower quadrant is $\phi_{\max} < 90°$.

$$\vec{V}_{gr} = (V \sin\theta + \frac{dV}{d\theta}\cos\theta)\vec{x} + (V\cos\theta - \frac{dV}{d\theta}\sin\theta)\vec{z}\,. \tag{11}$$

The ray direction is horizontal if the vertical (z) component of group velocity is zero:

$$\cos\theta - \frac{\sin\theta}{V(\theta)}\frac{dV}{d\theta} = 0\,. \tag{12}$$

In the absence of cusps, equation (12) has two solutions (different by ± 180 degrees) corresponding to the phase angle for the horizontal ray. For typical wavefronts with a monotonic change in the phase angle from the horizontal to the vertical ray, equation (12) determines the maximum phase angles for the two lower quadrants. Thus, we can use equation (12) to find the largest dip $\phi_{\max}$ for which we can record a specular reflection in the medium with the phase-velocity function $V(\theta)$ (assuming that the reflector starts at the surface).

In terms of NMO equation (4), the absence of phase angles corresponding to a certain range of dips results in the zero and negative values of the denominator and, consequently, in infinite or non-existent NMO velocity. To understand the physical meaning of the denominator D [equation (10)], note that it becomes identical to the left-hand side of equation (12) if we substitute $\theta = \phi$. This implies that D is proportional to the vertical component of the group-velocity vector corresponding to the phase angle ϕ equal to the reflector dip. Therefore, D vanishes if the zero-offset ray that corresponds to the phase-velocity vector normal to the reflector is horizontal (as for dip ϕ_{max} in Figure 1). In other words, NMO velocity becomes infinite if the zero-offset reflected ray (along with non-zero-offset rays) travels along the horizontal (common-midpoint) line. For larger dips, as explained above, specular reflections do not exist at all.

An example of a wavefront that contains the full range of phase angles in the quadrant of interest is shown in Figure 2. Although the wavefront in Figure 2 has the same shape as in Figure 1, the symmetry axis is now tilted away from the reflector; we would have had the same situation in Figure 1 had the reflector been located to the *left* of the source. For the case displayed in Figure 2, the phase-velocity vector of the horizontal ray [ϕ_{max} is a solution of equation (12)] points upwards, and it is possible to record specular reflections not only from any dip in the 0-90 degree range, but also for dips between 90 degrees and ϕ_{max}, if the aperture is sufficient. We conclude that anisotropy can make it possible to record reflections from overhang structures even in the absence of velocity gradient.

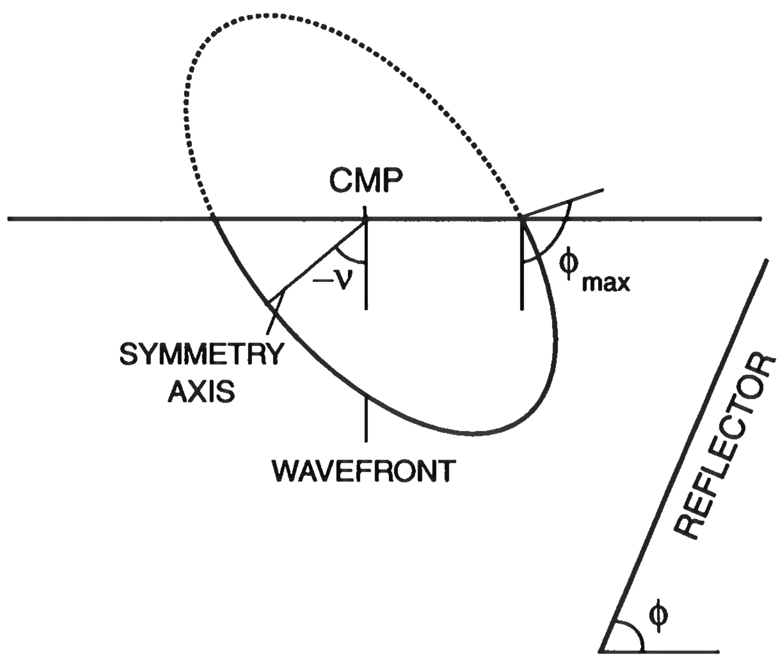

FIG. 2. The wavefront for the same TI medium as in Figure 1 but with a symmetry axis tilted away from the reflector. The direction of the wavefront normal in the lower right quadrant not only spans the full 0-90 degree range but also includes angles beyond 90 degrees. The angle ν between the symmetry axis and vertical is taken to be negative if the symmetry axis is tilted away from the reflector.

In the above discussion we assumed a homogeneous anisotropic medium. Some of our conclusions are still valid for the so-called factorized anisotropic media (e.g., Larner, 1993) with a vertical velocity gradient. In factorized models the ratios of the elastic constants are independent of spatial position and, consequently, the shape of the slowness surface and the relation between the phase and group velocity remains the same throughout the medium. As a result, for factorized TI media with the same shape of the slowness surface as for the model in Figure 1, the downgoing wavefield in the lower right quadrant still cannot contain phase (wavefront) angles exceeding $\phi_{\max}$. However, the upgoing wavefield formed due to the ray bending does include the missing phase angles in the range $\phi_{\max} < \theta < 90°$, as well as phase angles beyond 90 degrees. Therefore, in such a medium zero-offset reflections for dips $\phi_{\max} < \phi < 90°$ represent *turning* rays that exist only for appropriate spatial positions of the reflector with respect to the source.

NMO VELOCITY AS A FUNCTION OF DIP ANGLE

The results of Tsvankin (1995a) and Alkhalifah and Tsvankin (1995) indicate noticeable differences in the dependence of normal-moveout velocity for dipping reflectors on the dip angle and ray parameter. Although for the purposes of seismic processing it is more convenient to use the ray parameter as the argument, the dependence of NMO velocity on the dip angle still deserves a separate discussion. I begin with considering the special case of elliptical anisotropy, which is relatively easy to treat analytically. Then, NMO equation for general (non-elliptical) transverse isotropy with the axis of symmetry rotated at an arbitrary angle is simplified by means of the weak-anisotropy approximation. Comparison of the weak-anisotropy and exact numerical results elucidates the dip-dependence of NMO velocity for a range of TI models with a tilted symmetry axis.

Elliptical anisotropy

Transverse isotropy always means elliptical anisotropy for the SH–wave, while for $P-SV$–waves elliptical media represent just the subset of TI models that satisfy $\epsilon = \delta$. Although existing data indicate that elliptical anisotropy is not typical for TI formations, such as shales (Thomsen, 1986; Sayers, 1994), it is still instructive to examine this special case separately.

As shown in Appendix A, NMO velocity for P–waves in elliptical media with tilted elliptical axes is given by

$$V_{\rm nmo}(\phi) = \frac{V_{P0}}{\cos\phi}\sqrt{1+2\delta}\,\sqrt{1+2\delta\sin^2(\phi-\nu)}\left[1-2\delta\,\frac{\sin\nu\,\sin(\phi-\nu)}{\cos\phi}\right]^{-1}. \tag{13}$$

Equation (13) coincides with the normal-moveout equation by Uren et al. (1990) obtained using a different approach and presented in a different notation.

For a vertical symmetry axis ($\nu = 0$) equation (13) reduces to the expression discussed by Tsvankin (1995a):

$$V_{\rm nmo}(\phi) = \frac{V_{P0}\sqrt{1+2\delta}\,\sqrt{1+2\delta\sin^2\phi}}{\cos\phi}. \tag{14}$$

Substituting the P–wave phase velocity V_P, equation (14) can be rewritten as

$$\frac{V_{\rm nmo}(\phi)\cos\phi}{V_{\rm nmo}(0)} = \frac{V_P(\phi)}{V_{P0}}\,. \tag{15}$$

Equation (15) shows that if the elliptical axes are not tilted, the error in the cosine-of-dip dependence is determined directly by the phase-velocity variations.

If the reflector is horizontal ($\phi = 0$), equation (13) reduces to

$$V_{\rm nmo}(0) = V_{P0}\,\frac{\sqrt{1+2\delta}}{\sqrt{1+2\delta\,\sin^2\nu}} = V_{P0}\sqrt{1+\frac{2\delta\,\cos^2\nu}{1+2\delta\,\sin^2\nu}}\,. \tag{16}$$

Let us compare NMO velocity from equation (16) with the horizontal phase velocity

$$V_{hor} = V_{P0}\,\sqrt{1+2\delta\,\cos^2\nu}\,. \tag{17}$$

Equations (16) and (17) confirm the well-known fact that for a model with a vertical (or horizontal) elliptical axis ($\nu = 0$ or $\nu = 90°$), NMO velocity from a horizontal reflector is equal to the horizontal velocity. Although this is no longer strictly true for elliptical models with tilted axes, the difference between the two velocities is small since they coincide in the weak-anisotropy approximation. Indeed, for weak anisotropy ($\delta \ll 1$) we can drop the terms quadratic in δ, and

$$V_{\rm nmo}(0) = V_{P0}\,(1+\delta\,\cos^2\nu) = V_{hor}\,. \tag{18}$$

Thus, in elliptical media NMO velocity from a horizontal reflector remains close to the horizontal phase velocity, whether the elliptical axes are tilted or not.

Equation (13) for an arbitrary tilt of the symmetry axis can also be simplified under the assumption of weak anisotropy ($\delta \ll 1$):

$$V_{\rm nmo}(\phi) = \frac{V_{P0}}{\cos\phi}\left[1+\delta+\delta\,\sin^2(\phi-\nu)+2\delta\,\frac{\sin\nu\,\sin(\phi-\nu)}{\cos\phi}\right]\,. \tag{19}$$

Figure 3 shows that the tilt has a significant impact on the dip-dependence of NMO velocity. To separate the influence of the anisotropy, the curves in Figure 3 are normalized by the isotropic equation (2). When one of the elliptical axes is vertical ($\nu = 0$ or $\nu = 90°$), deviations from the cosine-of-dip relationship are entirely controlled by the phase-velocity variations [equation (15)]. Since the difference between the vertical and horizontal velocity for the model in Figure 3 is close to 20 percent, the anisotropy-induced error in the cosine-of-dip relationship for both $\nu = 0$ and $\nu = 90°$ is relatively small. In contrast, for the symmetry axis tilted at 45 and –45 degrees the anisotropic signature is much more pronounced. Note that at $\nu = 45°$ the NMO

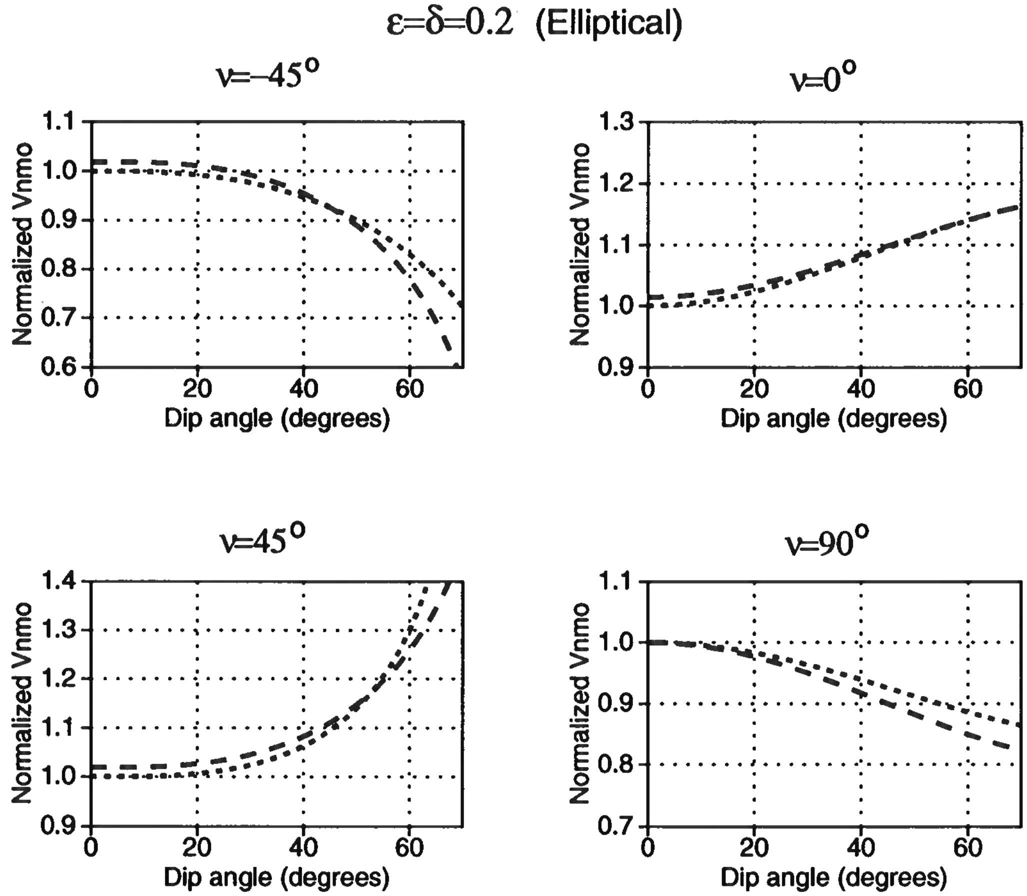

FIG. 3. The dip-dependence of P–wave moveout velocity for elliptical anisotropy (ϵ=δ=0.2). The dotted curve is the exact NMO velocity calculated from equation (13); the dashed curve is the weak-anisotropy approximation from equation (19). Both curves are normalized by $V_{\rm nmo}(0)/\cos\phi$, where $V_{\rm nmo}(0)$ is the exact NMO velocity for a horizontal reflector.

velocity goes to infinity at dips below 90 degrees since the section of the wavefront propagating towards the reflector does not contain the full range of the phase angles; this was discussed in detail in the previous section.

In general (non-elliptical) VTI media with $\epsilon \neq \delta$, equations for elliptical anisotropy are valid only for the SH–wave. Our moveout analysis for the elliptical P–wave remains entirely valid for the SH–wave if we replace V_{P0} with V_{S0} and δ with γ. For instance, the exact NMO equation (13) for the SH–wave takes the form

$$V_{\rm nmo}(\phi)\,[SH] = \frac{V_{S0}}{\cos\phi}\sqrt{1+2\gamma}\,\sqrt{1+2\gamma\sin^2(\phi-\nu)}\,\left[1-2\gamma\,\frac{\sin\nu\,\sin(\phi-\nu)}{\cos\phi}\right]^{-1}. \tag{20}$$

For SV–waves, the elliptical condition ($\epsilon = \delta$) implies that the phase and group velocity are independent of angle (however, SV–wave amplitudes can still be distorted by the anisotropy). Therefore, the NMO velocity for the SV–wave in elliptical media is described by the familiar isotropic cosine-of-dip dependence (2).

Weak transverse isotropy

The exact normal-moveout equation (4) is too complex (especially, when the symmetry axis is tilted) to separate the contributions of the anisotropy parameters to the NMO velocity. For the sake of qualitative moveout analysis, it is convenient to assume that the anisotropic coefficients are small and apply the weak-anisotropy approximation. The P–wave NMO velocity, linearized in the parameters ϵ and δ, is derived in Appendix B:

$$\begin{aligned} V_{\text{nmo}}(\phi)\cos\phi &= V_{P0}\{1+\delta+\delta\sin^2\bar{\phi}+3(\epsilon-\delta)\sin^2\bar{\phi}\,(2-\sin^2\bar{\phi}) \\ &+\frac{2\sin\nu\sin\bar{\phi}}{\cos\phi}\,[\,\delta+2(\epsilon-\delta)\sin^2\bar{\phi}]\}\,, \end{aligned} \tag{21}$$

where $\bar{\phi} = \phi - \nu$.

For $\nu = 0$, equation (21) reduces to the expression given by Tsvankin (1995a) for vertical transverse isotropy:

$$V_{\text{nmo}}(\phi)\ \cos(\phi) = V_{P0}\,[1+\delta+\delta\sin^2\phi+3(\epsilon-\delta)\ \sin^2\phi\,(2-\sin^2\phi)]\,. \tag{22}$$

The terms on the first line of equation (21) have the same form as VTI equation (22) but with the dip angle ϕ replaced by the difference $\phi - \nu$. The last term of equation (21) is a pure contribution of the tilt of the symmetry axis; note that it goes to zero not only for $\nu = 0$, but also for the dipping reflector normal to the symmetry axis ($\bar{\phi} = 0$). In the latter case, as demonstrated by Tsvankin (1995a), the dip-dependence of NMO velocity is described by the isotropic cosine-of-dip equation.

For another special case of a horizontal symmetry axis ($\nu = 90°$), NMO velocity (21) reduces to the following equation discussed in detail in the companion paper (this volume):

$$V_{\text{nmo}}(\phi)\ \cos(\phi) = V_{\text{nmo}}(0)\,[1-\delta\sin^2\phi+3(\epsilon-\delta)\ \sin^2\phi\,(\frac{4}{3}-\sin^2\phi)]\,. \tag{23}$$

For a horizontal reflector ($\phi = 0$) and an arbitrary tilt angle, the P–wave NMO velocity becomes

$$V_{\text{nmo}}(0) = V_{P0}\,[1+\delta-\delta\sin^2\nu+(\epsilon-\delta)\ \sin^2\nu\,(7\cos^2\nu-1)]\,. \tag{24}$$

Equation (24) shows that the contribution of tilt to the expression for the zero-dip NMO velocity is mostly controlled by the difference $\epsilon - \delta$. The ratio of the cosine-of-dip corrected NMO velocity [equation (21)] and the zero-dip value $V_{\rm nmo}(0)$ determines the error in the dip-dependence of NMO velocity caused by the anisotropy. The presence of a separate "tilt" term in equation (21), as well as the form of the argument $(\phi - \nu)$ in other terms, are indicative of a strong influence of tilt on the dip-dependence of the P–wave NMO velocity. Since the "tilt" term depends on ϵ and δ separately, we can no longer expect that deviations from the cosine-of-dip dependence will be controlled just by the difference between ϵ and δ, as was the case in VTI media. This conclusion is supported by the numerical results below.

To study the SV–wave NMO velocity, it is convenient to introduce parameter σ (Tsvankin and Thomsen, 1994):

$$\sigma \equiv \left(\frac{V_{P0}}{V_{S0}}\right)^2 (\epsilon - \delta)\,. \tag{25}$$

Then the weak-anisotropy approximation for SV–waves can be obtained from P–wave equation (21) by making the following substitutions (Tsvankin, 1995a): $V_{S0} = V_{S0}$, $\delta = \sigma$, and $\epsilon = 0$:

$$\begin{aligned} V_{\rm nmo}(\phi)\,\cos\phi &= V_{S0}\,\{1 + \sigma + \sigma\sin^2\bar{\phi} - 3\sigma\sin^2\bar{\phi}\,(2 - \sin^2\bar{\phi}) \\ &\quad + 2\sigma\,\frac{\sin\nu\sin\bar{\phi}\cos 2\bar{\phi}}{\cos\phi}\}\,. \end{aligned} \tag{26}$$

As discussed above, for the SH–wave anisotropy is elliptical, and the NMO velocity is given exactly by equation (20).

Numerical results for general transverse isotropy

Figures 4 and 5 show the comparison between the exact P–wave NMO velocity (4) and the weak-anisotropy approximation (21) at different tilt angles for two media with small and moderate values of the anisotropic parameters ϵ and δ. For both models, as expected, the accuracy of the weak-anisotropy approximation is quite satisfactory at all tilt angles.

It is interesting that for the model parameters used in Figure 5 the NMO velocity is identical for $\nu = \pm 45$, and also for $\nu = 0$ and $\nu = 90°$. This is explained by the vanishing ϵ for this model; if $\epsilon = 0$, the P and SV phase velocities are symmetric with respect to the 45-degree angle with the symmetry axis. In this case the phase velocity and, consequently, the NMO velocity do not change when we rotate the symmetry axis by 90 degrees.

Figures 4 and 5 also demonstrate that the behavior of P–wave NMO velocity is strongly dependent on the sign of $\epsilon - \delta$. For vertical transverse isotropy, the difference

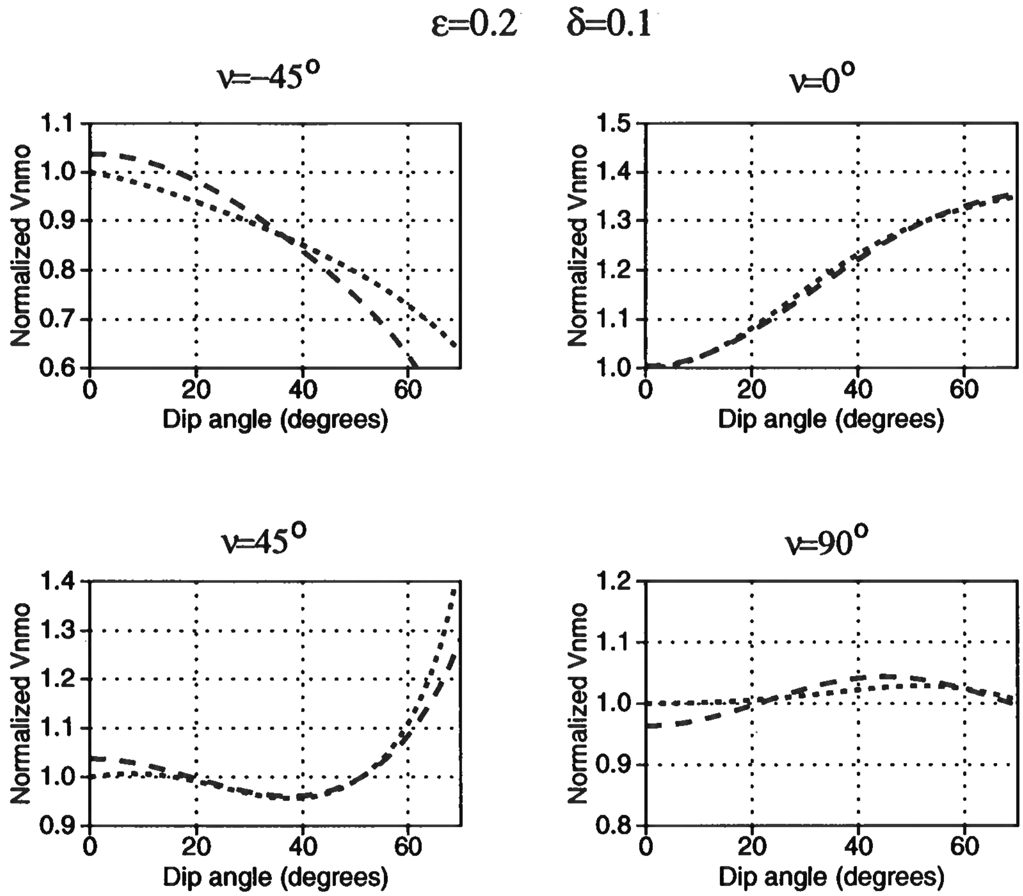

FIG. 4. Cosine-of-dip corrected P–wave normal-moveout velocity for ϵ=0.2 and δ=0.1. The dotted curve is the exact NMO velocity calculated from equation (4); the dashed curve is the weak-anisotropy approximation from equation (21).

$\epsilon - \delta$ is believed to be predominantly positive (Sayers, 1994; Tsvankin and Thomsen, 1994). The same is true for TI media with a horizontal symmetry axis due to a system of vertical parallel cracks (see the companion paper, this volume). Thus, we can assume that $\epsilon - \delta$ is typically positive for intermediate tilts as well. Note that $\epsilon - \delta > 0$ for transverse isotropy due to the interbedding of thin isotropic layers (Berryman, 1979), whether the layers are horizontal or tilted.

Figure 6 illustrates the influence of tilt on the P–wave NMO velocity for three models with a typical positive $\epsilon - \delta$. As predicted by the weak-anisotropy approximation, the tilt causes profound changes in the character of the NMO curves. If the symmetry axis is tilted towards the reflector ($\nu > 0$), the cosine-of-dip corrected NMO velocity remains almost flat in the range $0 < \phi < \nu$ and then increases for steep dips. A "negative " tilt $\nu < 0$ that corresponds to the symmetry axis tilted away from the reflector reverses the NMO curve and makes the cosine-of-dip corrected NMO velocity decrease with dip. This type of behavior for a vertical symmetry axis

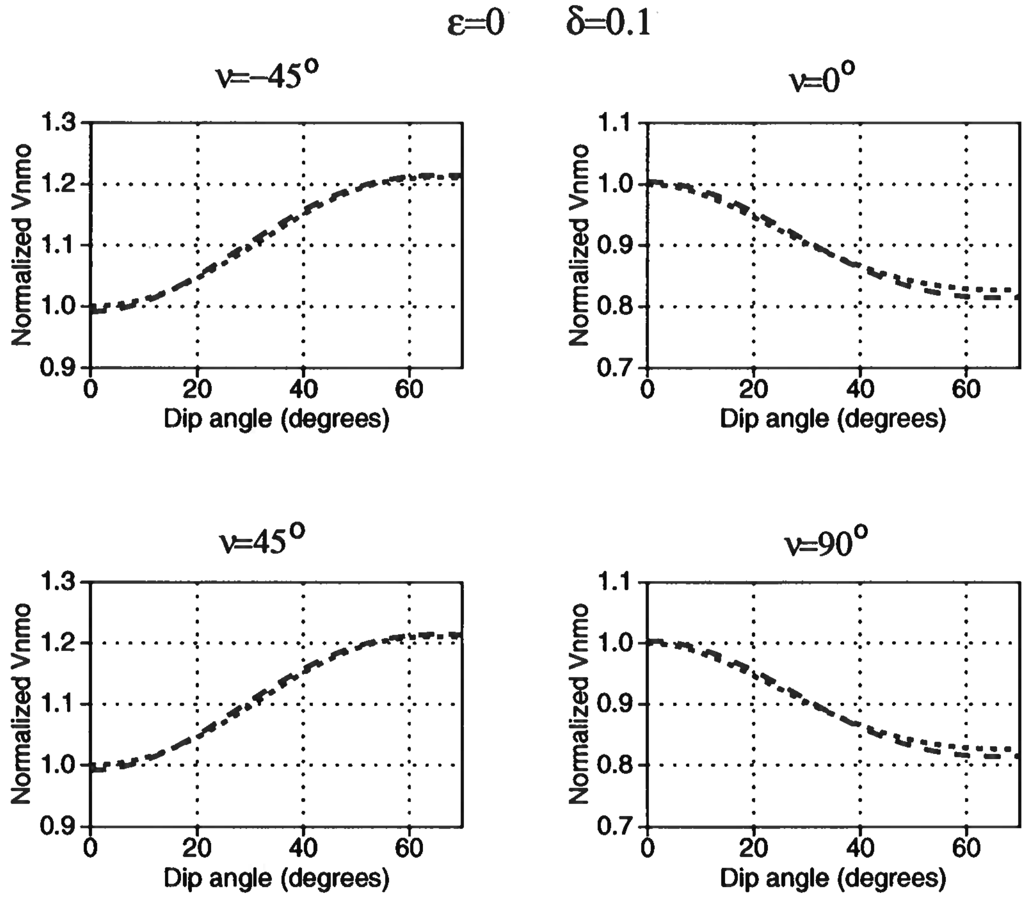

FIG. 5. Cosine-of-dip corrected P–wave normal-moveout velocity for ϵ=0 and δ=0.1. The dotted curve is the exact NMO velocity [equation (4)]; the dashed curve is the weak-anisotropy approximation (21).

can be caused by a negative value of $\epsilon - \delta$. Also, when the symmetry axis is tilted, the dip-dependence of the P–wave NMO velocity is no longer fully controlled by the difference $\epsilon - \delta$; the influence of the individual values of ϵ and δ is especially significant for a tilt of 45 degrees.

Deviations of the P–wave NMO curves from those for vertical transverse isotropy become pronounced at relatively mild tilts (Figure 7). Only for tilt angles up to $\pm(10-15)$ degrees the dip-dependence of NMO velocity is close to that for VTI media. A tilt of ± 20 degrees is sufficient to cause substantial changes in the NMO curves, with negative ν leading to a decrease in $V_{\rm nmo}$ with angle at steep dips. For $\nu = 20°$ the NMO velocity increases sharply at $\phi > 50°$ as the dip approaches the maximum phase angle $\phi_{\rm max}$ contained in the wavefront for this model (see the discussion above).

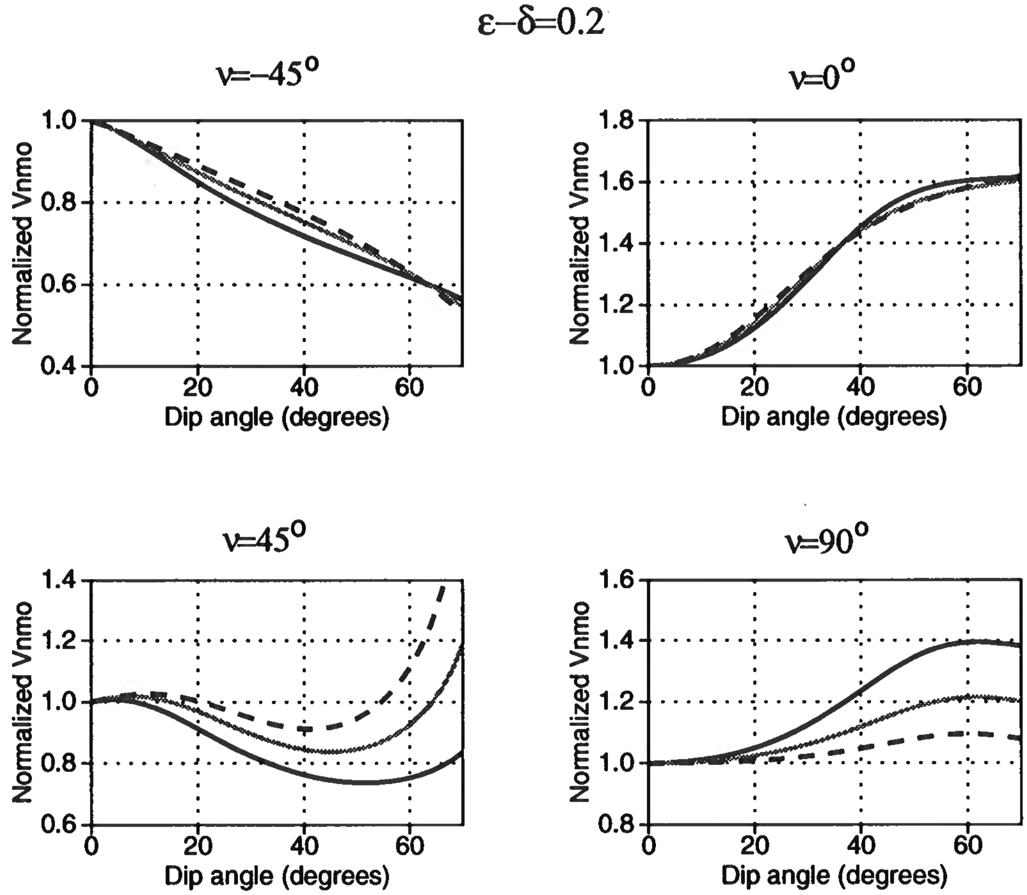

FIG. 6. The dip-dependence of the exact cosine-of-dip corrected P–wave NMO velocity for models with $\epsilon - \delta$=0.2: ϵ=0.1, δ=–0.1 (black); ϵ=0.2, δ=0 (gray); ϵ=0.3, δ=0.1 (dashed).

NMO VELOCITY AS A FUNCTION OF RAY PARAMETER

Since reflection data do not carry explicit information about the dip angle, for application in seismic processing equation (4) should be rewritten as a function of the ray parameter $p(\phi)$ corresponding to the zero-offset reflection:

$$p(\phi) = \frac{1}{2}\frac{dt_0}{dx_0} = \frac{\sin\phi}{V(\phi)}, \tag{27}$$

where $t_0(x_0)$ is the two-way traveltime on the zero-offset (or stacked) section, and x_0 is the midpoint position. The dip angle in equation (4) can be replaced by the ray parameter in a straightforward fashion using phase-velocity equations for transverse isotropy.

Let us begin by considering the simplest elliptically anisotropic model. In the previous section it was demonstrated that the tilt of the elliptical axes has a significant

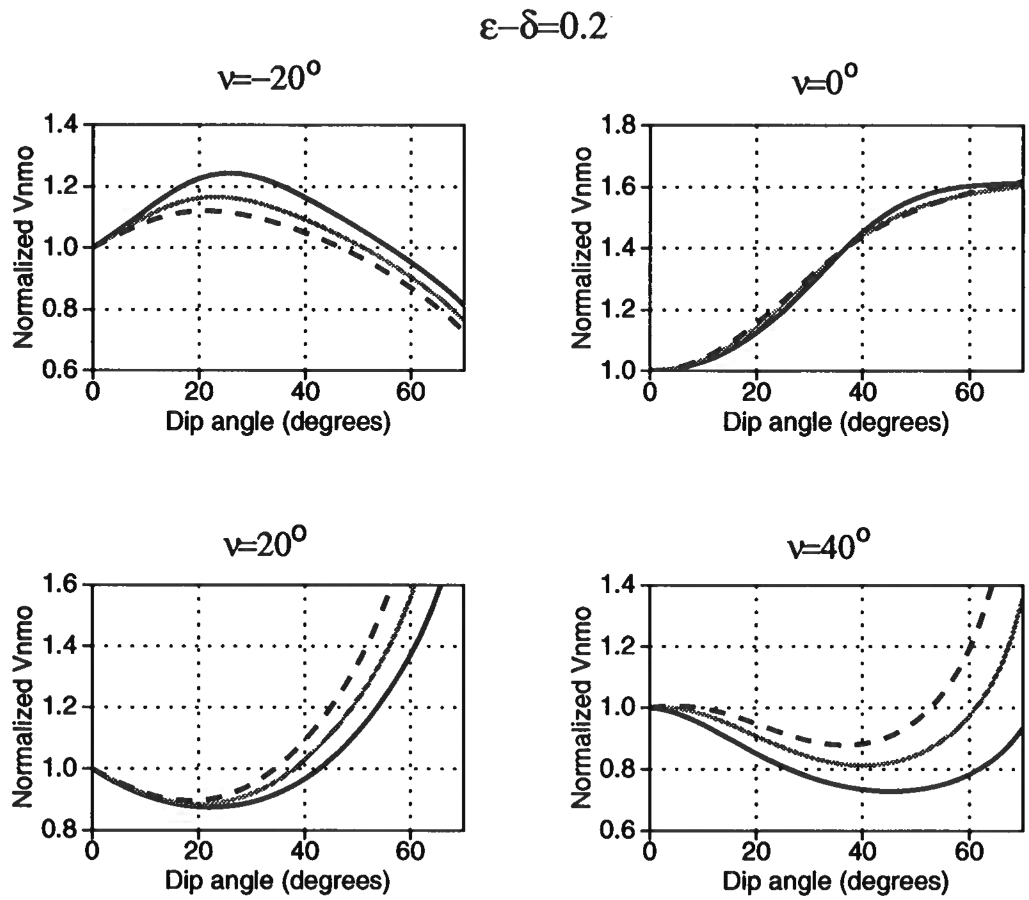

FIG. 7. The dip-dependence of the exact cosine-of-dip corrected P–wave NMO velocity at mild tilts. The models are the same as in Figure 6: ϵ=0.1, δ=−0.1 (black); ϵ=0.2, δ=0 (gray); ϵ=0.3, δ=0.1 (dashed).

influence on the dip-dependence of normal-moveout velocity. However, if represented through the ray parameter, the NMO velocity for elliptical anisotropy takes exactly the same form as in isotropic or VTI media (Appendix A):

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1 - p^2 V_{\rm nmo}^2(0)}} \,. \qquad (28)$$

It is somewhat surprising that the dependence of NMO velocity on the ray parameter in elliptically anisotropic models remains exactly the same as in isotropic media irrespective of the tilt of the elliptical axes. The contribution of anisotropy (including the tilt) in equation (28) is hidden in the values of the zero-dip NMO velocity $V_{\rm nmo}(0)$ [equation (16)] and the ray parameter p. Since the moveout for elliptical anisotropy with any orientation of the axes is purely hyperbolic (Uren et al., 1990), all isotropic time-related processing methods (NMO, DMO, time migration) are entirely valid in elliptical media. These conclusions, as well as equation (28), are fully applicable to the SH–wave because SH–wave anisotropy in transversely isotropic media is always elliptical.

Equation (28) also implies that deviations from the isotropic $V_{\rm nmo}(p)$ dependence for *any tilt* should be primarily controlled by the difference $\epsilon - \delta$ that quantifies the degree of "nonellipticity" of TI media. It is clear, for instance, that the anisotropic term in the weak-anisotropy approximation for $V_{\rm nmo}(p)$ should contain ϵ and δ only in the combination $\epsilon - \delta$. Otherwise, this term would not vanish for elliptical anisotropy that corresponds to $\epsilon = \delta$.

Next, I consider the parameters that control the dependence of the P-wave NMO velocity on p. The P-wave phase velocity can be represented as a product of the velocity in the symmetry direction (V_{P0}) and an anisotropic term dependent on the tilt ν and the anisotropic parameters ϵ and δ. Then, according to equation (27), the phase angle ϕ can be written as

$$\phi = f(pV_{P0},\, \epsilon,\, \delta,\, \nu)\,,$$

and the list of the parameters of $V_{\rm nmo}$ from equation (4) can be represented as

$$V_{\rm nmo} = V_{P0}\; f_1(\phi,\, \epsilon,\, \delta,\, \nu) = V_{P0}\; f_2(pV_{P0},\, \epsilon,\, \delta,\, \nu)\,.$$

Therefore, the NMO velocity divided by the zero-dip value can be represented by

$$\frac{V_{\rm nmo}(p)}{V_{\rm nmo}(0)} = f_3(pV_{P0},\, \epsilon,\, \delta,\, \nu) = f_4(pV_{\rm nmo}(0),\, \epsilon,\, \delta,\, \nu)\,. \qquad (29)$$

Equation (29) shows that the normalized NMO velocity is independent of V_{P0}, while the contributions of the ray parameter and the zero-dip NMO velocity are absorbed by the term $pV_{\rm nmo}(0)$. Essentially, changes in $V_{\rm nmo}(0)$ (for fixed anisotropic coefficients and tilt) just stretch or squeeze the NMO velocity curve expressed through the ray parameter. Therefore, in the calculations below I use $pV_{\rm nmo}(0)$ as the argument and study the influence of the parameters ϵ, δ and ν on the NMO velocity.

Since the function $V_{\rm nmo}(p)$ is of primary importance in DMO processing, in the following I will call the dependence of the P–wave NMO velocity on the ray parameter "the dip-moveout (DMO) signature." Hereafter, the NMO velocity is calculated as a function of $pV_{\rm nmo}(0)$ from equation (4) and is normalized by the isotropic expression (28) to demonstrate the distortions in the DMO signature caused by the anisotropy.

Figure 8 proves that the influence of the shear-wave velocity V_{S0} on the P–wave normal moveout, ignored in equation (29), is indeed small. Some difference between the NMO velocities for the two models that span a wide range of V_{P0}/V_{S0} ratios is noticeable only for a horizontal symmetry axis ($\nu = 90°$). However, this separation

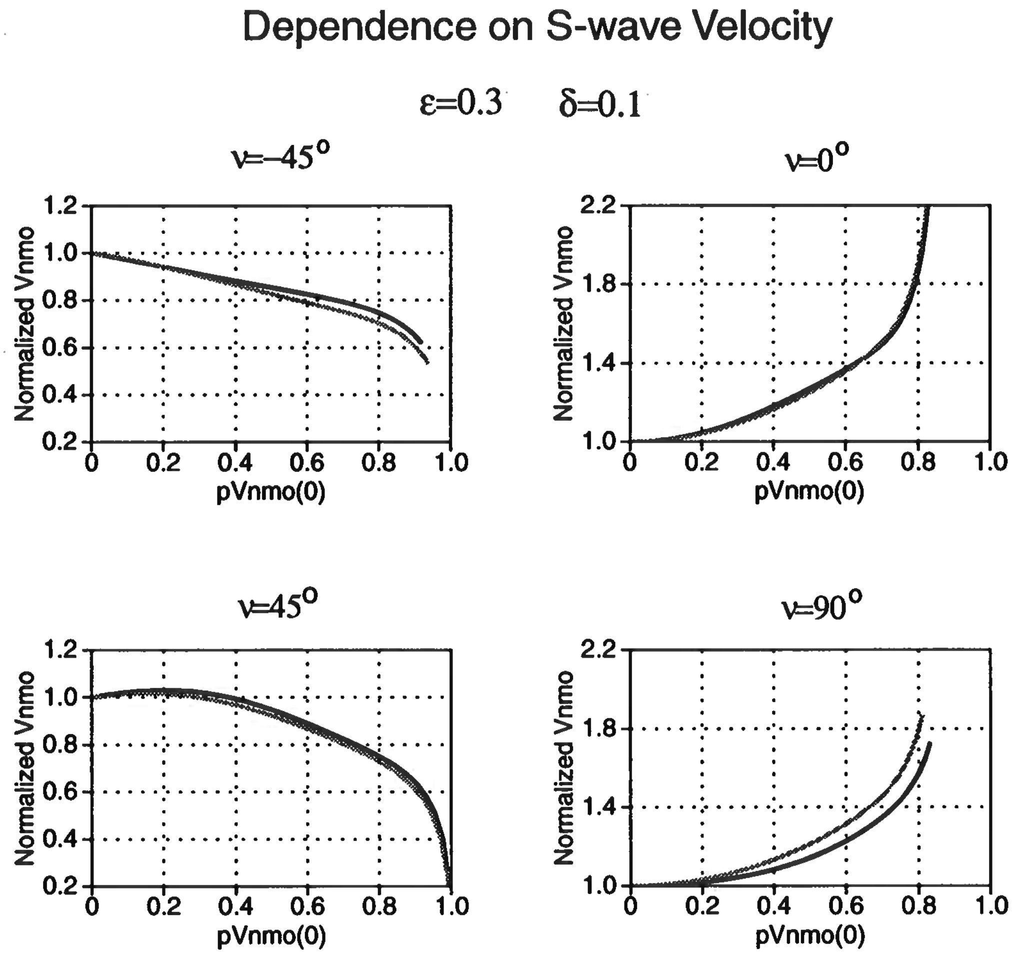

FIG. 8. The influence of V_{S0} (the shear-wave velocity in the symmetry direction) on the P–wave normal-moveout velocity. NMO velocity is computed from equation (4) and is normalized by the isotropic expression (28). The curves correspond to models with V_{P0}/V_{S0}=1.5 (black) and V_{P0}/V_{S0}=2.5 (gray); for both media, ϵ=0.3, δ=0.1.

between the curves is mostly caused by a small influence of V_{S0} on the zero-dip NMO velocity $V_{\rm nmo}(0)$. The difference in $V_{\rm nmo}(0)$ leads to a small horizontal shift between the curves which gets amplified when the NMO velocity is divided by the isotropic equation (28).

The normalization by equation (28), equivalent to the replacement of the true dip with an "apparent" dip angle (Larner, 1993; Tsvankin, 1995a), leads to substantial changes in the character of the NMO curve. Comparison of Figure 9 with Figure 6 shows that these changes are especially pronounced for positive ν that correspond to the symmetry axis tilted towards the reflector. For instance, at a tilt of 45 degrees and the 45-degree dip, the value of $pV_{\rm nmo}(0)$ that represents the sine of the apparent dip angle is close to 0.9, which is far different from $\sin\phi$ = 0.71. With a further increase in dip, $pV_{\rm nmo}(0)$ approaches unity (both for ν = 45° and ν = −45°), and the

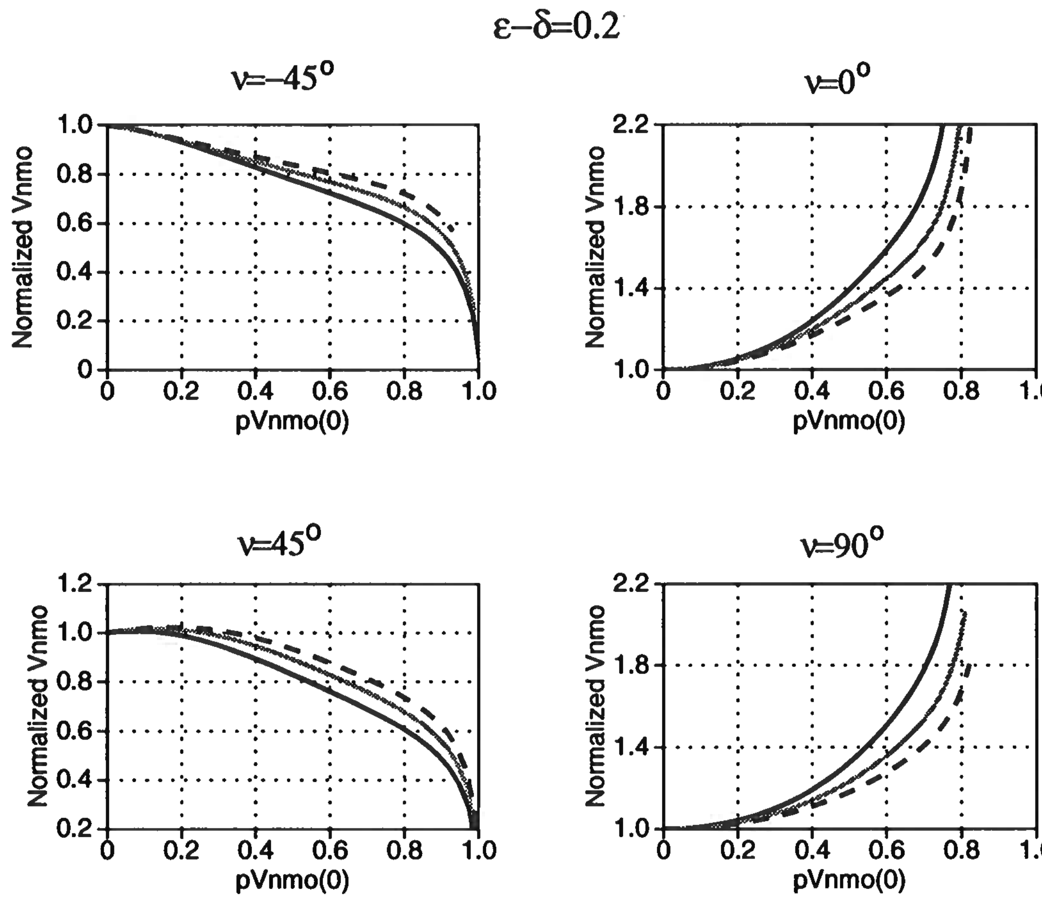

FIG. 9. Normalized P–wave NMO velocity as a function of the ray parameter. The models correspond to $\epsilon - \delta$=0.2: ϵ=0.1, δ=–0.1 (black); ϵ=0.2, δ=0 (gray); ϵ=0.3, δ=0.1 (dashed).

isotropic expression (28) goes to infinity at dips well below 90 degrees, leading to the small values of the normalized NMO velocity (Figure 9).

Note that the influence of the anisotropy on the P–wave NMO velocity is almost the same for vertical ($\nu = 0$) and horizontal ($\nu = 90°$) orientations of the symmetry axis, if the ray parameter is used as the argument. Another important feature of the NMO curves in Figure 9 is the reversal of the P–wave signature at intermediate tilt angles ($\nu = \pm 45°$). Even a tilt of $\pm$ 20 degrees is sufficient to eliminate a sharp increase in the normalized NMO velocity typical for vertical transverse isotropy (Figure 10). This conclusion holds for other transversely isotropic models with typical small and moderate values of ϵ and δ. Therefore, mild tilts of the symmetry axis in both directions reduce the influence of the anisotropy on $V_{\text{nmo}}(p)$. For a tilt of $\pm$ 20 degrees, the DMO signature is almost isotropic for a wide range of reflector dips.

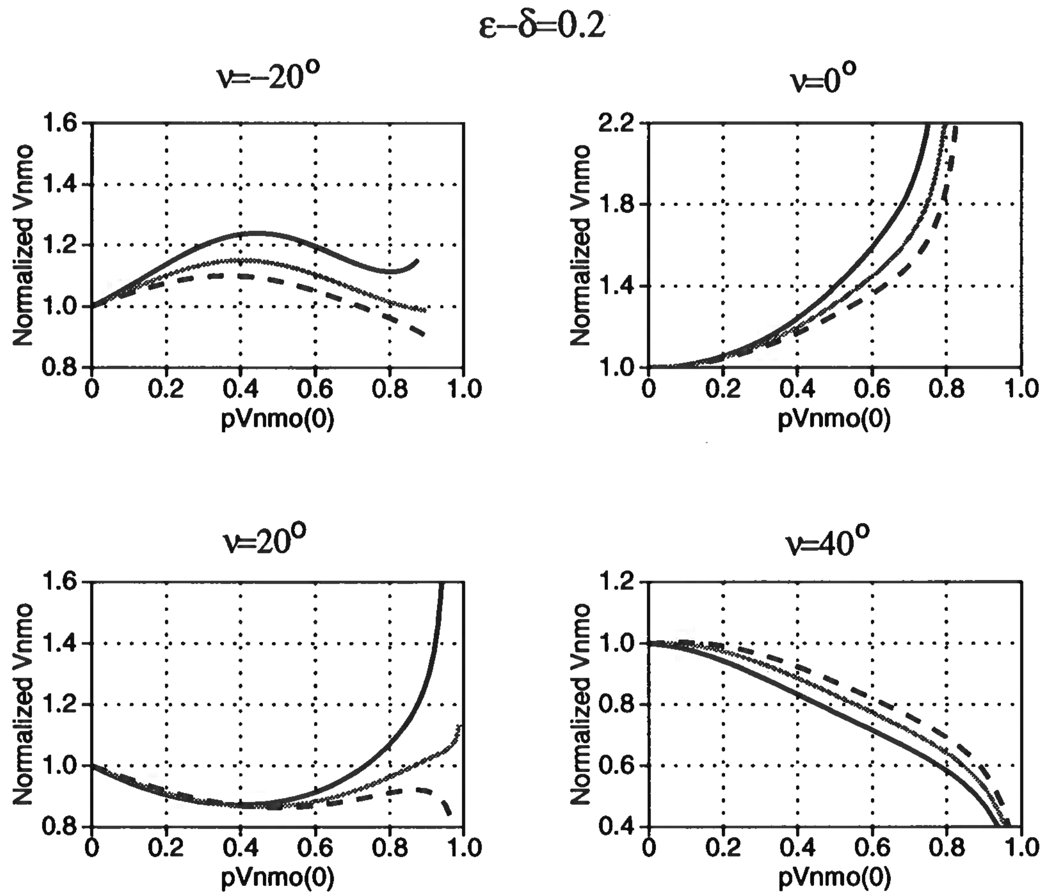

FIG. 10. Normalized P–wave NMO velocity for relatively mild tilt angles. The dips range between 0 and 70 degrees. The models correspond to $\epsilon-\delta$=0.2: ϵ=0.1, δ=–0.1 (black); ϵ=0.2, δ=0 (gray); ϵ=0.3, δ=0.1 (dashed).

PARAMETER η FOR TILTED AXIS OF SYMMETRY

For vertical transverse isotropy, the dependence of the P–wave NMO velocity on the ray parameter is controlled by a combination of the anisotropic coefficients that Alkhalifah and Tsvankin (1995) denoted as η [equation (3)]. This result makes it possible to reduce the number of parameters in time-related P–wave processing to just two ($V_{\rm nmo}(0)$ and η), which greatly facilitates the practical implementation of processing and inversion algorithms in VTI media.

If the symmetry axis is tilted, the function $V_{\rm nmo}(p)$ (for the P–wave) depends on $pV_{\rm nmo}(0)$, ϵ, δ, and the tilt ν [equation (29)]. The question to be addressed next is whether the influence of ϵ and δ on the NMO velocity is still absorbed by the parameter η. Note that for small δ the parameter η is close to the difference $\epsilon-\delta$, so the plots for constant $\epsilon-\delta$ given in the previous section contain a partial answer to this question. For instance, in Figure 9 we do see some separation between the curves

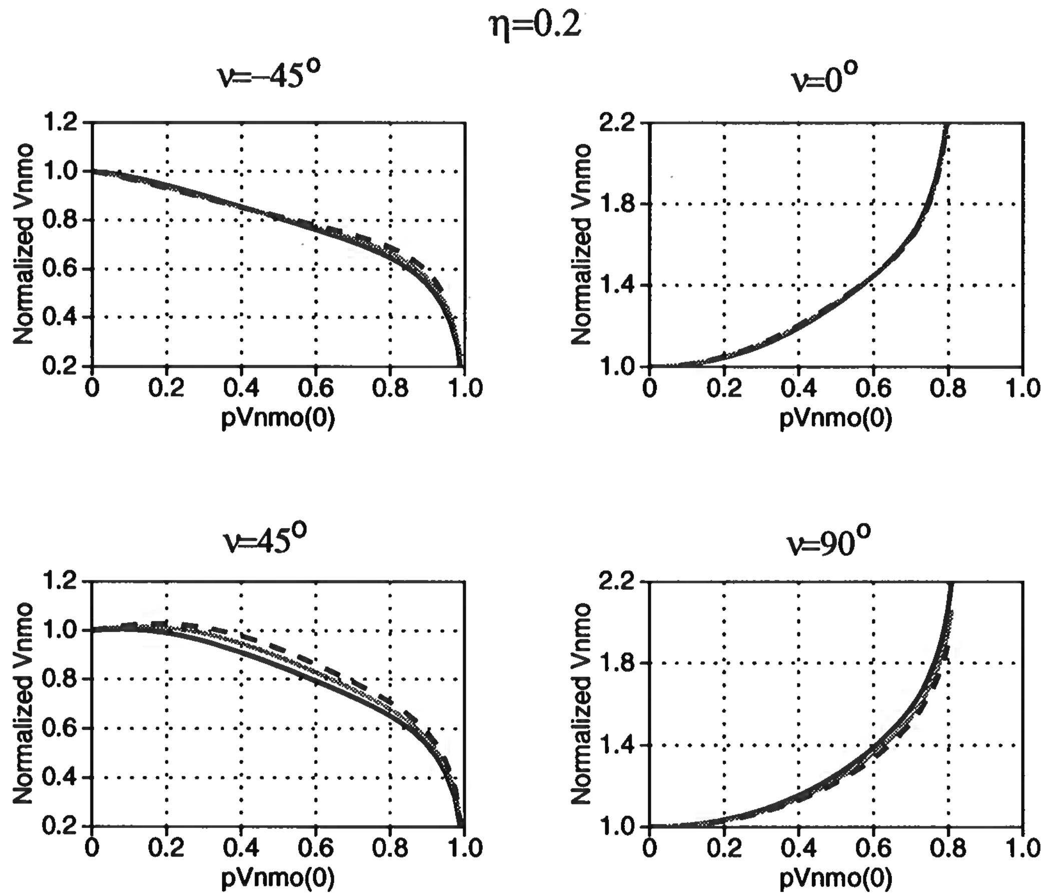

FIG. 11. Normalized P–wave NMO velocity for models with the same $\eta = 0.2$: $\epsilon = 0.1$, $\delta = -0.071$ (black); $\epsilon = 0.2$, $\delta = 0$ (gray); $\epsilon = 0.3$, $\delta = 0.071$ (dashed).

with the same $\epsilon - \delta$, but the general behavior of the NMO velocity for fixed $\epsilon - \delta$ is similar. Figure 11 reproduces the result from Figure 9, but this time the values of δ were adjusted to make $\eta = 0.2$ for all three models. Evidently, the curves moved much closer to each other, although a perfect coincidence of the NMO velocities for the full range of dips was achieved only for VTI media ($\nu = 0$). Since a detailed discussion of the signature for horizontal transverse isotropy is given in the companion paper (this volume), in the following I focus on intermediate tilt angles.

Figure 12 shows the normal-moveout velocity at the same tilt angles as in Figure 11 but for variable values of η. While for vertical and horizontal orientations of the symmetry axis η has a pronounced influence on the NMO-velocity curves, at intermediate tilt angles the resolution in η is considerably lower. For $\nu = \pm 45°$ the difference between the curves corresponding to $\eta = 0.2$ and $\eta = 0.3$ is relatively small (Figure 12).

As discussed above, for weak anisotropy deviations of the P–wave NMO velocity

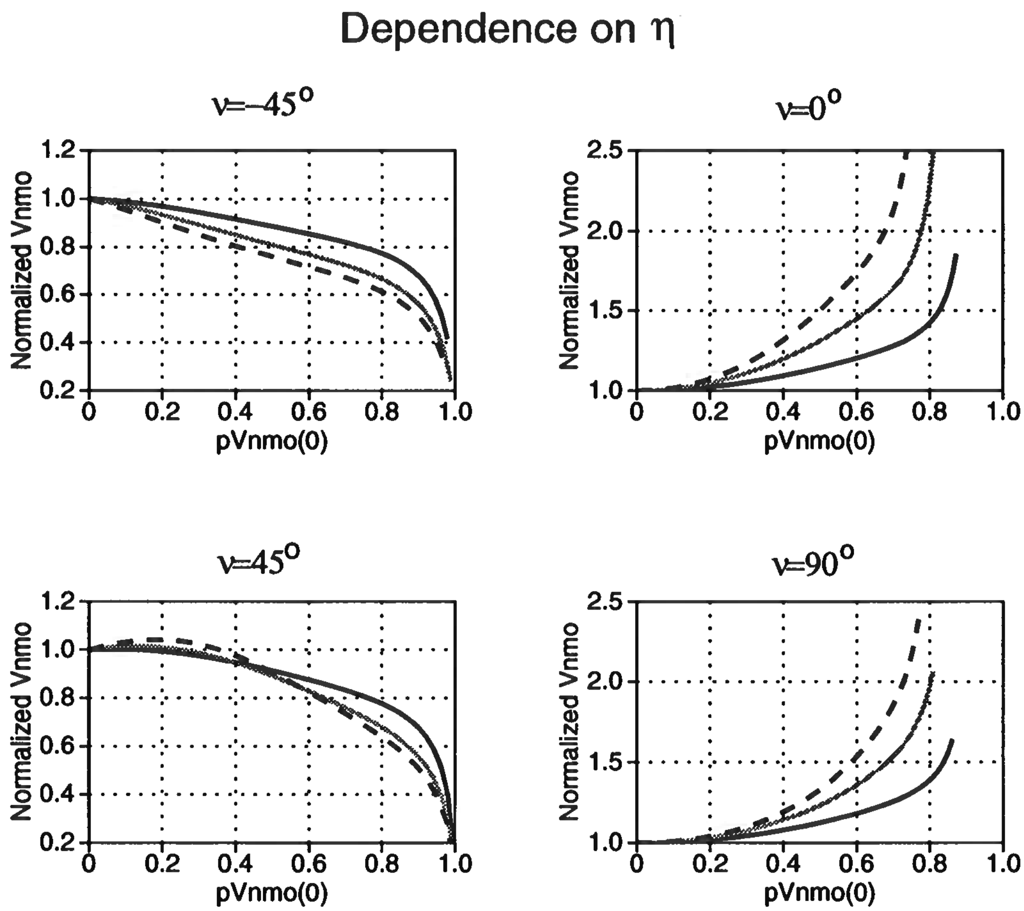

FIG. 12. Normalized P–wave NMO velocity for models with different η: η=0.1 (black); η=0.2 (gray); η=0.3 (dashed). For all models, $\delta = 0$ and $\epsilon = \eta$.

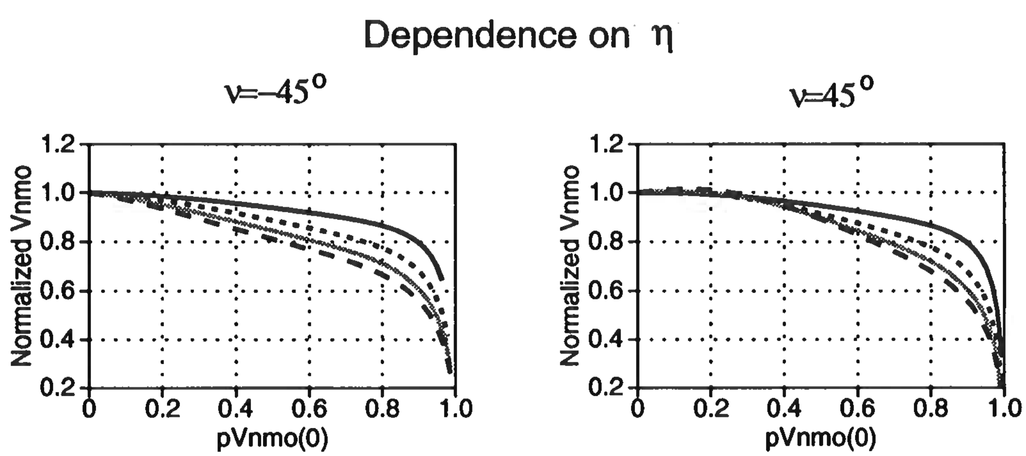

FIG. 13. Sensitivity of P–wave NMO velocity to η at intermediate tilt angles. The curves correspond to η=0.05 (black); η=0.1 (dotted); η=0.15 (gray); η=0.2 (dashed). For all models, $\delta = 0$ and $\epsilon = \eta$.

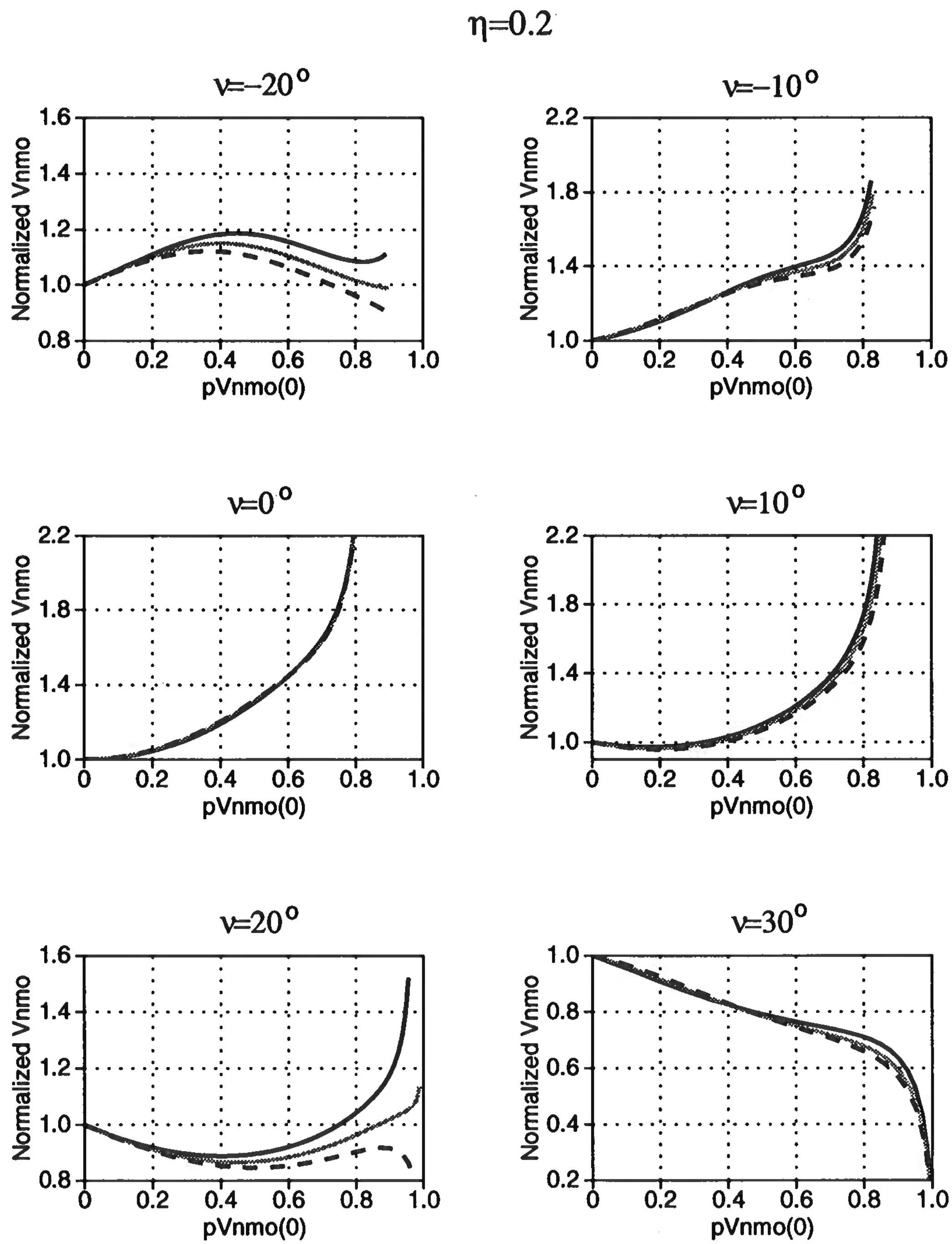

FIG. 14. Normalized P–wave NMO velocity for mild tilt angles and η=0.2: $\epsilon = 0.1$, $\delta = -0.071$ (solid); $\epsilon = 0.2$, $\delta = 0$ (gray); $\epsilon = 0.3$, $\delta = 0.071$ (dashed).

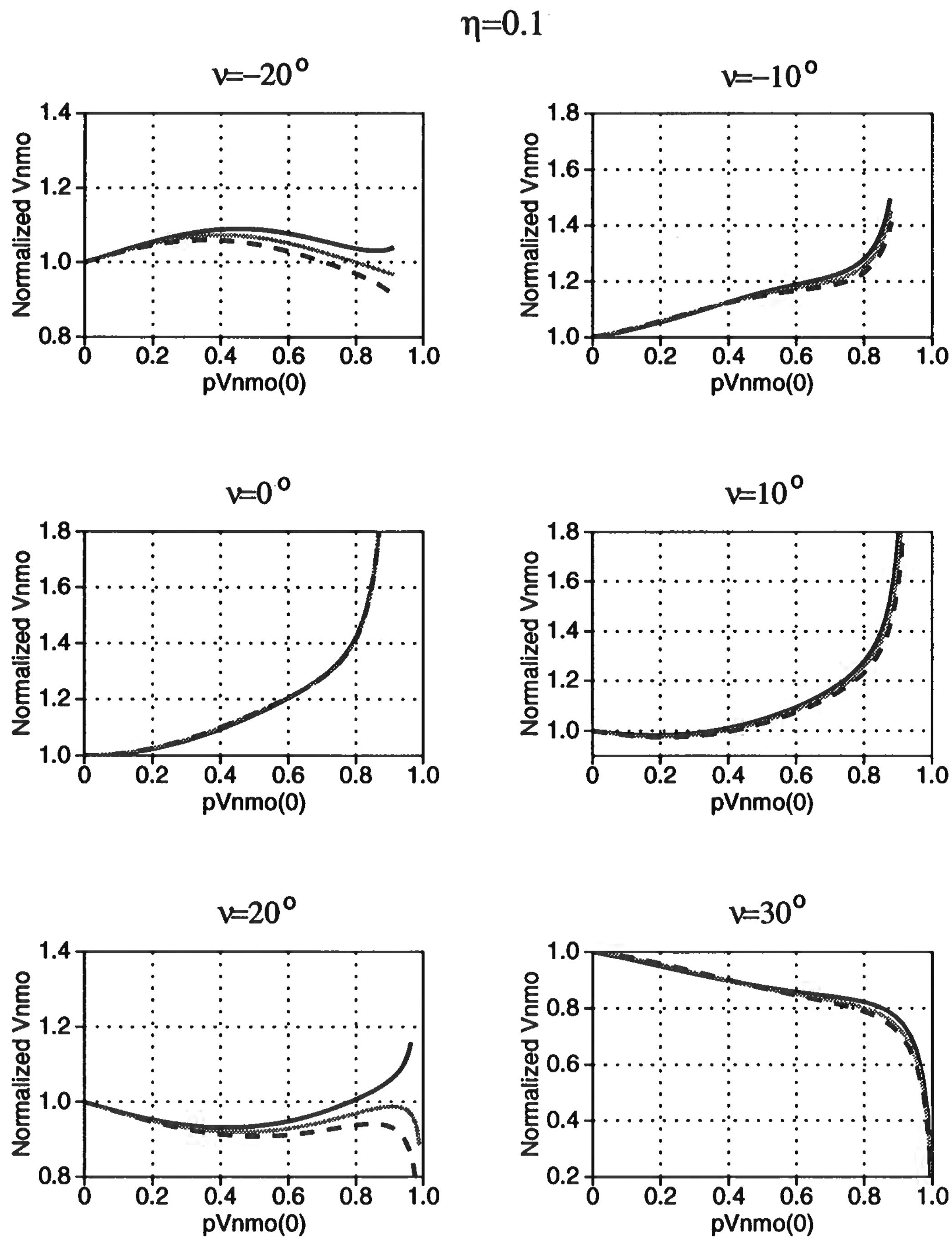

FIG. 15. Normalized P–wave NMO velocity for mild tilt angles and η=0.1: ϵ=0.1, δ=0 (black); ϵ=0.2, δ=0.083 (gray); ϵ=0.3, δ=0.167 (dashed).

from the isotropic dependence (28) are proportional to the difference $\epsilon - \delta$, which is close to η for small ϵ and δ. This is illustrated by Figure 13 that shows that at intermediate tilts the normalized NMO velocity is almost proportional to η for small values of this parameter. However, with increasing η this is no longer the case, and the curves for $0.15 < \eta < 0.3$ are relatively close to each other (Figures 12 and 13). Apparently, at $\nu = \pm 45°$ the influence of the terms quadratic in the anisotropic coefficients leads to a more complicated relation between NMO velocity and η. We conclude that with increasing η the NMO velocity at intermediate angles becomes much less sensitive to this parameter, as well as to the coefficients ϵ and δ. While for relatively small η the inversion of the NMO velocity for η and dip-moveout processing can be performed in same fashion as in VTI media (provided the tilt is known), the approach developed for vertical transverse isotropy cannot be used for values of η on the order of 0.15 and higher.

The profound differences between the dip-dependence of NMO velocities for vertical transverse isotropy and TI media with a tilted axis of symmetry makes it necessary to study the transition between the two signatures at mild tilt angles. As we have seen in the previous section, a tilt of 20 degrees is sufficient to completely eliminate the increase in the normalized NMO velocity typical for vertical transverse isotropy. Figures 14 and 15 show that the tilt should not exceed 10-15 degrees for the character of the normalized P–wave NMO velocity to be close to that in VTI media. For small tilt angles within this range, the NMO velocity grows with p much faster than in isotropic media, and this increase is largely governed by η. However, it should be mentioned that even for $\nu = \pm 10°$ the anisotropic signature is less pronounced than for VTI media and the η control over the DMO signature is less tight. At tilt angles of about ± 20 degrees, $V_{\text{nmo}}(p)$ becomes even less dependent on η with the DMO signature being relatively close to isotropic for a wide range of dips. For larger tilts, the signature gets reversed, and the normalized NMO velocity decreases with p.

DISCUSSION AND CONCLUSIONS

Transverse isotropy with a tilted axis of symmmetry may be typical, for instance, for sedimentary formations near the flanks of salt domes. Here, the behavior of normal-moveout velocity was studied in the plane of TI media that contains the symmmetry axis. In addition to distorting the values of NMO velocities, the influence of tilt leads to profound changes in the structure of the reflected wavefield. For homogeneous TI models with a symmetry axis tilted towards the reflector and typical values of the anisotropic parameters, it is impossible to generate specular zero-offset reflections for a certain range of steep dips that depends on the shape of the wavefront. If the medium is factorized with vertical velocity gradient, the "missing" dips can be imaged only using *turning* rays, although the corresponding reflectors are sub-vertical. In a different situation, typical for the symmetry axis tilted away from the reflector, anisotropy can produce specular zero-offset reflections from overhang structures in the absence of velocity gradient. These phenomena may cause serious complications in the imaging of such steep structures as salt domes or volcanic intrusions.

The dependence of normal-moveout velocity on the tilt angle was studied using the equation of Tsvankin (1995a) valid for any strength of the anisotropy. A concise NMO approximation, obtained from the exact equation in the limit of weak anisotropy, helps to understand the influence of the tilt and anisotropic parameters on the NMO velocity for all wave types. While the "tilt" term in the expression for the normal-moveout velocity from horizontal reflectors is mostly determined by the difference between ϵ and δ, the influence of tilt on the dip-dependence of NMO velocity is much more complex.

For purposes of seismic inversion and processing, NMO velocity from dipping reflectors should be studied as a function of the ray parameter p (we call the dependence $V_{\text{nmo}}(p)$ the "DMO signature"). For vertical transverse isotropy, the P-wave DMO signature is controlled by just two effective parameters – the zero-dip NMO velocity $V_{\text{nmo}}(0)$ and the anisotropic parameter η. The same two coefficients determine the time-migration impulse response and, therefore, are sufficient for all time-related processing methods (Alkhalifah and Tsvankin, 1995). The reduction in the number of independent parameters made it possible to develop practical and efficient methods of P-wave velocity analysis and time-domain processing in VTI media (e.g., Anderson and Tsvankin, 1995). Therefore, it is important to find out whether η or another effective parameter still absorbs the influence of the anisotropic coefficients for TI models with a tilted symmetry axis.

If the medium is elliptically anisotropic ($\epsilon = \delta$, $\eta = 0$) with tilted elliptical axes, the dependence $V_{\text{nmo}}(p)$ is shown to be described by the same equation as in isotropic media. Since the reflection moveout in elliptical media is purely hyperbolic, all isotropic time-related processing methods (NMO, DMO, time migration) are entirely valid for elliptical anisotropy with any orientation of the elliptical axes. Time-to-depth conversion, however, requires knowledge of the vertical velocity that cannot be found from moveout data alone. It should be mentioned that the NMO velocity from horizontal reflectors in elliptical models remain close to the horizontal phase velocity.

Since for equal values of ϵ and δ the function $V_{\text{nmo}}(p)$ is the same as for isotropy or elliptical anisotropy, deviations from the isotropic DMO signature in the weak-anisotropy approximation are determined by the difference $\epsilon - \delta \approx \eta$. The numerical results confirm that for relatively small values of ϵ and δ the P-wave DMO signature depends just on η and the tilt angle. However, due to the inflence of the higher-order anisotropic terms at intermediate tilt angles, this conclusion does not hold if the coefficients ϵ, δ, or η exceed 10-15 percent.

On the whole, the dependence of the NMO velocity on the ray parameter has the same character as for vertical transverse isotropy only for a narrow range of tilt angles corresponding to near-vertical and near-horizontal orientations of the symmetry axis. As shown in the companion paper (this volume), for horizontal transverse isotropy the NMO velocity is controlled by the parameter η of the "equivalent" VTI medium, which is relatively close to the generic value of η. For mild tilts away from vertical, the behavior of the P–wave NMO velocity is similar to that in VTI media: $V_{\rm nmo}$ increases with p much faster than in isotropic (or elliptically anisotropic) media and is tightly controlled by η (at a fixed tilt ν). However, the comfortable limit for applying the approaches developed for VTI media is only about $\nu = \pm 10°$. A tilt of ± 20 degrees is sufficient to eliminate the anisotropic DMO signature almost entirely and make the NMO velocity much less dependent on η. At larger tilts of $\pm(30-55)°$, the signature gets reversed, with $V_{\rm nmo}(p)$ increasing slower than in isotropic media. Although at these "intermediate" tilt angles the NMO curves for a fixed η are relatively close to each other, with increasing η the NMO velocity becomes much less sensitive to this parameter, as well as to the coefficients ϵ and δ.

Provided that the tilt angle is known, the dependence $V_{\rm nmo}(p)$ can be reliably inverted for η only for the the symmetry axis close either to vertical or to horizontal. Also, η can be obtained in the $\pm(30 - 55)°$ tilt range, but only for small values of the anisotropic parameters. An interesting implication of the above results is the possibility to use the DMO signature in the inversion for the orientation of the symmetry axis. In general, this inversion would require a 3–D azimuthal analysis of reflection traveltimes on survey lines with different direction. However, if the plane that contains the symmetry axis has been determined, the strong dependence of the P–wave NMO velocities on the tilt angle sometimes can be used to constrain the tilt. For instance, if the formation has a known value of η (say, a typical positive value for shales), the disappearance of the anisotropic DMO signature is indicative of a tilt of about ± 20 degrees. If the signature gets reversed (i.e., the normalized NMO velocity decreases with p), the tilt should be on the order of ± 30 degrees or more. Note that the influence of the anisotropy on the P–wave function $V_{\rm nmo}(p)$ can also be reduced by a positive vertical-velocity gradient (Larner, 1993; Tsvankin, 1995). Hence, the estimation of the tilt angle would be impossible without properly accounting for vertical inhomogeneity.

The substantial magnitude of deviations of the P–wave DMO signature from the isotropic one in several ranges of tilt angles means that conventional isotropic DMO cannot be applied to many typical transversely isotropic models. Some existing anisotropic dip-moveout algorithms based on the exact NMO equation (4) can be directly used in the symmetry plane of TI media with a tilted symmetry axis. For instance, the Hale-type DMO method of Anderson and Tsvankin (1995) is designed for any symmetry plane in arbitrary-anisotropic media. The main problem in the application of anisotropic DMO to TI media with a tilted axis of symmetry is estimation of the anisotropic parameters. Although the DMO signature is not fully controlled by η and the tilt may not be exactly known, it may still be possible to find an "effective" transversely isotropic model by fitting measurements of $V_{\rm nmo}(p)$ to the theoretical curves computed from equation (4). The anisotropic parameters and the tilt angle provide extra degrees of freedom in the NMO equation, thus making this approach feasible. However, the parameters recovered from the fitting procedure would represent just one possible anisotropic model that would be suitable for DMO processing but not necessarily for poststack migration, or for interpretation of the amplitude variation of offset (AVO) response.

Although the above analysis was performed for a single transversely isotropic layer, the results can be extended to vertically inhomogeneous media by using the generalized Dix equation of Alkhalifah and Tsvankin (1995). This equation is valid in symmetry planes of any anisotropic model that consists of a dipping reflector beneath a vertically statified overburden. Each layer, for instance, can be transversely isotropic with an in-plane symmetry axis tilted at an arbitrary angle. The normal-moveout velocity for such a model will be represented by a combination of the single-layer NMO velocities that were discussed in this paper.

ACKNOWLEDGMENTS

I wish to thank members of the A(anisotropy)-Team at CWP for useful discussions and Jack Cohen for his review of the manuscript.

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APPENDIX A: NMO VELOCITY FOR ELLIPTICALLY ANISOTROPIC MEDIA

Here, we obtain normal-moveout velocity for elliptically anisotropic media with tilted in-plane elliptical axes using equation (4). To satisfy the assumptions behind equation (4), we also assume the incidence plane to be the dip plane of the reflector.

For elliptical anisotropy, *P*-waves are described by a single anisotropic parameter δ ($\epsilon = \delta$). If the symmetry axis makes the angle ν with vertical, the *P*–wave phase-velocity function is given by

$$V_P(\theta) = V_{P0}\sqrt{1 + 2\delta \sin^2 \bar{\theta}}\,, \tag{A-1}$$

$\bar{\theta} = \theta - \nu$, V_{P0} is the *P*–wave phase and group velocity along the symmetry axis. Note that both θ and ν may be either positive or negative; we will assume that the positive direction is counterclockwise from the vertical axis that points downward.

The derivatives of the phase velocity needed to evaluate the NMO velocity (4) are obtained from equation (A-1) as

$$\frac{dV_P(\theta)}{d\theta} = \frac{V_{P0}\,\delta\,\sin 2\bar{\theta}}{(1 + 2\delta\,\sin^2\bar{\theta})^{1/2}}\,, \tag{A-2}$$

$$\frac{d^2V_P(\theta)}{d\theta^2} = 2V_{P0}\,\delta\,\frac{\cos 2\bar{\theta} - 2\delta\sin^4\bar{\theta}}{(1 + 2\delta\,\sin^2\bar{\theta})^{3/2}}\,. \tag{A-3}$$

Substituting equations (A-1)–(A-3) into NMO formula (4) yields

$$V_{\mathrm{nmo}}(\phi) = \frac{V_{P0}}{\cos\phi}\sqrt{1+2\delta}\,\sqrt{1 + 2\delta\sin^2(\phi-\nu)}\,\left[1 - 2\delta\,\frac{\sin\nu\,\sin(\phi-\nu)}{\cos\phi}\right]^{-1}\,. \tag{A-4}$$

Next, let us express NMO velocity as a function of the ray parameter p. The function $V_{\mathrm{nmo}}(p)$ for elliptical anisotropy with a vertical symmetry axis has the same form as in isotropic media (Tsvankin, 1995a; Alkhalifah and Tsvankin, 1995):

$$V_{\mathrm{nmo}}(p) = \frac{V_{\mathrm{nmo}}(0)}{\sqrt{1 - p^2V_{\mathrm{nmo}}^2(0)}}\,, \tag{A-5}$$

with

$$V_{\mathrm{nmo}}(0) = V_{P0}\sqrt{1+2\delta}\,.$$

Here, we will prove that the isotropic relationship (A-5) between NMO velocity and the ray parameter holds in elliptically anisotropic media irrespective of the tilt of the elliptical axes. It is easier to carry out this proof by transforming equation (A-5) into equation (A-4) rather than the other way around.

Using the phase-velocity equation (A-1), we represent the ray parameter as

$$p = \frac{\sin\phi}{V(\phi)} = \frac{\sin\phi}{V_{P0}\sqrt{1 + 2\delta\,\sin^2(\phi-\theta)}}\,. \tag{A-6}$$

The zero-dip NMO velocity $V_{\mathrm{nmo}}(0)$ for arbitrary tilt of the symmetry axis can be found from equation (A-4):

$$V_{\mathrm{nmo}}(0) = \frac{V_{P0}\sqrt{1+2\delta}}{\sqrt{1+2\delta\sin^2\nu}}\,. \tag{A-7}$$

Substituting equations (A-6) and (A-7) into the isotropic equation (A-5) for $V_{\mathrm{nmo}}(p)$, we get

$$V_{\mathrm{nmo}}(\phi) = \frac{V_{P0}\sqrt{1+2\delta}\,\sqrt{1+2\delta\sin^2(\phi-\nu)}}{d}\,, \tag{A-8}$$

$$d = \{1 + 2\delta\,[\sin^2\nu + \sin^2(\phi-\nu)] + 4\delta^2 \sin^2\nu\, \sin^2(\phi-\nu) - (1+2\delta)\,\sin^2\phi\}^{1/2}.$$

The denominator d can be simplified further:

$$d = \cos\phi - 2\delta\,\sin\nu\,\sin(\phi-\nu),$$

and equation (A-8) reduces to expression (A-4) for $V_{\rm nmo}(\phi)$.

APPENDIX B: WEAK-ANISOTROPY APPROXIMATION FOR NMO VELOCITY

For weak transverse isotropy ($\epsilon \ll 1$, $\delta \ll 1$, $\gamma \ll 1$), the NMO equation (4) can be significantly simplified by retaining only the terms linear in the anisotropic coefficients ϵ, δ (for the P– and SV–waves), and γ (for the SH-wave). First, we expand equation (4) in the anisotropic terms $\frac{1}{V(\phi)}\frac{d^2V}{d\theta^2}$ and $\frac{\tan\phi}{V(\phi)}\frac{dV}{d\theta}$, which turn to zero in isotropic media. Dropping the quadratic and higher-order terms in this expansion, we get

$$V_{\rm nmo}(\phi) = \frac{V(\phi)}{\cos\phi}\left[1 + \frac{1}{2V(\phi)}\frac{d^2V}{d\theta^2} + \frac{\tan\phi}{V(\phi)}\frac{dV}{d\theta}\right]. \qquad \text{(B-1)}$$

For P–waves, the phase-velocity function, fully linearized in the anisotropic coefficients ϵ and δ, was given by Thomsen (1986). In the case of the symmetry axis tilted at the angle ν from vertical, the weak-anisotropy approximation for the phase velocity becomes

$$V_P(\theta) = V_{P0}\,(1 + \delta\sin^2\bar\theta\cos^2\bar\theta + \epsilon\sin^4\bar\theta), \qquad \text{(B-2)}$$

$\bar\theta = \theta - \nu$.

The derivatives of V_P (B-2) to be used in the NMO equation (4) are

$$\frac{dV_P(\theta)}{d\theta} = \dot V(\theta) = V_{P0}\,\sin 2\bar\theta\,(\delta\cos 2\bar\theta + 2\epsilon\sin^2\bar\theta), \qquad \text{(B-3)}$$

$$\frac{d^2V_P(\theta)}{d\theta^2} = \ddot V(\theta) = 2V_{P0}\,[\,\delta\cos 4\bar\theta + 2\epsilon\sin^2\bar\theta\,(1 + 2\cos 2\bar\theta)]. \qquad \text{(B-4)}$$

Since the phase velocity and its derivatives [equations (B-2) – (B-4)] should be evaluated at the angle $\theta = \phi$, they become functions of the variable $\bar\phi = \phi - \nu$. It is convenient to rewrite equation (B-1) as

$$V_{\rm nmo}(\phi)\cos\phi = V(\bar{\phi})\left[1+\frac{\ddot{V}(\bar{\phi})}{2V(\bar{\phi})}+\tan\bar{\phi}\,\frac{\dot{V}(\bar{\phi})}{V(\bar{\phi})}+(\tan\phi-\tan\bar{\phi})\,\frac{\dot{V}(\bar{\phi})}{V(\bar{\phi})}\right]. \quad \text{(B-5)}$$

The term

$$C(\bar{\phi}) = V(\bar{\phi})\left[1+\frac{\ddot{V}(\bar{\phi})}{2V(\bar{\phi})}+\tan\bar{\phi}\,\frac{\dot{V}(\bar{\phi})}{V(\bar{\phi})}\right] \quad \text{(B-6)}$$

has exactly the same form as $[V_{\rm nmo}(\phi)\cos(\phi)]$ for a vertical axis of symmetry ($\nu = 0$, $\bar{\phi} = \phi$). The weak-anisotropy approximation for VTI media was obtained by Tsvankin (1995a):

$$V_{\rm nmo}(\phi)\cos\phi = V_{P0}\,[\,1+\delta+\delta\sin^2\phi+3(\epsilon-\delta)\sin^2\phi\;(2-\sin^2\phi)]\,. \quad \text{(B-7)}$$

Replacing ϕ with $\bar{\phi}$ in equation (B-7) allows us to represent $C(\bar{\phi})$ from equation (B-6) as

$$C(\bar{\phi}) = V_{P0}\,[\,1+\delta+\delta\sin^2\bar{\phi}+3(\epsilon-\delta)\sin^2\bar{\phi}\;(2-\sin^2\bar{\phi})]\,. \quad \text{(B-8)}$$

Using equations (B-2) and (B-3), we find the weak-anisotropy approximation for the remaining term in equation (B-5).

$$(\tan\phi-\tan\bar{\phi})\,\frac{\dot{V}(\bar{\phi})}{V(\bar{\phi})} = \frac{2\sin\nu\sin\bar{\phi}}{\cos\phi}\,[\,\delta+2(\epsilon-\delta)\sin^2\bar{\phi}\,]\,. \quad \text{(B-9)}$$

Substituting equations (B-8) and (B-9) into (B-5) yields

$$\begin{aligned} V_{\rm nmo}(\phi)\cos\phi &= V_{P0}\,\{1+\delta+\delta\sin^2\bar{\phi}+3(\epsilon-\delta)\sin^2\bar{\phi}\,(2-\sin^2\bar{\phi}) \\ &\quad +\frac{2\sin\nu\sin\bar{\phi}}{\cos\phi}\,[\,\delta+2(\epsilon-\delta)\sin^2\bar{\phi}\,]\}\,. \end{aligned} \quad \text{(B-10)}$$

Equation (B-10) is the weak-anisotropy approximation for the P–wave NMO velocity fully linearized in the anisotropic parameters ϵ and δ.