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Principal Investigators: Norman Bleistein, Jack K. Cohen, John A. DeSanto and Frank G. Hagin

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Center for Wave Phenomena
Colorado School of Mines
Golden, Colorado 80401
(303)273-3557

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Introduction

This is a final report for the project, The Application of Inverse Methods to the Ocean Environment, supported by the Selected Opportunities Research Program of the Office of Naval Research at the Colorado School of Mines. The research has been carried out in the Center for Wave Phenomena in the Mathematics Department. The SRO has partially supported four faculty members: Norman Bleistein, Jack K. Cohen, John A. DeSanto and Frank G. Hagin. Over the course of the program, seven students were also partially supported by this program.

The SRO program has had an extremely positive effect on graduate education and scholarly activity at the Colorado School of Mines. It has been a catalyst for generating support for related research, so that, in total, we have supported between five and seven graduate students through each year of the SRO program. Five graduate courses directly related to our research program have been introduced. Six students have completed -- or are about to complete -- graduate degrees including three PhDs (details are given in Appendix A). In addition, many distinguished scholars have visited the Center in support of our research activities (see Appendix B).

During the three years spanned by the SRO, we have made major progress in the theory and application of inversion methods. At the start of the SRO, our inversion techniques, although well founded, were restricted to idealized data and physical assumptions. At the conclusion of the SRO, the theory and techniques had been extended to encompass most of the standard data collection procedures and were based on much more realistic physical assumptions. This progress is outlined in the next section. Our research efforts have been reported in technical reports, journal articles and proceedings (see Appendix C) and have been presented at national and international meetings including invited talks and invited articles (see Appendix D).

The SRO program at the Colorado School of Mines consisted of two major projects in inverse scattering. The first of these is reflector imaging for seabed mapping and seismic exploration. The second is ocean profile inversion. We now turn to a discussion of these projects.

Research Background and Current Status

There have been two major research projects in inverse scattering under the Selected Research Opportunities Program. The first of these is reflector imaging for seabed mapping and seismic exploration. The second is ocean profile inversion. We will discuss those programs herein that order. Each project also had a significant effort in modeling of the type of experiments for which inversion theory and computer algorithms arebeing developed. The first project was carried out under the direction of the principal investigators Bleistein, Cohen and Hagin; the second project was led by DeSanto.

Reflector Imaging for Seabed Mapping and Seismic Exploration

Our research group is committed to the practical solution of inverse problems. We have been developing stable algorithms for inverse problems for over ten years. In this section, we give a short summary of our work before the period covered by the SRO (i.e., before Fall 1983) and then we continue with a moredetailed account of the last three years.

Our inversion techniques take the form of integral operators that can be viewed as back propagation or back projection operators of spatially and frequency filtered data. These data are propagated with acoustic or elastic parameters of a background or reference medium. The more complex this background medium is, the more difficult the determination and computation of the kernel of the integral operator becomes. The kernel can be further complicated by the structure of the source/receiver configuration.

At the start of the Selected Research Opportunities Project, our inversion operators applied to "backscatter" or "zero-offset" data in a constant density medium with a constant reference or background sound speed.

At the end of the Selected Research Opportunities Project, we were able to invert data with variable background sound speed and density for any of the source/receiver configurations used in seismic surveys. We have applied our inversion techniques to synthetic and field data for a subset of configurations for which computer code has been written: depth-dependent background speed, zero-offset, background speed depending on depth and one transverse variable, zero-offset, constant background speed, common or fixed offset; constant background, common (or single) source, multi-receiver. We have also developed a corresponding suite of direct modeling techniques and attendant computer code to provide the necessary synthetic data. In addition, we have a project in process on velocity analysis, whereby optimal background propagation speeds are determined. Another project deals with direct modeling and inversion of full elastic data. These are all described in further detail below.

Two-and-One-Half Dimensions

In most seismic surveys and in many other comparable inverse problems, data is collected along a line on the boundary of the medium of interest. From such data, it is generally not possible to compute a three dimensional inversion for the structure of the medium. To create a model consistent with such a data collection scheme, we assume that the earth varies only with depth and the one transverse variable along the line of data collection. However, we still model our wave propagation as three dimensional. We call this combination of three dimensional propagation in a medium with only two dimensional variation <u>two-and-one-half dimensional</u>. Much of the migration/inversion literature uses two dimensional wave propagation. However, this conflicts with the three dimensional sources used in actual data acquisition and so we have always derived inversion algorithms for three dimensional wave propagation. Thus, we develop modeling and inversion techniques primarily in three dimensions and two-and-one-half dimensions. (Only recently, we have received a data set from an experiment carried out on the surface of a steel cylinder. The source for this experiment is best modeled as a line source. For this problem, we have specialized our results to two dimensional inversion.)

The computer outputs presented here are all for two-and-one-half dimensional modeling and inversion. However, we have tested both the constant background and c(z) background zero-offset inversions on synthetic three dimensional data sets.

Despite the emphasis on two-and-one-half dimensions, we are aware of the pitfalls of this approach [Cohen and Bleistein, 1983].

Earlier Research

Our early research in inverse problems was motivated by the results of N. N. Bojarski [1974] who formulated a fundamental integral equation which became our main tool for studying the inverse source problem. Our work in this area was reported in Bleistein and Cohen [1977a].

Simultaneously, we began research on another theory of Bojarski's [1967], addressing the problem of imaging a scattering obstacle from high frequency far field scattering data [Bleistein, 1976, Mager and Bleistein, 1978, Cohen and Bleistein, 1979a]. We soon began applying these results to the problem of imaging flaws in solids [Bleistein and Cohen, 1977b, 1980] which arises in nondestructive testing. While the nondestructive testing aspect of inversion has not been pursued by our group during the last few years, other researchers have achieved success by exploiting our methods [Langenberg, Brück and Fischer, 1983, Höller, Langenberg and Schmitz, 1984, Langenberg, Fischer, Berger and Weinfurter, 1985].

During 1976 we began a fruitful line of research on the problem of inversion of data gathered in response to sources on the surface of a layered half space. In our model the layers need not be horizontal in the model. This model is applicable both to seismic inversion for seabed mapping and for exploration for hydrocarbons. It also has application to nondestructive evaluation of solids, to medical imaging and to vertical seismic profiling.

The objective of our methods is to image the interfaces and obtain estimates of reflection strength across them, from which one can proceed further to estimate changes in earth parameters across the interfaces.

Born Inversion

Our work in this area began with a formulation involving plane wave sources [Cohen and Bleistein, 1977] and was followed by work employing the more realistic model of point source probes [Cohen and Bleistein, 1979b]. This latter paper is often cited since it gave a practical algorithm for the seismic "backscatter" or "zero-offset" problem. Here, we inverted for a perturbation of sound speed, relative to a constant background sound speed. This paper has become the basis for further development by both us and other researchers.

Our method has come to be known as "Born inversion" because the perturbation approach is similar to the Born approximation in potential scattering. Although small variations in sound speed is a basic premise of this method, we have found that it has broader applicability. In particular, we applied our algorithm to Kirchhoff approximate data from a single reflector in a constant background medium. We find by asymptotic analysis that, when the background velocity is chosen as the velocity in the upper medium, the method will properly locate the reflector and accurately estimate reflection strength for any size jump in velocity across the reflector. This type of verification has persisted throughout our work as we have extended our method to morecomplex background structure and to various source/receiver configurations used in practice.

The Singular Function

Although we began our analysis by seeking perturbations in the sound speed itself, we have modified our output so that it produces an array of Dirac delta functions with support on each of the surfaces of discontinuity of the velocity field. These surfaces arejust the reflectors in the subsurface. The scaling of each delta function is proportional to the reflection strength of that reflector. We call the Dirac delta function with support on the surface the singular function of the surface [Cohen and Bleistein, 1979a, Bleistein, 1984, Bleistein, Cohen and Hagin, 1985] and we call the array of scaled singular functions the reflectivity function of the subsurface.

This approach to the inverse problem is motivated by the fact that seismic surveys, on land or over the seabed, produce bandlimited data that are also high frequency data for most of the length scales of interest. From near zero-offset high frequency data, one cannot detect trends in the earth parameters, but only discontinuities. Such discontinuities aremost easily detected as bandlimited delta functions. In Cohen and Bleistein [1979a], we developed a rigorous asymptotic theory for the transition from bandlimited high frequency data for a function to determination of the singular functions of its surfaces of discontinuities.

Early Theoretical Studies

In 1978, Frank Hagin joined the research group and began exploring the stability of the class of inversion algorithms being developed by Bleistein and Cohen. We had empirically observed the stability of these algorithms. However, since an integral equation of the first kind was being inverted, the theoretical issue of stability had to be addressed to lay a foundation for continued work. We soon recognized that our inversion equation was moreclosely related to the well-conditioned problem of inverting Fourier transforms than to the ill-conditioned problem of inverting integral equations with compact operators. In Hagin [1980, 1981a, 1981b] and Gray and Hagin [1982] the stability issue was laid to rest for the one dimensional inverse problem. Moreover, many of the concepts developed in one dimension carried over to three dimensional inverse problems [Hagin and Gray, 1984]. This research also introduced the theme of variable reference speed which has become important in our current work.

Summary, Preselected Research Opportunities Project

In summary, before the commencement of the SRO grant in Fall 1983, the group was well grounded in the basics of inversion techniques as applied to the wave equation in simple three dimensional settings. We had developed a research level computer program for inverting "backscatter" data to determine perturbations from a constant reference background. In addition to these accomplishments, clear direction was seen for several lines of research; these were outlined in the Selected Research Opportunities Project proposal.

Research Since 1983

We now describe our progress from October 1, 1983 to January 31, 1987.

The constant reference speed assumption of our earlier work has important applications. However, the algorithm produced inversions that deteriorated unacceptably for structures whose cumulative velocity change was large. Recursive applications of the algorithm can alleviate this problem to a degree by using different reference velocities in different regions. A better solution is to develop inversion algorithms which allow the ab initio inclusion of as much of the known velocity structureas possible. Such variable reference speed schemes hold the potential for improved accuracy and economy.

Secondly, the backscatter experiment, although an important theoretical model, can only be approximated by the standard "stacking" of the actual data. There are well known situations for which this approximation is poor.

Correction by Postprocessing

As a first attempt to improve the inversion obtained when there are large variations of the background speed, we developed a postprocessing scheme that corrected for errors in the linear theory when there were large variations in propagation speed across interfaces. In this approach, it is necessary to find regions in which several major reflectors are nearly parallel (not necessarily horizontal). In such regions, the algorithm described in Hagin and Cohen [1984] can be used to correct for the major errors inherent in the constant reference speed algorithm. When applicable, this algorithm provides an inexpensive way to refine the inversion and can provide dramatic improvement in both location and parameter estimates.

An example of this method is provided in Figures 1 and 2 taken from the Hagin and Cohen paper. Figure 1 shows the output of the constant reference inversion algorithm. One can see from the figure that the first reflector and velocity increment are properly reconstructed. The output is seen to degrade with depth, both in reflector mapping and velocity estimation. Figu1re shows the results of applying the refinement algorithm and the fit is seen to be nearly perfect, even though the velocity increment is more than 150%. Furthermore, the cpu time for the refinement is only about 2% of the cpu time for the initial inversion.

Background Speed Depending on One Variable: c(z)

As suggested above, inversion schemes with a variable reference speed have clear advantages over those based on one or more constant reference speeds. In Bleistein and Gray [1985], such an algorithm was derived for the case of zero-offset data, under the assumptions of a constant density and a depth-dependent reference velocity, c(z). A key step in the derivation was the early use of the "high frequency" assumption (which was already being used at the implementation stage, as noted above).

The philosophy of this paper now pervades our entire research program. An inversion algorithm is an integration over source/receiver pairs in which the kernel of the integral operator uses ray-theoretic (WKBJ) travel times between output points at depth and the source/receiver points, and ray theoretic amplitudes consistent with the background reference speed. The kernel of the operator contains another factor also expressed in terms of functions related to ray theory. The output is the reflectivity function of the subsurface, thus providing a reflector map in which the peak amplitude on each reflector provides a means for estimating reflection strength.

The computer code developed by Gray has become a production line code at Amoco. It is particularly well suited for imaging flanks of salt domes in otherwise horizontally stratified media, such as in the Gulf of Mexico. Figures 2A-C, taken from the Bleistein and Gray paper, show this capability. Figure 2A is a geologic model of a salt dome intruding into an otherwise horizontally stratified geologic structure. Figure 2B shows a zero-offset time section for this model generated using a finite difference scheme developed by Dan Whitmore at Amoco Production Company. Figure 2C is the result of applying Gray's program to this synthetic data. It can be seen that the flank of the structure is well defined up to the vertical. (This program does not image reflectors beyond vertical because, when it was written, turned rays were not incorporated into the code.) In contrast, a constant background inversion would steepen slanting images in the time section Figure 2B but could not image reflectors to near vertical. The improvement in location of the c(z) algorithm can be traced ultimately to the bending of rays along their trajectory, allowing the observed data to be projected back along curved trajectories to their origin on the reflectors. The lack of refraction in a constant background algorithm will mislocate the reflectors by projecting the data back along straight ray paths. The new algorithm requires only a modest increase in cpu time over the constant background code [Bleistein and Gray, 1985]; the cpu times for inversion with a c(z) background are comparable to the computer codes for constant background k-f migration following Stolt [1978].

While the Bleistein-Gray c(z) algorithm produced improved reflector mapping, it was soon discovered that it did not provide the correct magnitude of jump in velocity for curved reflectors. Cohen and Hagin [1985] approached the inversion with a c(z) reference speed from a somewhat different point of view. They succeeded in finding an inversion operator which correctly estimated the jump in velocity across a single reflector given accurate synthetic high frequency data. The structure of their inversion operator was similar to the Bleistein-Gray operator. Consequently, the virtues of the code for that algorithm carried over to the new one. However, the derivation of the new algorithm put the determination of inversion operators on a new footing. In this approach the general problem of finding an inversion operator was reduced to that of finding a suitable "completeness" relation. This idea suggested a systematic approach to developing further inversion algorithms -- for example, extensions to c(x,z) and c(x,y,z) reference speeds, as well as extensions to variable density and nonzero-offset source/receiver configurations.

Computer Implementation

Computer code implementing the Cohen and Hagin result was written by one of our graduate students, Brian Sumner. When the background sound speed depends only on one variable it is possible to write down solutions for the relevant quantities of the inversion process as integrals with respect to z. Sumner's code assumes a background sound speed that is piecewise linear in z and continuous. In this case, analytic solutions for all of the ray theoretic components of the inversion kernel arepossible. Furthermore, such a background allows for connecting available velocity picks as a function of depth in a straightforward manner.

Tests on synthetically generated data, as well as on field data, have been carried out. We present here a field data example from data gathered over a saltdome in the seabed. This particular data set was provided to us by Golden Geophysical. The data had no usable amplitude because of standard seismic preprocessing -- dip moveout correction and spherical spreading correction -- applied to the data. The former takes no account of amplitude and the latter is only an approximate correction for amplitude variations with time. However, as with all high frequency asymptotic techniques, amplitude errors (which are slowly varying on the wave length scale) do not effect location of images, but only intensity on them. Thus, it is still possible to use the inversion algorithm to image reflectors, although we could not hope to recover reflection coefficients from peak amplitude.

Figure 4A shows the data set with automatic gain applied. (If this werenot done, the deeper reflection events would have too small an amplitude to be seen on the plot.) The long diffraction tail from the saltdome, moving diagonally across the figure, is evident.

Figure 4B is the output of the inversion algorithm. The diffraction tail has been gathered up onto the saltdome. Also, the termination of the horizontal reflectors provides a well defined boundary to the flank of the saltdome. Unfortunately, the preprocessing tends to degrade the reflections from the flank of the saltdome, itself. Thus, with this type of data, we can confirm the validity of the method only through the proper gathering of the diffraction tail and the well defined terminations of the horizontal reflectors.

Common Offset Inversion and Another Approach to Inversion

This inversion assumes that the source and receiver are separated by a fixed (common) offset. As originally developed by Sullivan and Cohen [1987], the method imaged reflectors and estimated an acoustic reflection coefficient from the peak amplitude of the output. Their model assumed a constant background speed and constant density. When the data can be modeled as being polarized, it is possible to interpret the reflection coefficient as being fully elastic.

We present an example of application of this method to synthetic data. Figure 5A shows common offset data for sources and receivers located 800 ft apart over a reflector near 2000 ft depth. Figure 5B shows the reconstruction of the reflector. The location is nearly exact. The peak amplitude on the reflector agreed with the theory to three decimal places.

Next we present the results of applying this method to field data provided by Conoco. The data came from an ocean region in which the seabed was nearly horizontally layered. The objective of the study was to analyze the reflection strength along a particular reflector. For Conoco, the objective was to use variations in elastic parameters as a hydrocarbon indicator.

Figures 6A and 6B show inversions in the same region from data sets in which the offset was 885 ft and 1985 ft, respectively. The reflectors at 4900 ft and 5100 ft in the center of the figures are nearly identical on the two plots, although produced from the two different offset data sets and with the same average constant background sound speed. This confirms that, for this example, at least, application of a constant background speed produces a reasonable result.

On each output, the amplitude varies horizontally along the interface at 4900 ft. This change in reflection strength is a function of the change in the three elastic parameters across the interface. Given the outputs from three different common offset data sets, the change in elastic parameters can be determined. Given additional offsets the error arising from noise in the data can be reduced.

Applying Sullivan and Cohen, along with Bleistein [1987a], Roger Parsons [1986] was able to show that the change in reflection strength is mainly caused by density variations which, in turn, are due to differences in gas saturation. We have confirmed Parsons' results.

The method of these authors suggests another approach to the general inverse problem. An integral operator as a sum over traces is assumed. The phase of the integral operator is taken to be the sum of travel times in the background medium from source to output point and from receiver to output point. The amplitude is left to be determined.

The method is applied to Kirchhoff approximate data for a single reflector. The output is then a multi-fold integral over frequency, the observation surface and the reflecting surface. The spatial integrals are all approximated by the method of stationary phase. The remaining frequency domain integral is required to be a bandlimited singular function of the reflecting surface, multiplied by a spatial scaling factor. It is possible to chose the weighting factor of the original kernel so that the scaling factor is exactly the geometrical optics reflection coefficient.

Kirchhoff Inversion

Beylkin [1985] presented a result that unified the theory of high frequency inversion in complex background structure, with complex source/receiver configurations. Beylkin's result is framed in the context of pseudo-differential operators and generalized Radon transforms. However, the key insight was compatible with the approach taken by us. With Beylkin's technique one can directly obtain, in principle, the required kernel of the integral inversion operator for virtually all models of experiments for the acoustic wave equation. Beylkin's approach is consistent with the high frequency assumptions used by us. Thus, we were able to modify Beylkin's inversion operator to determine the reflectivity function of the subsurface by using our singular function theory [Cohen and Bleistein, 1979b]. That is, we obtain an output as an array of singular functions of the subsurface reflectors with peak amplitudes proportional to the reflection strength on each reflector [Cohen, Hagin and Bleistein, 1986, Bleistein, Cohen and Hagin, 1987, Bleistein, 1987a, 1987b].

We now call this theory "Kirchhoff inversion" for two reasons. First, it has the structure of Kirchhoff migration [Schneider, 1978]. Second, we test the method analytically by applying it to Kirchhoff approximate data from a single reflector in the complex background structure and verify the claims made above by asymptotic analysis of the multi-fold integral. This is true for any size jump in acoustic parameters across the reflector. Thus, as with the inversions in less complex structures discussed earlier, the method has broader applicability than its basis in perturbation theory would suggest.

Three Dimensional Inversion with c(x,y,z) Background

In Cohen, Hagin and Bleistein [1986], we describe Kirchhoff inversion with a general c(x,y,z) background with common source or common receiver data gathered on a curved surface. This last feature may reduce the need for certain types of "static" corrections. These results and the earlier common offset algorithm for constant reference described in Cohen and Sullivan [1987] are our first inversions to dispense with the zero-offset (backscatter) assumption.

Two-and-One-Half Dimensional Inversion with c(x,z) Background

Combining this new approach with the results on two-and-one-half dimensional propagation presented in Bleistein [1986], we have derived computationally feasible solutions [Bleistein, Cohen and Hagin, 1987] to the two-and-one-half dimensional inverse problem in the same generality as for the three dimensional problem of the previous paragraph.

Here, the necessary computations are straightforward when the background propagation speed is constant or a function of z, or even a function of x and z. Thus, we are able to write down inversion algorithms for the following source/receiver configurations: (i) common source or (ii) common receiver, and (iii) fixed (common) offset between source and receiver. In all three cases, the upper surface can be curved. Furthermore, we can show that the output will produce a reflector map of the subsurface and an estimate of reflection strength for all configurations.

Computer Implementation, c(x,z) Background, Two-and-One-Half Dimensions

Another graduate student, Paul Docherty, is developing an inversion consistent with this theory. At present, he has a computer implementation with proper phase but not with proper amplitude. Consequently, his program will locate reflectors but not yet estimate reflection strength. Thus, he now has a c(x,z) background migration program. Incorporation of correct amplitude into his program is now in progress.

To determine proper amplitude for a broad range of background sound speeds requires a fairly sophisticated ray tracing and direct modeling computer code, with c(x,z) background. Docherty has developed such a program. It will be described below. First, we present two examples of his current migration code.

Figure 7A shows a model with three layers and an initial velocity of 6000 ft/sec and increments of 1000 ft/sec at each interface. Zero-offset data was generated by a finite difference scheme. Figure 7B depicts the output of Docherty's algorithm when a nearly correct background velocity is chosen. Gaps in the output arise from both specular ray paths that reach the surface outside the receiver array and ray paths that have passed through caustics in the subsurface. When this migration example was carried out, the computer implementation did not include rays that passed through caustics.

Proper location of the lowest layer confirms the validity of an algorithm which properly accounts for refractions. As a comparison, Figure 7C shows the output from a conventional constant background migration. This demonstrates the need for a c(x,z) algorithm in some complex media.

Figure 8A shows a salt dome model with ray paths from one particular horizon. Figure 8B is output from Docherty's algorithm applied to synthetic data for this model. Figure 8C is the output for the same data from the c(z) algorithm. Both algorithms give comparable results in the horizontal layers and on the flanks of the salt dome. However, Docherty's algorithm more accurately depicts the horizontal layer directly below the salt dome. The reason for this is that a c(z) background speed cannot characterize the lateral changes in speed across the salt dome. Docherty has also applied his algorithm to field data, including the example of Figures 4A and 4B. He produced essentially the same image as did Sumner. The quality of the data below the saltdome was not good enough to show a difference between these two programs on this data set.

Determining Earth Parameters

The geometrical optics reflection coefficient determined by Kirchhoff inversion is angularly dependent. The angle is the opening angle between a specular pair of rays from the output point at depth to a particular source/receiver pair on the surface. That opening angle is a priori unknown. In Bleistein [1987a, 1987b], a method for determining that opening angle is derived. It requires that two inversion operators be applied to the data simultaneously. The second operator differs from the first in only a minor way. On the reflector (i.e., at peak amplitude), the ratio of the outputs of the two operators is in known proportion to the cosine of the unknown angle. Thus, the angle is determined. This result was first checked by Sullivan on the synthetic example of Figures 5A and 5B. He obtained three place accuracy on the cosine of the opening angle at several points of the reflector.

In a constant density medium, once the angle is known, the jump in propagation speed across the interface can be determined from the reflection coefficient. To determine the jumps in two parameters in a variable density medium, one needs these results for two opening angles. If the reflection coefficient is known at more angles, redundancy of the data could be exploited to reduce the effects of noise. It is this latter technique that was used by Parsons [1986] to determine both propagation speed contrasts and density contrasts across a reflector.

Asymptotic Analysis of Kirchhoff Inversion

The primary objective of the paper by Bleistein [1987a] was to confirm by classical means -- asymptotic analysis by multi-dimensional stationary phase -- the validity of three dimensional Kirchhoff inversion in constant density, variable background sound speed medium for common source, common receiver or common offset data. The determination of the opening angle in the angularly dependent reflection coefficient is a result of this classical analysis of our modification of Beylkin's inversion, not readily accessible in the pseudo-differential operator approach.

The analysis of that paper made the extension to the variable density case transparent. Those results can be found in Bleistein [1987b].

Elastic Inversion

As a PhD thesis project, Sumner is developing the extension of Kirchhoff inversion to elastic waves. Here, the inversion operator will be a sum of tensor integral operators applied to vectors of observed data. Each tensor operator has a phase corresponding to a mode converted or mode preserved wave propagation. He is attempting to determine the amplitude tensors via the method of Sullivan and Cohen, described above.

Sumner is developing the two-and-one-half dimensional inversion as well as the three dimensional inversion and developing computer code for the former.

Velocity Analysis in Two-and-One-Half Dimensions

Velocity analysis refers to the technique used to determine the background propagation speeds used for migration and inversion. The method requires many inversions carried out on a relatively coarse subset of the output data grid.

Seismic surveys consist of redundant data in that many sources are used and an array of receivers records data for each source. Thus, a particular point on a reflector will provide a reflection event on many different time traces, with Snell's law being satisfied at that point for each particular source/receiver pair, but with a different angle of incidence/reflection.

This redundancy of data provides a means of determining a first approximation of propagation speed to be used in subsequent inversions. The basis of this method is to use a common offset data set and invert with a suite of background speeds, then use the output in a modeling program to generate a synthetic zero-offset data set. The process is repeated for each common offset available in the complete data set, thus obtaining many equivalent zero-offset traces. The traces are then plotted along side one another for comparison purposes. For the "correct" background speed, the same reflection event will appear at the same time on each trace, independent of the offset of the original data set. For the wrong background speed, this will not be the case; the different lengths of propagation paths will lead to a different arrival time of the equivalent zero-offset event.

The example of Figure 9 demonstrates this method applied to a data set gathered over the seabed. The data was provided to us by Golden Geophysical. Figure 9A shows one offset (1056 ft) of a suite of common offset data sets. The inversion based zero-offset mapping scans the input data to find the optimum velocity field. Note the prevalence of the reflection off the dome, which cuts across the horizontal beds. Conventional velocity analysis would estimate different velocities for the flat and dipping events, even though the ray paths travel through the same medium.

Figures 9B-D are velocity scans for the indicated location. Each shot contributes a zero-offset trace for this location. Figures 9B-D plot the contribution from each shot side by side for three candidate velocity functions. Horizontal alignment of events on these displays shows that the velocity is correct (i.e., the zero-offset traces from each shot are identical). Figure 9B shows a velocity function ($v = v_0 + kt$) which is too slow. Figure 9C shows a function which is approximately correct. Figure 9D shows a function which is too fast.

Displays similar to Figures 9B-D were generated for sixteen candidate velocity functions. These data are then compared for semblance (i.e., statistical similarity) as a function of velocity. This generates a semblance matrix. This matrix gives semblance for each velocity as a function of travel time. Figure 9E shows such a display. Comparing Figure 9E with the suite of displays similar to Figures 9B-D, reliable dip-independent velocities can be estimated. As noted at the beginning of this section, these velocities can then serve as inputs to other inversion programs.

Computation of Kernel for Three Dimensional Kirchhoff Inversion

As mentioned above, the inversion kernel contains functions related to ray theory. In particular, it involves the amplitude and phase of the WKBJ Green's functions for source point being the output of the algorithm and observation point being either a source or receiver point of the data set. It also contains the magnitude of the sum of gradients of the travel time functions. Finally, it contains a certain determinant which is the essential feature of the Beylkin result. This determinant has the interpretation of a ratio of surface areas in different parameter domains. One area is taken on the surface of p-vectors, the other on the surface of some independent parameter set used to characterize the source/receiver configuration. In Bleistein [1987c] it is shown how this ratio can be computed from results of ray tracing, which must be done to determine the other components of the kernel, anyway. The determination of this determinant by this method requires negligible additional cpu over what is already needed for the determination of the Green's functions themselves. We believe this is the most computer-efficient method of determining this factor for a general background in three dimensional inversion.

Averaging for Image Enhancement

The purpose of this project is to exploit the redundancy of most experimental data sets for noise reduction and image enhancement. As noted above, a typical data set consists of many common shot experiments, each of which provides only partial coverage of the subsurface and, in practice, can be noisy. Thus, one might compute a weighted sum over shots of the output data to enhance the ouput image of the data. Alternatively, one could reorder the data as common receiver data, invert it, then compute a weighted sum over receiver points, for each output point.

The result of each of these approaches would be a sum over all sources and receivers. By imposing the criterion that the two resulting average outputs be the same, the weighting factors in the two averages are determined. What results is an inversion that is symmetric in sources and receivers.

Figure 10A shows the impulse response for a common shot inversion. That is, this is the response from a single impulse of data on the common shot record. It represents the weighting of the data across its candidate reflectors. The asymmetry of the operator can be seen in the figure. Figure 10B shows the impulse response for the weighted average operator. The symmetry of this operator in source and receiver location is apparent from the figure.

This work is recent [Bleistein and Jorden, 1987] and the method has not yet been tested on field data.

Modeling Projects for Seabed Mapping and Seismic Exploration

Modeling is a necessary adjunct to any project on inversion. There are three reasons for this. First, inversion operators contain the Green's function of the direct problem as part of the integration kernel. Thus, solutions of direct scattering problems with the same complexity as the model of the inverse problem being studied are a necessary prerequisite for studying the inverse problem. Second, modeling and its computer implementation provide a means of generating synthetic data for testing of the inversion theory as it is being developed. Third, important insights into the nature of the inverse problem are gained from studying direct scattering problems. One cannot study the former without studying the latter.

Two-and-One-Half Dimensional Wave Propagation

Two-and-one-half dimensional wave propagation has been mentioned repeatedly in the discussion of our research program. Bleistein [1986] describes the basic elements of the analytic representation of the WKBJ Green's function and the Kirchhoff approximate acoustic wave upward scattered from a reflector in two-and-one-half dimensional propagation.

It is assumed that the sound speed and density depend only on two variables (x,z), while the propagation is in three dimensions. The fields are to be evaluated in the plane, $y = 0$. It is shown in this paper that the fields can be computed totally in two dimensions with a spreading factor accounting for the geometrical spreading out-of-plane. This factor has the dimensions of length-squared over time and is differentially related to arclength through a factor of c, the propagation speed.

This paper has become the basis for much of the two-and-one-half dimensional modeling and inversion carried out by our group.

Ray Tracing and Modeling in a Two-and-One-Half Dimensional c(x,z) Medium

As an adjunct to the project on two-and-one-half dimensional inversion in a medium with a c(x,z) background, Docherty [1985, 1987] has been engaged in a project to develop the attendant ray tracing and modeling algorithm. Building on Keller and Perozzi [1983] and Fawcett [1983], Docherty has developed an algorithm and computer code to do ray tracing in a model consisting of several layers of constant velocity bounded by general surfaces.

The ray tracing program described in Docherty [1985] had difficulties with regions of triplication of travel times caused by caustics of the ray family in the subsurface. The ray tracing program in the current modeling program [Docherty, 1987] has largely overcome these problems in an automated way.

This modeling method determines in-plane rays, phase, amplitude, reflection and transmission coefficients, properly taking account of out-of-plane spreading. This spreading occurs because the propagation is three dimensional, even though the propagation speed depends on only two variables. It is possible to compute the necessary components of the wave field factors by computations in two dimensions only [Bleistein, 1986]. Phase changes associated with caustics and postcritical reflections are taken into account, as well.

The computer program that was developed will generate ray plots, list ray coordinates and travel times and provide a shot record of observed data. The ray plots are often helpful in interpreting events on shot records. In model data, they may indicate regions of the model where the program has failed to find any rays or it has encountered an unforeseen pathology. In either model or field data, ray plots provide an interpretation tool for understanding anomalies of the observed field.

The basic procedure used here is a continuation scheme, where one iterates to go from an approximate solution to a "nearby" solution. Typically, the approximate solution is exact for some "nearby" set of data; e.g., slightly different source/receiver pair, or slightly different propagation parameters. Such a continuation procedure can break down when there is more than one ray solution between two points, corresponding to multiple branches of the travel time curve. These problems have been largely overcome by combining a shooting scheme and a continuation procedure and automating the synthesis of the two.

This modeling program has several components. First, it can generate a wavelet and its Hilbert transform. Linear combinations of these two can produce a wavelet with an arbitrary constant phase shift. Such combinations are useful to model data that has passed through a caustic, for example.

A second component of the program is the modeling of an earth environment. This is done by passing a cubic spline through the data points defining each interface.

The next stage is a ray plotting program for primary rays only. This program will march through a specified source/receiver set. For example, given a common shot geometry, the program will march through receiver locations calculating ray paths and monitoring take-off angles of the rays. Typically, when there is ray triplication that is skipped by the program, it shows up as an unusually large increment in this take-off angle as compared to previous increments. The program will then return to the previous position and search through smaller increments in take-off angle to detect rays that have passed through a caustic and emerge at the surface at receivers already covered by previous ray trajectories. Figure 11A is an example of a ray family with a caustic.

At the next level, ray paths with single or multiple reflections can be specified. Figure 11B shows an example of single reflections from many layers, while Figure 11C shows an example with multiple reflections.

Finally, the program will produce a record of response from a specified set of propagation paths. Figure 11D is an example of this. Here, there was a single shot below the uppermost surface -- typical of an ocean survey -- and many receivers, over the same earth model of the previous figures. Trace 36 is the zero-offset position. On this trace, the first arrival is a primary reflection from the first reflector, followed by its source ghost reflecting first off the upper surface and then off the first reflector. Next is the primary from reflector two. The next event is the reverberation in layer one, followed by the reflections from reflectors three and four. Clearly, any combination can be built up in this manner.

Modeling with a Hybrid Kirchhoff Ray Method

Some modeling problems lend themselves more naturally to a hybrid approach that combines ray theory with a generalized Kirchhoff approximation applied to the Kirchhoff integral representation of a wave reflected or transmitted from an interface. This is the case, for example, when the shape of a particular interface causes an involved caustic structure in the ray paths, while the direct ray paths between the given surface and source and receiver points do not suffer from problems as severe.

To generate model data, rays are traced down to a prescribed reflector through a specified medium and the ray-theoretic Green's functions necessary for the Kirchhoff approximation are computed on that interface. A sum approximating the Kirchhoff integral is then carried out.

Figures 12A and 12B show the output of a shooting ray scheme for models with velocity lenses and truncated reflectors. Note that the shooting method does not breakdown at caustics.

Preliminary work on this project was reported in Sullivan [1984]. That paper was based on the wave equation datuming method of Berryhill [1979]. Sullivan extended Berryhill's work in two ways. First, he introduced a correct two-and-one-half dimensional amplitude adjustment based on Bleistein [1986]. Second, he developed a hybrid ray-theoretic Kirchhoff technique. This technique allowed him to account for multiple transmission and reflection effects in the theory and in the resulting computer algorithm. His ray tracing technique is partially based on Docherty's [1987].

As an example of synthetic data generated by Sullivan's modeling program, Figure 12C shows backscatter output from a single synclinal reflector. The syncline is sufficiently deep that the rays reflected from it form a caustic below the observation surface -- a buried focus -- and a triplication of responses on the traces near the center of the figure. The short time response near the top of the figure reproduces the source wavelet. The later response, which has passed through the caustic, exhibits the well known phase shifting of such responses and the impulse of the source wavelet has been transformed into a doublet.

Compressional Wave Reflection Coefficient for Acoustic and Elastic Models

A study by another graduate student, Chris Liner, points out the need for ultimately considering elastic wave propagation to obtain meangingful estimates of earth parameters -- even for sound speed and density contrasts. This studies wave reflection from a plane interface, both for acoustic and elastic models of the wave propagation, under the assumption of compression plane wave incidence. The objective was to study the compressional wave reflection coefficient for the two models of wave propagation. The models were normalized so that the reflection coefficients were equal (to .46) at normal incidence and the difference in reflection coefficients were computed.

Figure 13A shows the difference in reflection coefficients for an example in which the compressional wave speed contrast is 2.5x and the Poisson ratio for the two media was .25. One can see that at 10^{o}, less than half the critical angle of 23.6^{o} for this example, the difference in the reflection coefficients is already about 10%. The result with the layers interchanged is shown in Figure 13B.

Figure 13C shows the result for a model with compressional speeds like the first case, but with Poisson's ratio of .45. This is a "soft" solid, nearly acoustic (Poisson ratio of .5). Here, it can be seen that the difference in reflection coefficients is less than 10% beyond the critical angle. At the other extreme, Figure 13D shows the difference in reflection coefficient for the same compressional speed contrast, but a Poisson ration of .10, a "hard" solid. Here, the error is seen to exceed 10% at less than 10^{o}.

This study provides evidence of the need to extend our inversion theory to an elastic model of wave propagation if we are to consider source/receiver configurations of sufficiently wide offset to provide well conditioned data for inversion for more than one parameter.

Ray Theoretic Elastic Modeling

A necessary prerequisite to elastic wave inversion is ray theoretic elastic modeling. In particular, the method requires a ray theoretic Green's dyadic and a generalized Kirchhoff approximation using that Green's dyadic in the integral representation of the upward scattered wave.

The Green's dyadic, in the generality we require, was not available in the open literature. It was derived by Cohen [1987]. Exploiting this result, Sumner was able to write down a Kirchhoff approximation for the upward scattered wave from a single reflector, with a known inhomogeneous elastic medium above the reflector. This latter work will be reported in Sumner's PhD thesis, along with the Kirchhoff elastic wave inversion theory.

Higher Order Inversions

Hagin has begun investigating the feasibility of using the pseudo-differential operator (PDO) theory to generate more accurate inversion schemes. In Beylkin [1985], it was demonstrated that the PDO theory provides a framework for the formulating of asymptotic inversions. In this and later papers by Beylkin and CWP researchers, only the first term of the asymptotic expansion of the inverse operator was used, and only the first term of the WKBJ expansion of the Green's function making up the original integral equation was used. If more terms (e.g., at least two) are used consistently, it is not known if the resulting inversion algorithm will be superior in any measurable way. Moreover, given that in most real experiments the measured data is band limited (in particular the lower frequencies muted) it is not clear whether or not such higher order algorithms can provide useful information.

Investigations on these questions are being done with Bruce Zuver (PhD candidate, University of Denver). Second order (in high frequency) inversion schemes are being studied. Although this investigation is preliminary, some interesting observations can already be made. First, second order schemes are complicated to the extent that, in practice, some ingenuity or careful numerical approximations must be applied in order to use them in a practical problem. Second, the "canonical" problem (constant reference speed and back-scatter experiment) does lead to a relatively simple second order inversion algorithm which could be implemented. Moreover, this analysis provides some useful insights into the whole process of asymptotic inversions in this setting. For example, a pattern is established that relates the order of asymptotic expansion to the local behavior of perturbations due to a reflecting surface. This information could be used, e.g., to improve the accuracy of first order algorithms and will be of use ultimately in deriving a new generation higher order inversions.

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Ocean Sound Speed Profile Inversion

Background

In temperate deep waters the main characteristic of the sound speed in the ocean is a minimum value at what is termed the channel axis. Above this value the sound speed increases mainly due to temperature, and near the ocean surface the value is strongly seasonal dependent. Below this value the sound speed increases almost monotonically due to its dependence on hydrostatic pressure. This sound speed structure results in the channeling of acoustic energy in the ocean and enables it to propagate for thousands of miles. Some simple examples are illustrated in Fig. 15 [Kaczkowski, 1986]. A further discussion can be found in DeSanto [1979, 1985].

The inverse problem arises in our case when the results of acoustic measurements are used to predict the sound speed. Since the major variability of the sound speed is in depth (with a smaller variability in range), any inversion algorithm must start with a guess (or background) sound speed which depends on depth. If for example we chose a constant background sound speed, the resulting ray paths would be straight lines, and the propagated acoustic energy would not exhibit characteristic convergence effects. To a convenient background sound speed is a profile consisting of two linear segments (bilinear profile). An example is illustrated in Fig. 16 with an associated ray trace (illustrating refraction) in Fig. 17 [Kaczkowski, 1986]. The latter also illustrates a key difference between our problem and the seismic prospecting methods discussed above. It is that the direction of propagation of the acoustic energy (range) is orthogonal to the direction of profile variability (depth) for ocean problems.

We have treated two approaches to this problem. The first is called the Fourier inversion method [DeSanto, 1984; Boden, 1985; Boden and DeSanto, 1986]. In this method we derive a linear transformation between the scattered data and a correction to the sound speed profile based on an assumed (guess) background profile. Using approximation methods, the kernel of the transform can be evaluated to yield a form whereby the transformation becomes a Fourier transform. This is then inverted to find the profile correction. The Fourier inversion method is thus a direct inversion method, and using it the profile is recovered as a function of depth. The data is sampled at a single receiver as a function of frequency (or wavenumber).

The second approach is based on the method of damped least squares [Kaczkowski, 1986]. It is indirect in the sense that it uses the iteration of forward propagation models with assumed profiles to zero in on the correct profile. The profile is recovered here not as an explicit function of depth but rather in terms of its parameter values (e.g. upper and lower slope, depth of the channel axis, etc.). Here the data is sampled as a function of depth (vertical array) or as a function of range (horizontally).

Fourier Inversion

The theoretical results on this problem have been summarized in other publications [DeSanto, 1984; Boden, 1985; Boden and DeSanto, 1986]. We do briefly summarize the ideas below in order to put the progress in context.

The problem is to infer the intervening sound speed profile from propagated data at a single receiver as a function of frequency or wavenumber (k). Briefly, the method is finite-band frequency inversion. The analysis begins with the Bessel transform representation of the acoustic field as an integral over scaled horizontal wavenumber and is taken as the perturbation parameter or correction. Subtracting the "incident" field (for which we have a specific functional form in this inhomogeneous region) yields an integral transform relation between the scattered field data (as a function of k) and the correction for the refraction index. Asymptotic evaluation of the transform kernel permits us to interpret the relation as a quasi-Fourier pair. "Quasi" refers to the fact that the phase contains the WKB phase terms of our Green's functions, one for propagation from source to intermediate state and the second from the intermediate state to the receiver. We integrate over the intermediate states. The phase is not linear but can be made monotonic at large enough ranges (typically beyond 10 km) and this is the key to the Fourier-like inversion which follows.

The method is used to generate data as a forward propagation algorithm using the true (or known) sound speed or refraction index profile. To do the inversion, first guess a background (and depth-dependent) profile, and integrate over the data bandwidth to produce the correction. Some examples of the data and an inversion in the upper ocean region for a small profile anomaly are presented in Figs. 18-21. In Figs. 18 and 19 we illustrate the magnitude of the scattered field data as a function of wavenumber (frequency). This illustrates the slight spectral differences which occur when a small profile bump or anomaly occurs superimposed on an upper ocean linear profile. The latter is illustrated in the top pictures in Figs. 20 and 21 for the index of refraction (inverse of sound speed). In the middle illustrations in these latter figures are indicated the guesses for the index of refraction and its slope. The latter is more illustrative in that the profile slope stands out more clearly. In the lower pictures, the recovered values of refraction index and slope are illustrated. Note that the inversion is nearly a perfect noise-free reconstruction of a small profile anomaly. This is an update of the inversion in Boden [1985] using a recently developed integration routine to directly reconstruct the profile from the data considered as a straight Fourier transform in the WKB phase.

The status of the theory and computational implementation is as follows. Aside from the basic theoretical development to generate the integral transform relation between data and the profile correction, there is an additional development involving asymptotics on the transform kernel K. Depending on the geometry of the source/receiver arrangement and the profile (one turning point at the channel axis) we have several regions in which to do the asymptotics on K, and each gives a different functional value for K. Our example in the figures was for an inversion in the upper ocean, above the receiver depth. This is the region of greatest profile variability. The example has the receiver above the source and both above the channel axis. An analogous inversion could be obtained in the lower ocean, between the reciprocal depth of the receiver and the bottom. For a source and receiver on the channel axis these are thus the only two regions and reconstruction is complete. But for the geometry in our examples it is not possible to invert in the mid-ocean regions because of the lack of structure in the data. An example of the data between source and receiver is illustrated in Boden [1985]. We are presently considering alternative asymptotics in these regions. There are several turning depths encountered in the mid-regions and this substantially complicates the analysis.

The scientific question involved in the profile inversion problem is whether the sound speed profile can be recovered from propagated acoustic field measurements. We have demonstrated that we can do this in the upper and lower regions of the ocean in a self-consistent manner. That is, we use our algorithm as a transform pair, first as a direct algorithm to generate the data using the true profile, then as an inverse algorithm to predict the true profile using the data and a profile guess. A second question is what is the best set of data to invert? Our algorithm contains data parametrically as a function of frequency, range, source depth and receiver depth. The simplest inversion, because it can be treated as a quasi-Fourier transform, is in frequency. That is: an inversion using a single source and single receiver at fixed range for a set of frequencies. The best inversion using this method seems to be for a center frequency of about 50-60 Hz and large bandwidth. Put simply as a counting problem, the inversion for a vector of sound speed values requires a data vector as a function of frequency. Another data vector for a single frequency as a function of receiver range (horizontal array) can also be treated as a quasi-Fourier transform. Inversions using depth- and range-dependent data are discussed in the damped least squares section below.

Damped Least Squares Inversion

This section is based on the work of student, P. Kaczkowski [1986]. This is a different approach to the ocean sound speed profile inversion problem than the Fourier method. The latter was a direct inversion method whereas this method uses an iteration of forward propagation models. The Fourier method was a transform relation between the scattered data as a function of frequency to recover the profile in functional form as an explicit function of depth. (Actually the transformation in the Fourier method expressed the data as a function of either frquency, depth or range. The frequency inversion was the one which could be treated using Fourier methods). Here the method uses as data the propagated acoustic field as either a function of depth (vertical array) or range (horizontal array). The method per se does not depend on the type of data used, but the only propagation codes which were developed were for single frequency and the number of data points in frequency necessary to apply the method required a prohibitively large number of computer runs. Finally, the profile here is recovered parametrically. It is parameterized in terms of simple algebraic functions, for example a linear segment. What is recovered in this case is the slope value. The two parameterizations used were for a bilinear profile and a Munk profile. The parameters recovered for the bilinear profile were the upper and lower slopes, the depth of the channel axis, and the sound speed at this depth. For the Munk profile the parameters were the axis speed and depth, the channel width (a measure of spread of this parabola-like profile) and the hydrostatic gradient.

The procedure is as follows. The value of the propagated field as a function of depth or range is related to source and medium parameters using some model. Here we chose both ray theory and the parabolic equation as our direct propagation models. The true data is generated using these models and the true source and medium parameters. One then guesses the source distribution and medium geometry and compares the field generated using them to the true field. Using successive iterations these source and medium estimates are improved until a good match is obtained. There is an efficient way to automate this using the Marquardt-Levenberg algorithm, which is linearized. The squared error between observed and true data is minimized and an algorithm to correct the guess is derived. The method involves a matrix inversion and if the latter is singular, a damping parameter is introduced, and an optimal value of this can be found. Alternatively a singular value decomposition, the matrix, is introduced. The damped least squares algorithm relies on finding successive parameter values which lead :o better fits of the mean squared error (MSE).

An example of synthetic range- and depth-dependent data (transmission loss) used in the inversions is illustrated in Fig. 22. Idealized error curves using the parabolic equation program to generate the transmission loss are illustrated in Fig. 23. Both the mean squared error (MSE) and its differential value are illustrated. Real error curves using range- and depth-dependent transmission loss are illustrated in Figs. 24 and 25, and differentiated error curves in Figs. 26 and 27.

The general results for this method are as follows. Depth-dependent data generally gave better solutions. This was important in that it gave an indication of how to sample a realistic ocean problem in order to acquire the best data set for an inversion. Second, one-parameter inversions worked better than two-parameters inversions, etc. The algorithm was less robust the more parameters to invert and the poorer the guess. An example of a successful one-parameter inversion (slope) is illustrated in Fig. 28. Each frame illustrates the data to be fit (+) with the model output for the current parameter value. The lower plots marked error is the difference between data and current model output. Other examples of both successful and unsuccessful inversions and possible methods to improve the latter are contained in Kaczkowski (1986).

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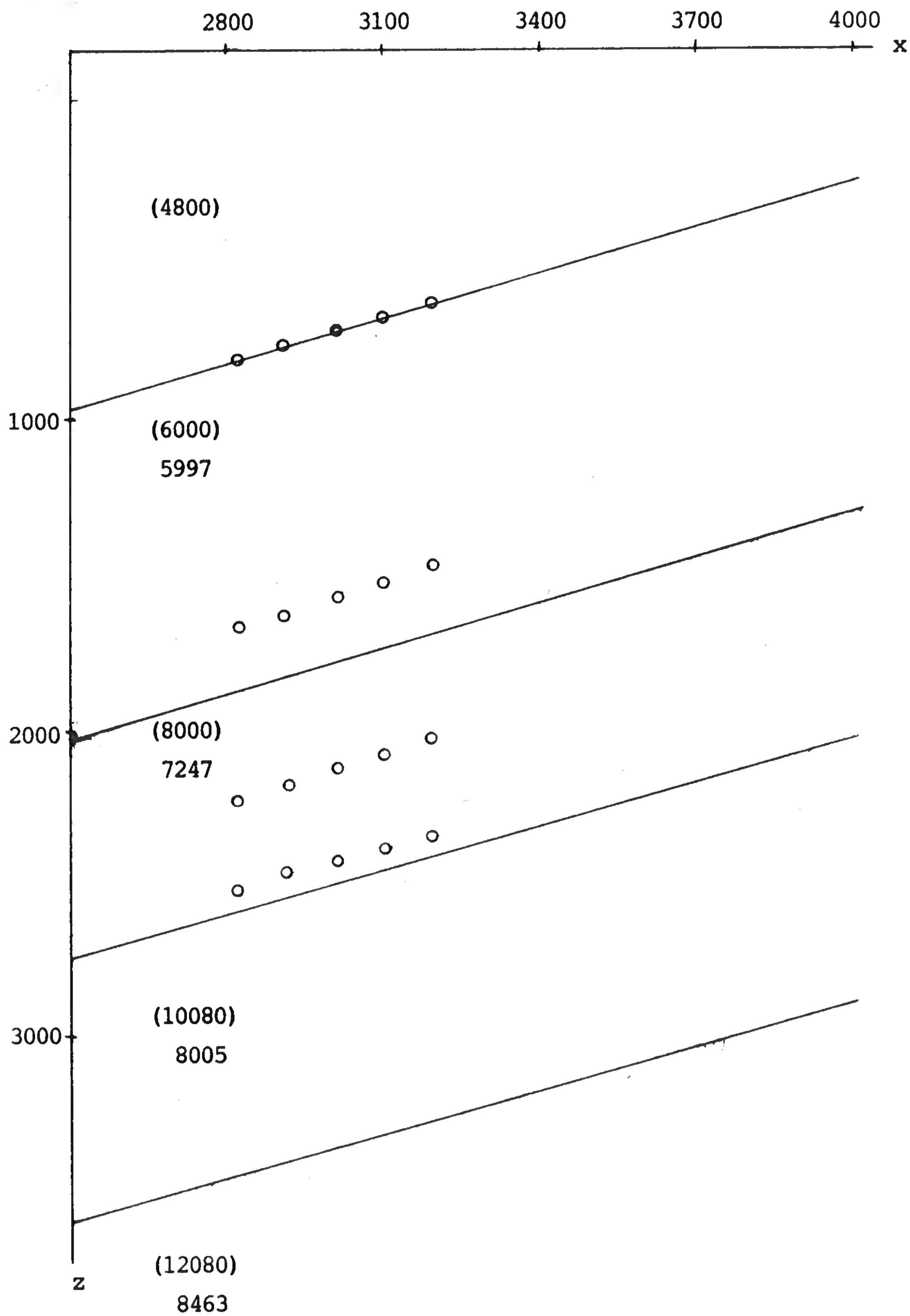
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4000
x
(4800)
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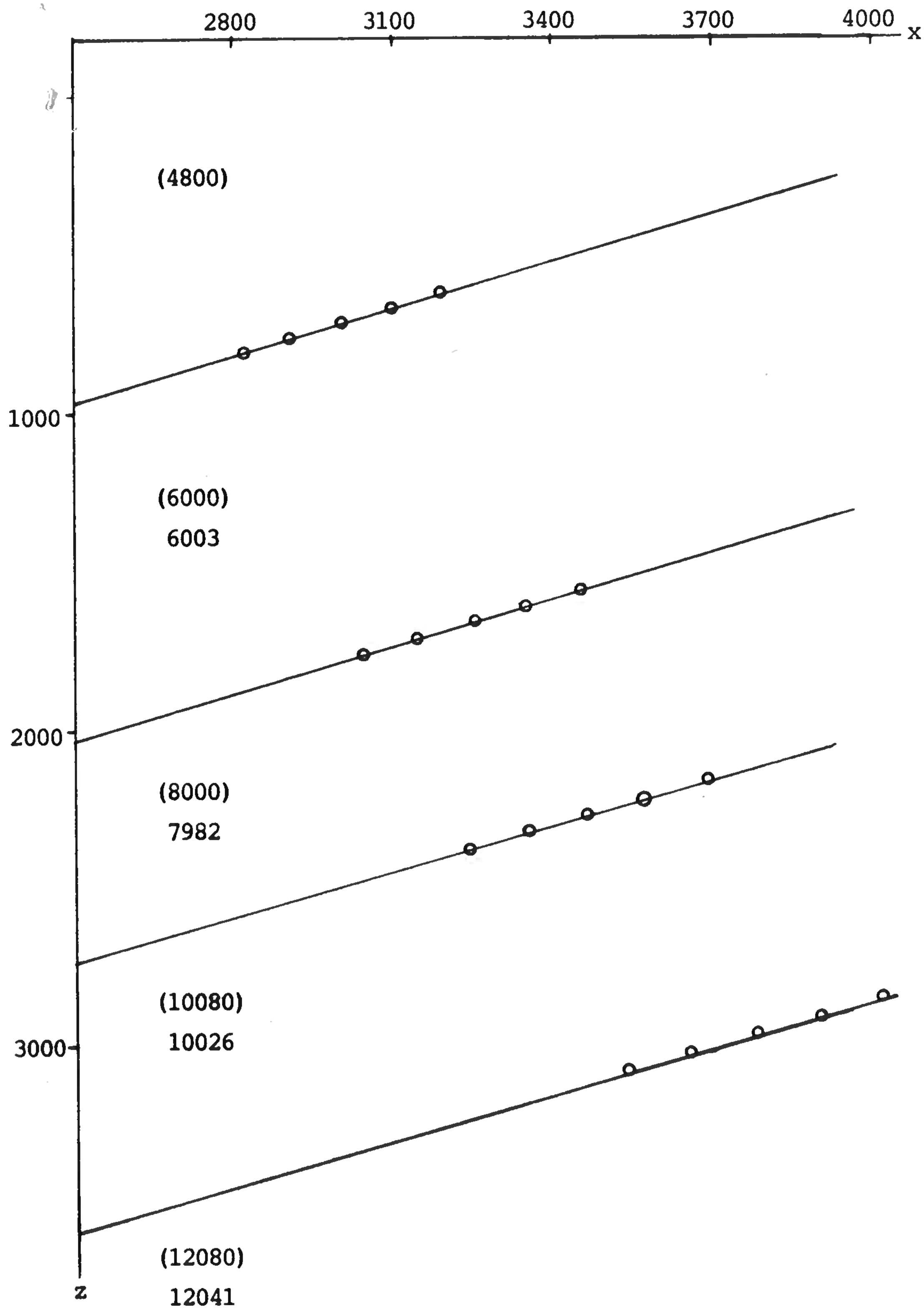
8005
z
(12080)
8463

FIGURE 1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z

FIGURE 2

Geologic Model

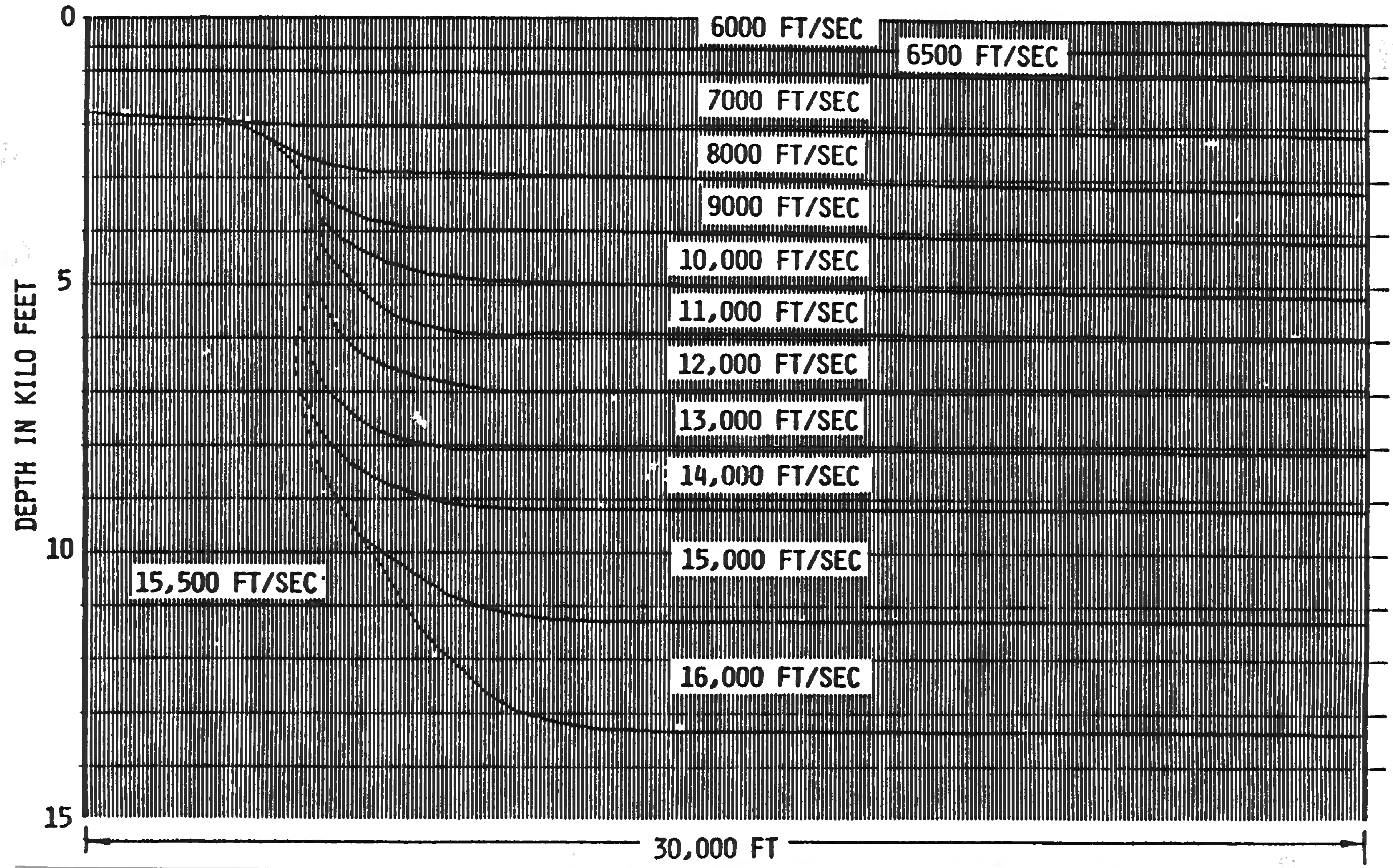


FIGURE 3A

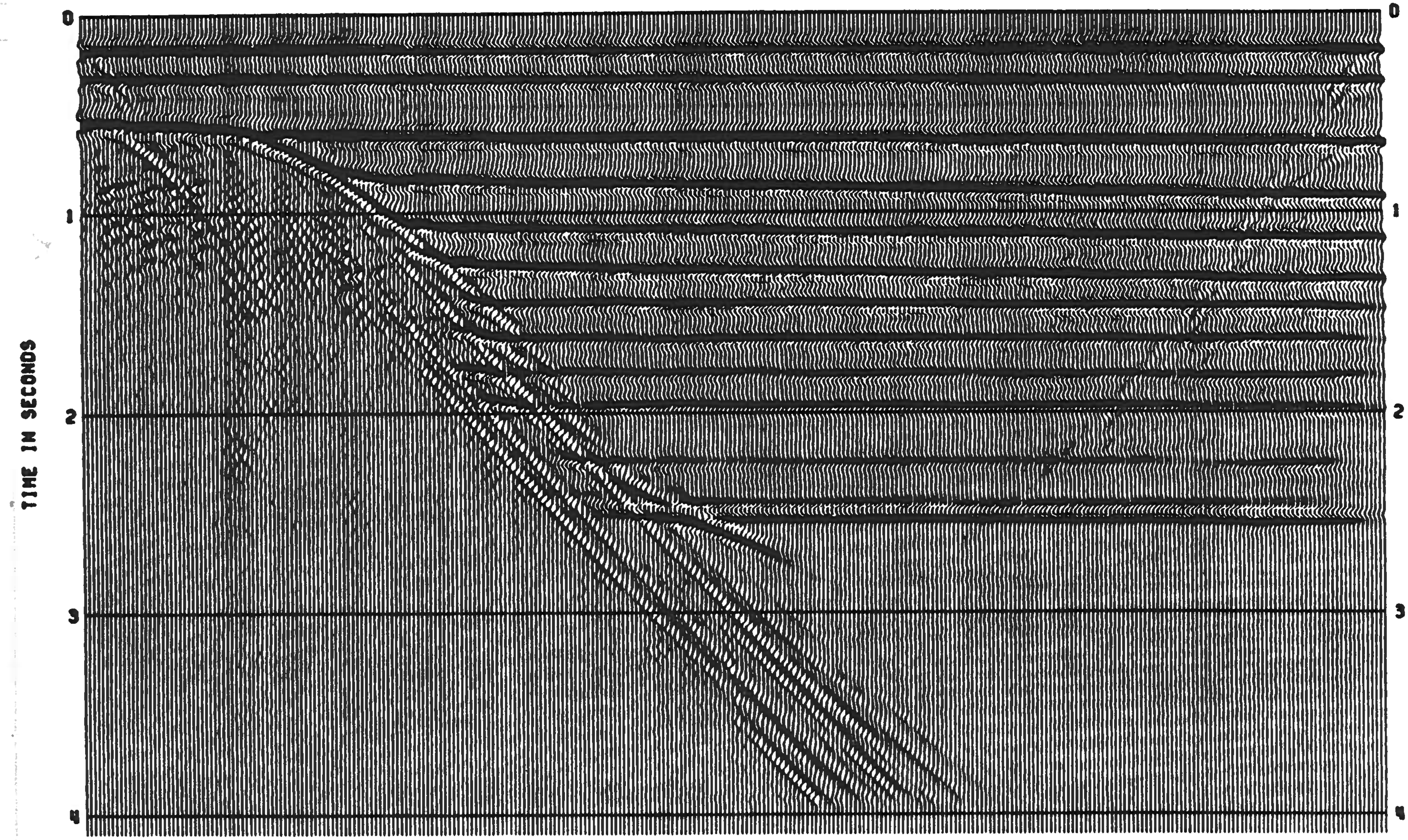


FIGURE 3B

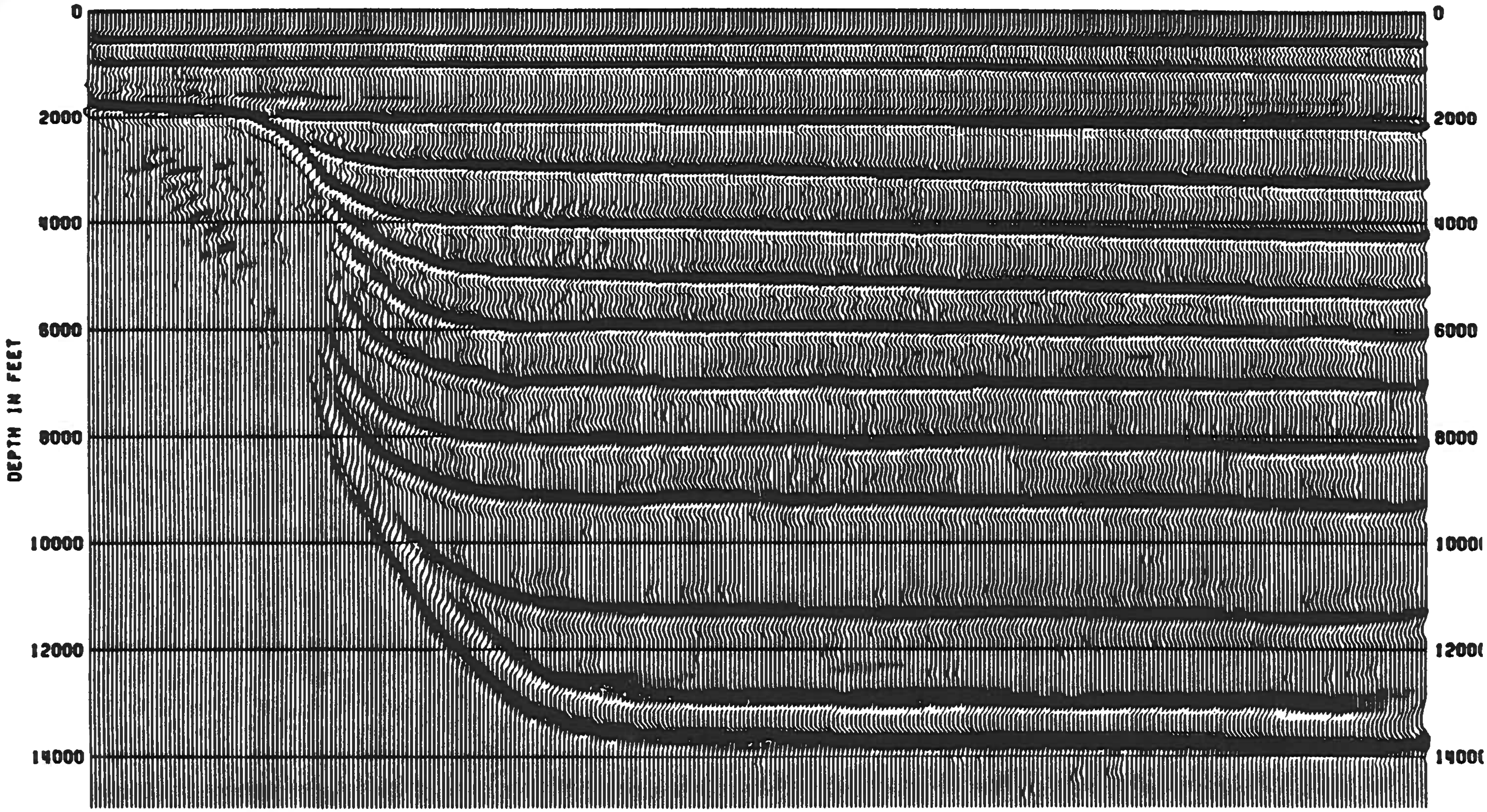
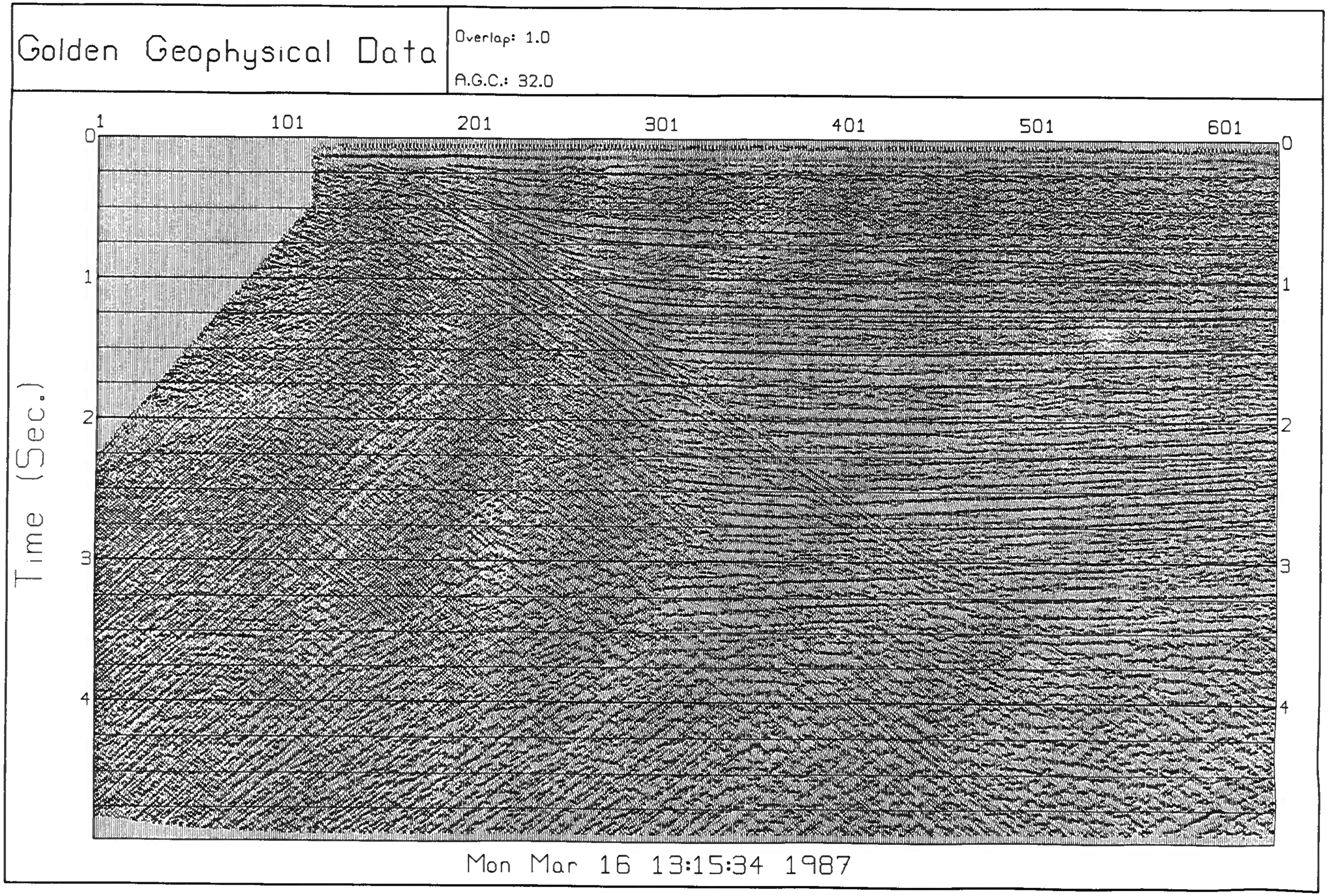


FIGURE 3c



Golden Geophysical Data
Overlap: 1.0
A.G.C.: 32.0
1
101
201
301
401
501
601
0
1
2
3
4
Time (Sec.)
Mon Mar 16 13:15:34 1987

FIGURE 4A

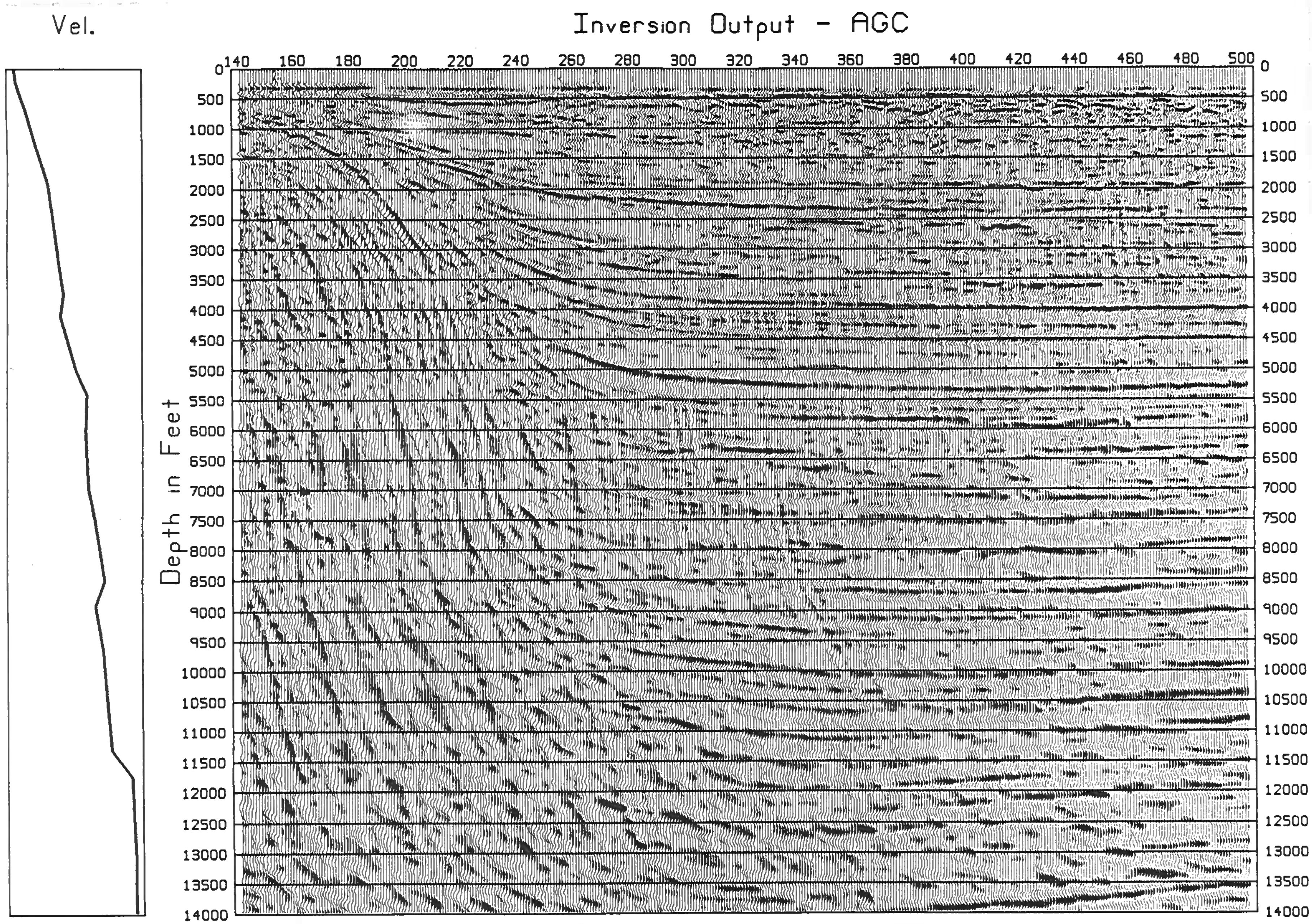

Vel.
Inversion Output - AGC
Depth in Feet
140
160
180
200
220
240
260
280
300
320
340
360
380
400
420
440
460
480
500
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
9500
10000
10500
11000
11500
12000
12500
13000
13500
14000

FIGURE 4B

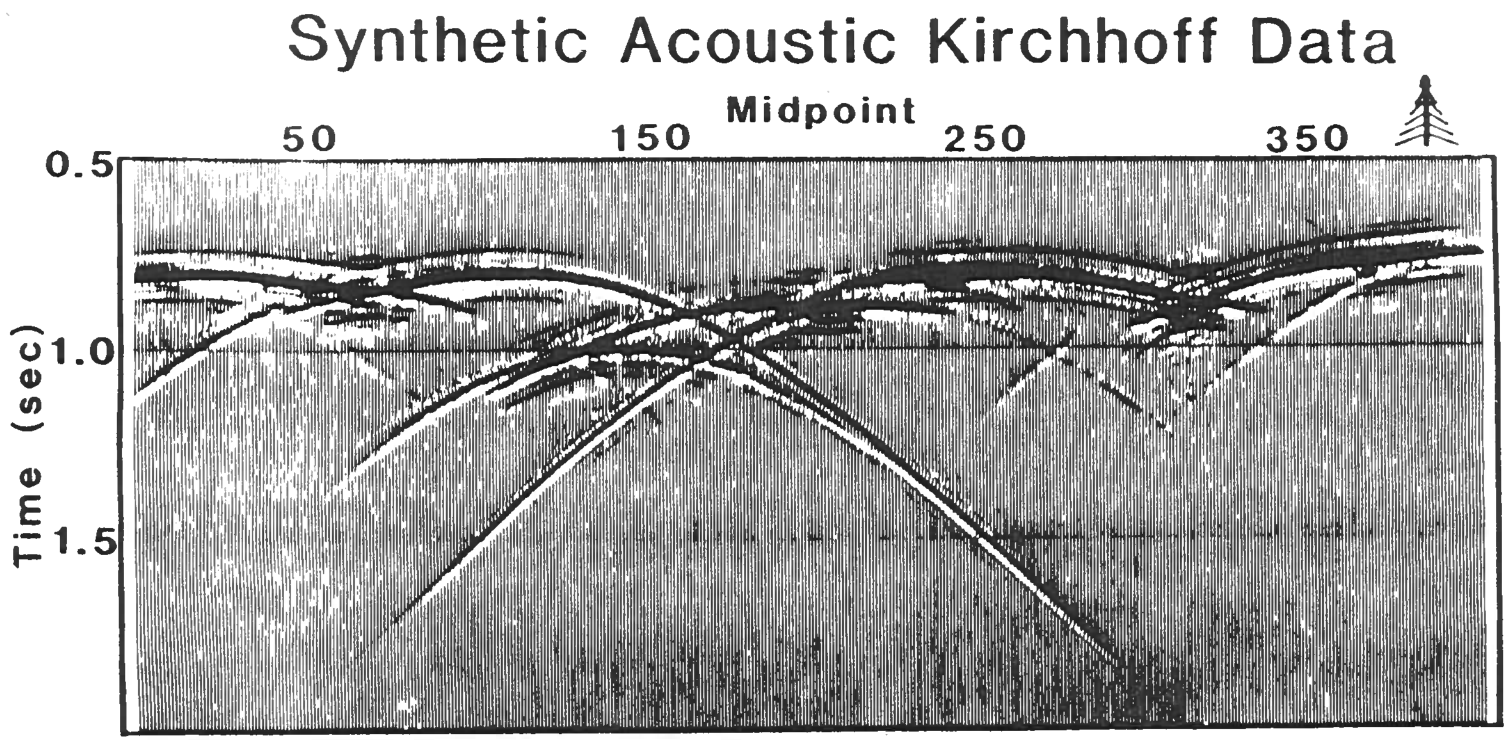

Synthetic Acoustic Kirchhoff Data
Midpoint
50
150
250
350
0.5
1.0
1.5
Time (sec)

FIGURE 5A

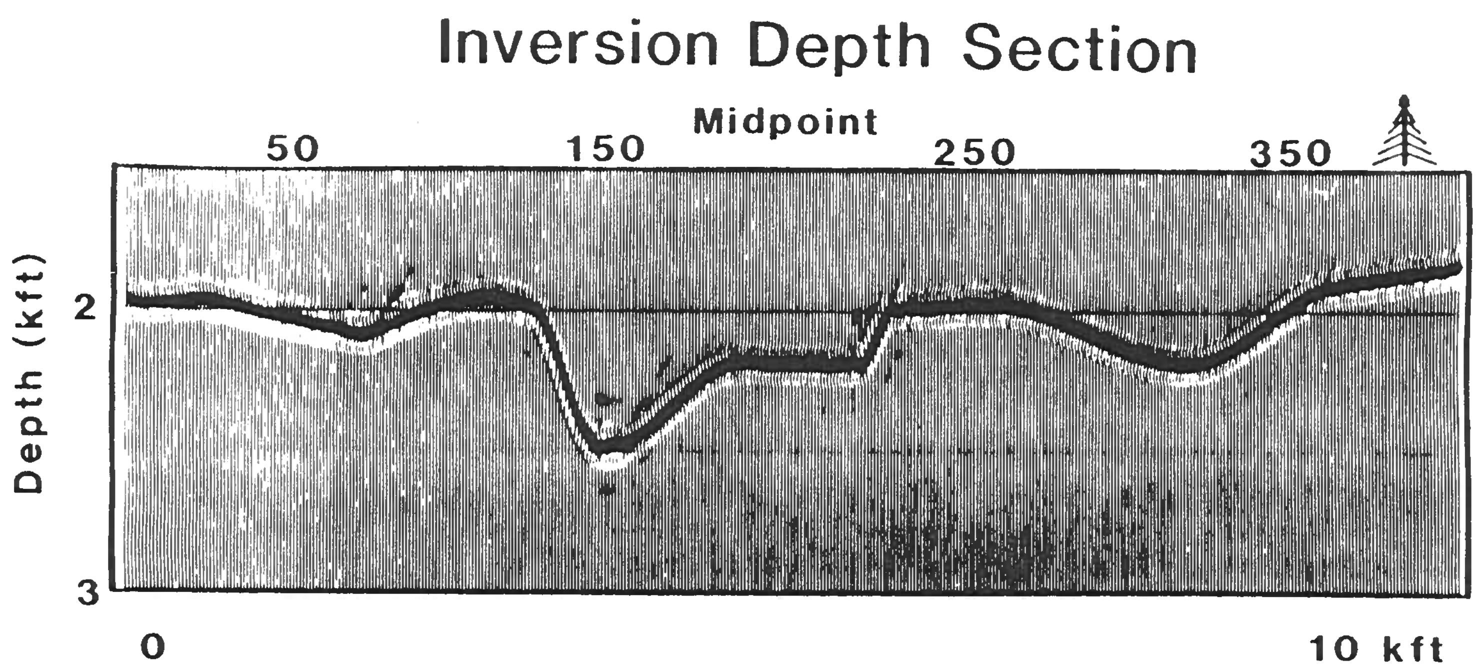

Inversion Depth Section
Midpoint
50
150
250
350
Depth (kft)
2
3
0
10 kft

FIGURE 5B

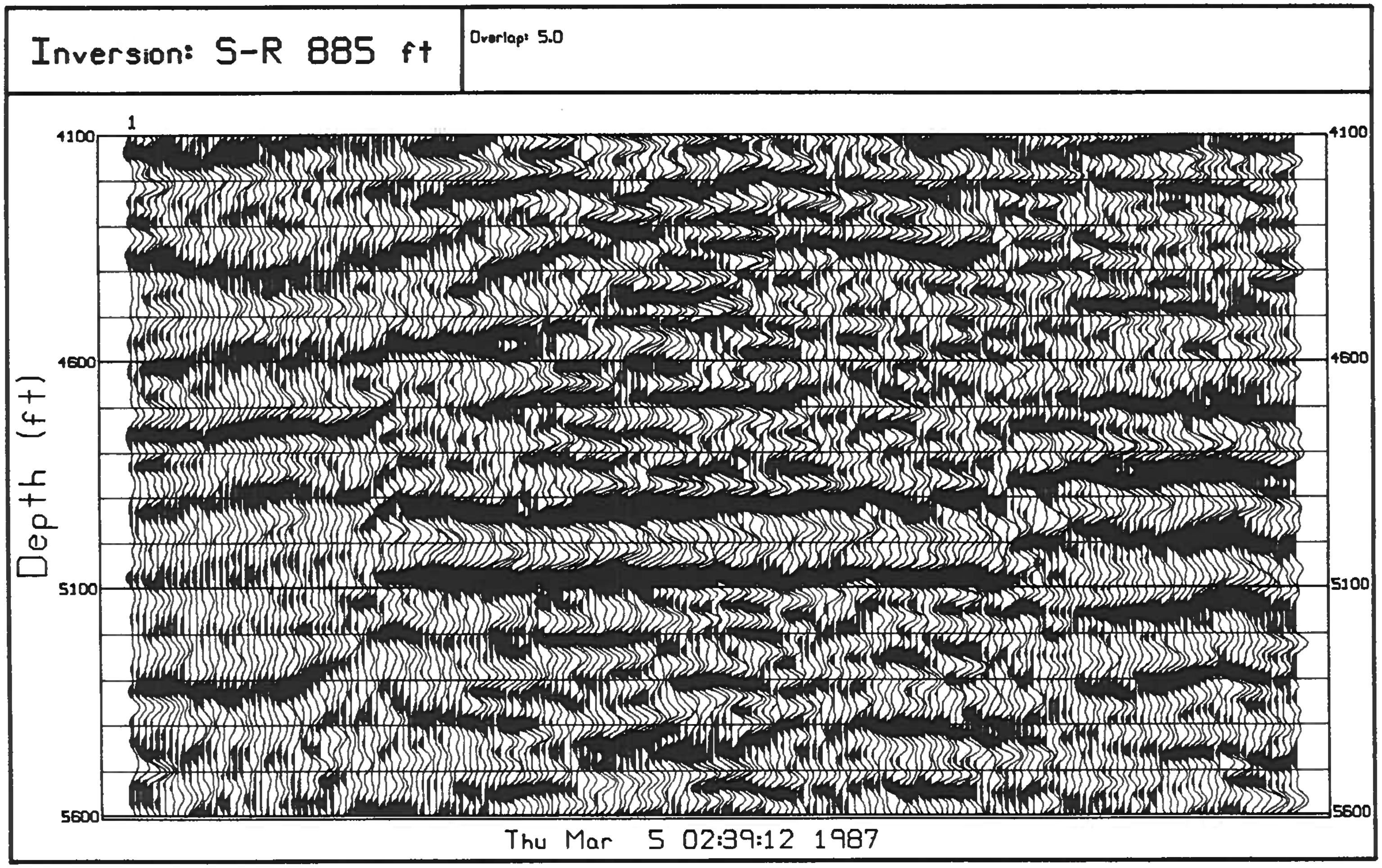

Inversion: S-R 885 ft
Overlap: 5.0
1
4100
4600
5100
5600
Depth (ft)
Thu Mar 5 02:39:12 1987

FIGURE 6A

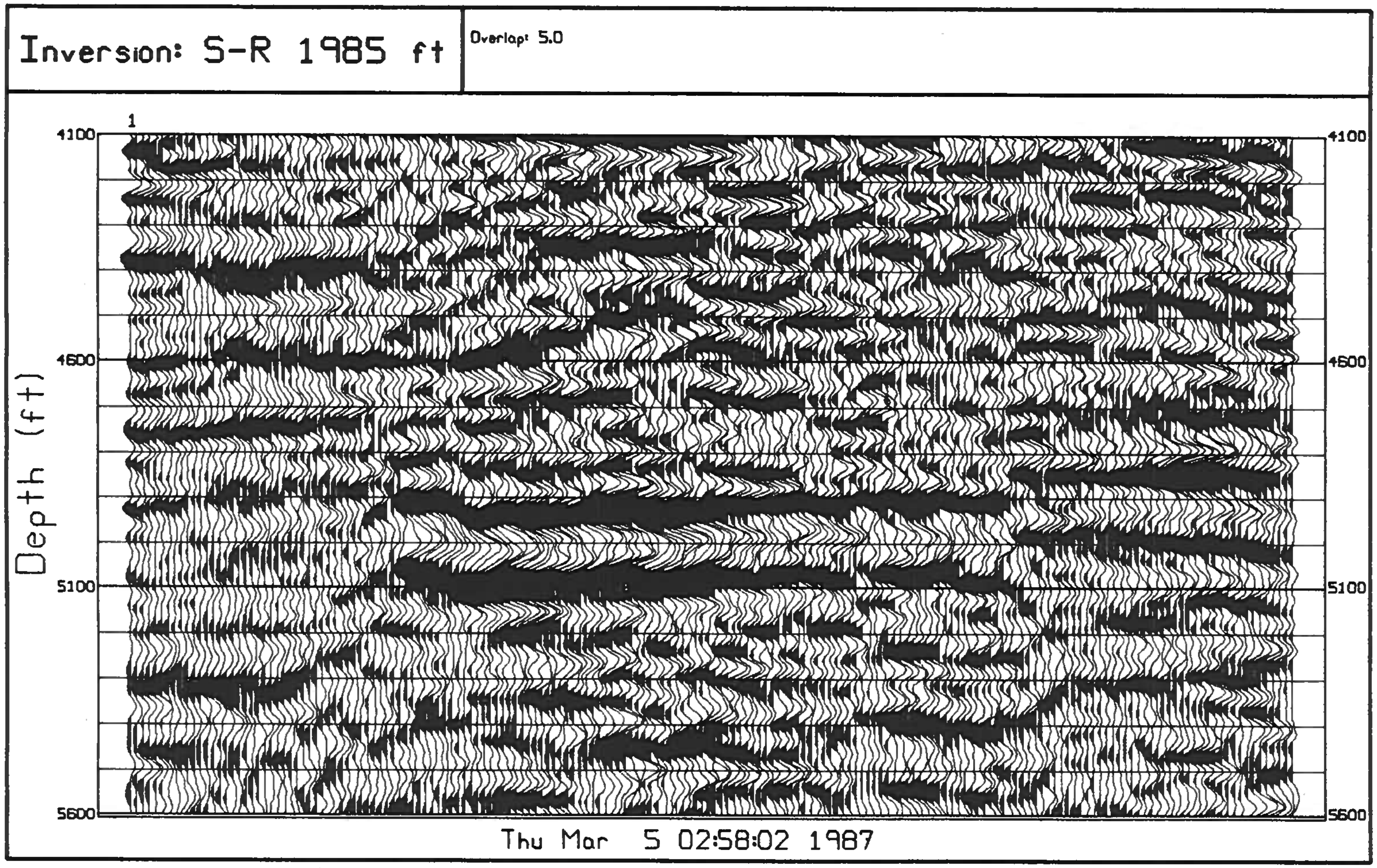

Inversion: S-R 1985 ft
Overlap: 5.0
1
4100
4600
5100
5600
Depth (ft)
4100
4600
5100
5600
Thu Mar 5 02:58:02 1987

FIGURE 6B

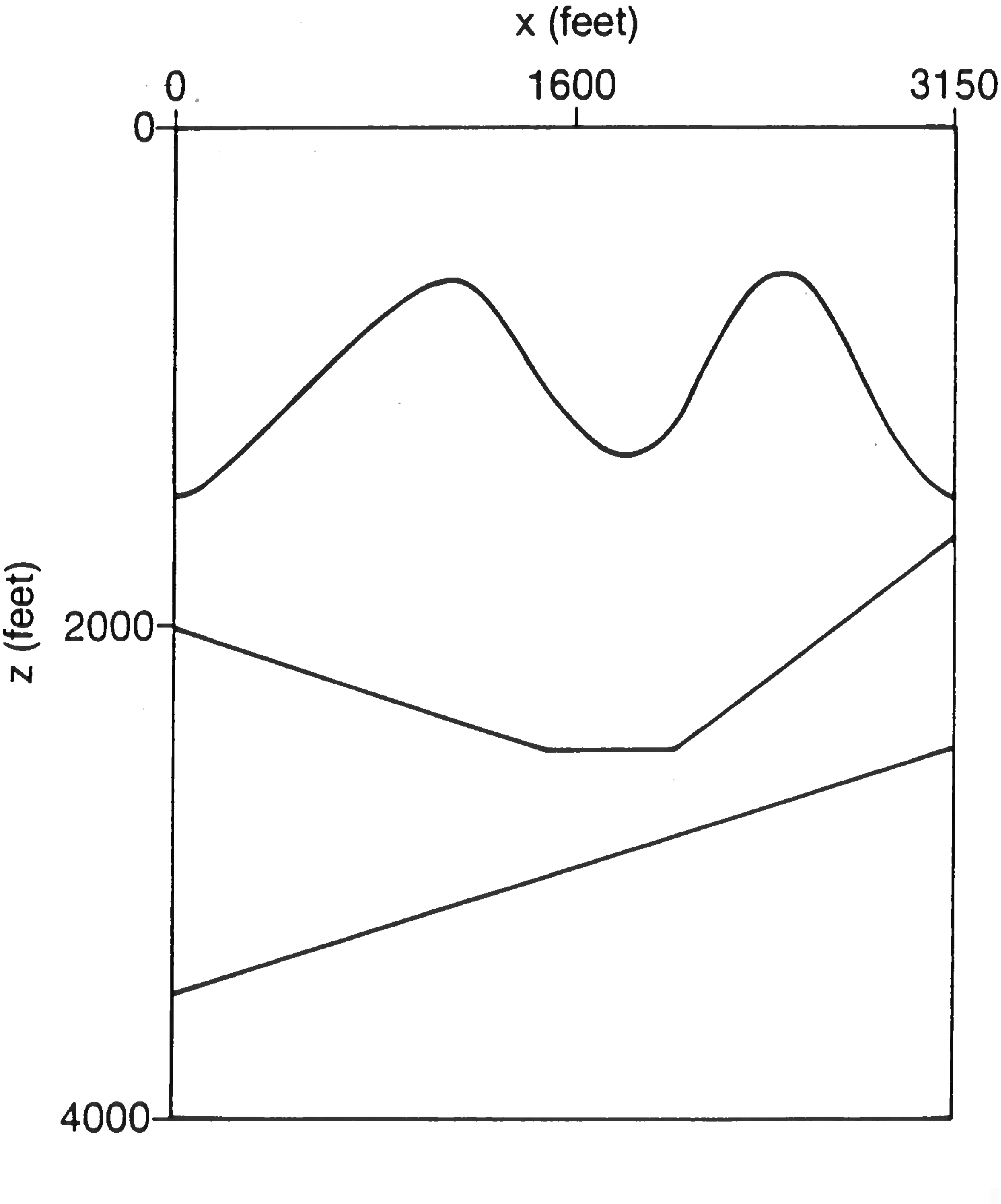


FIGURE 7A

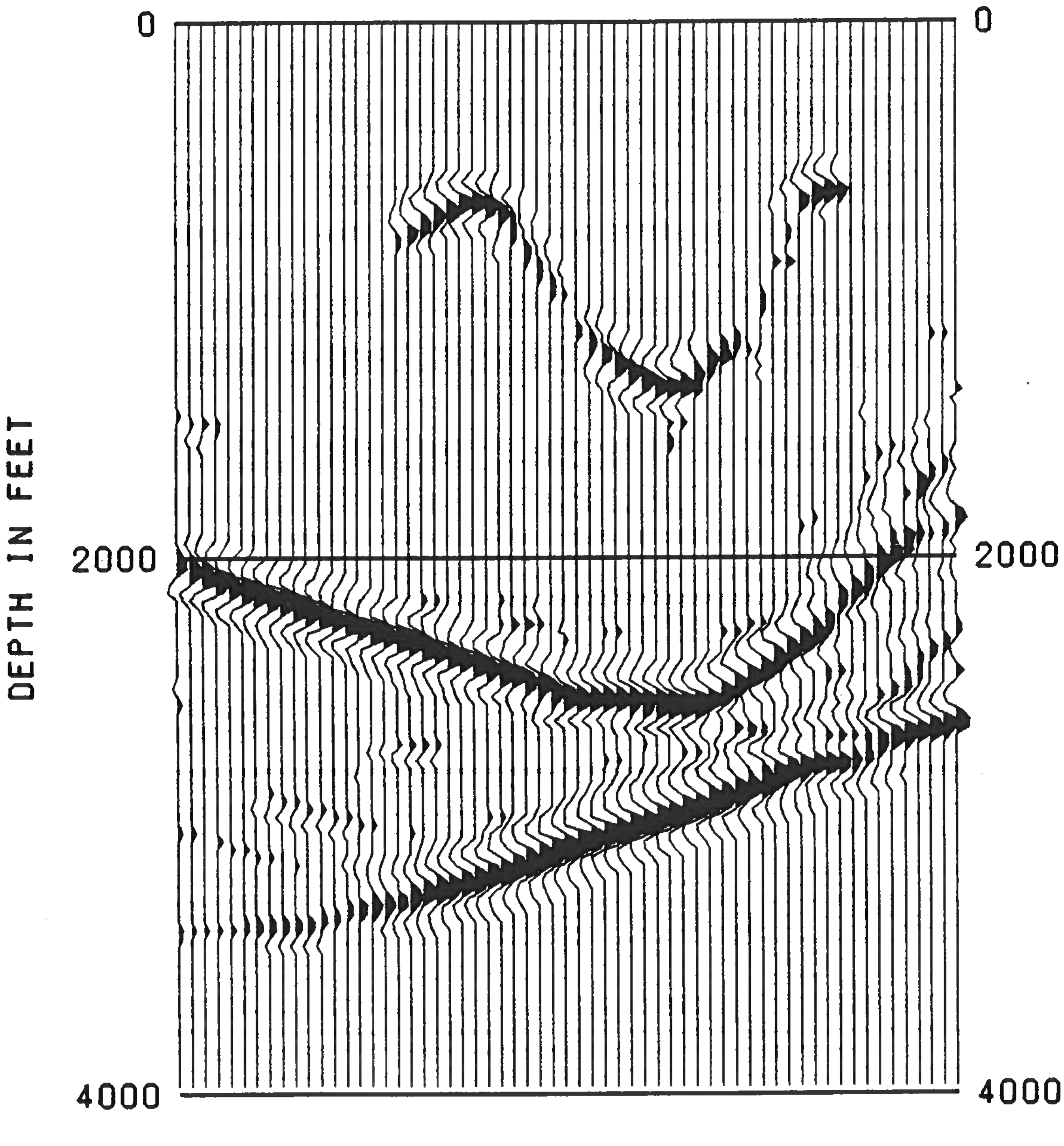


FIGURE 7B

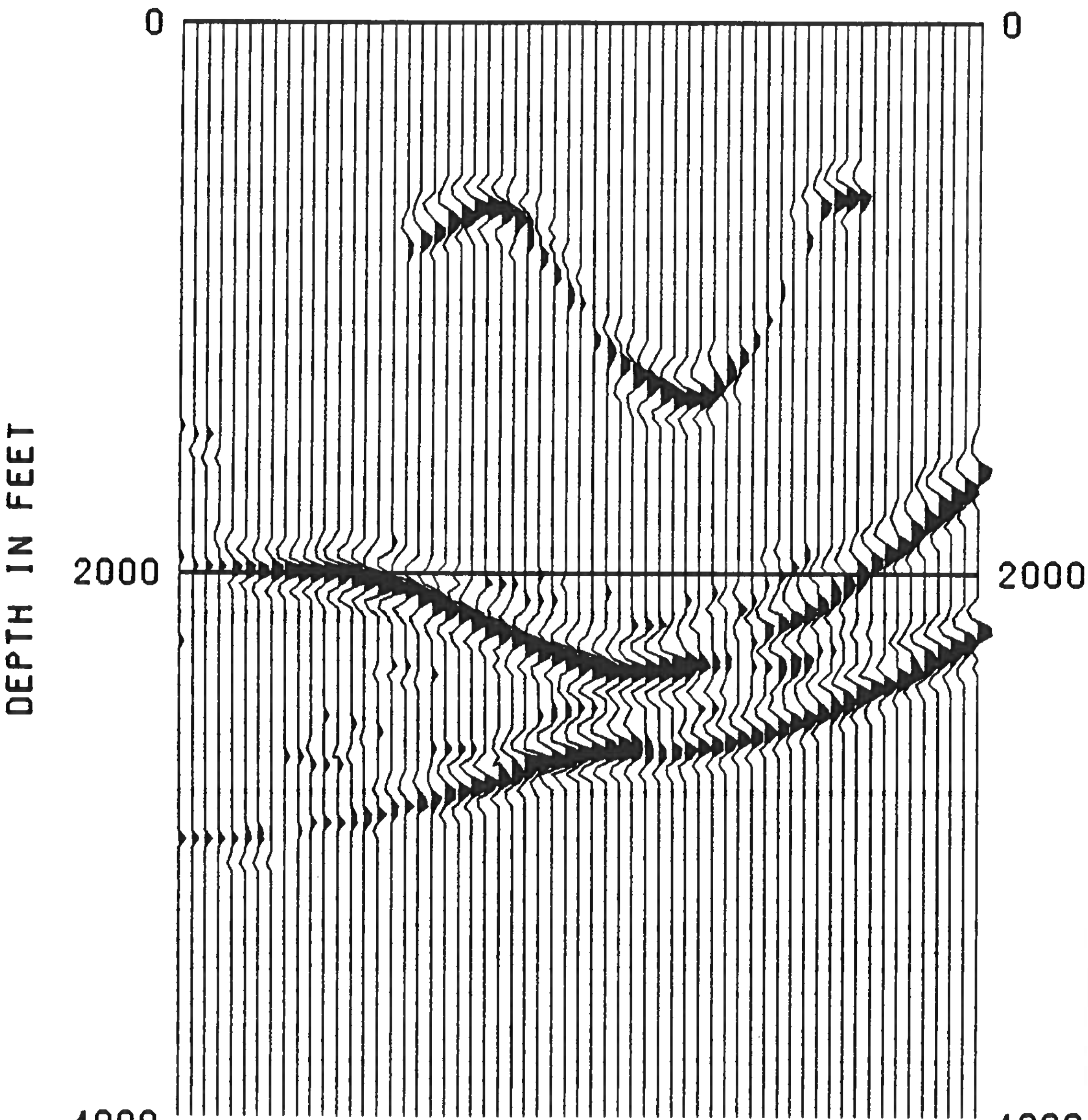


FIGURE 7c

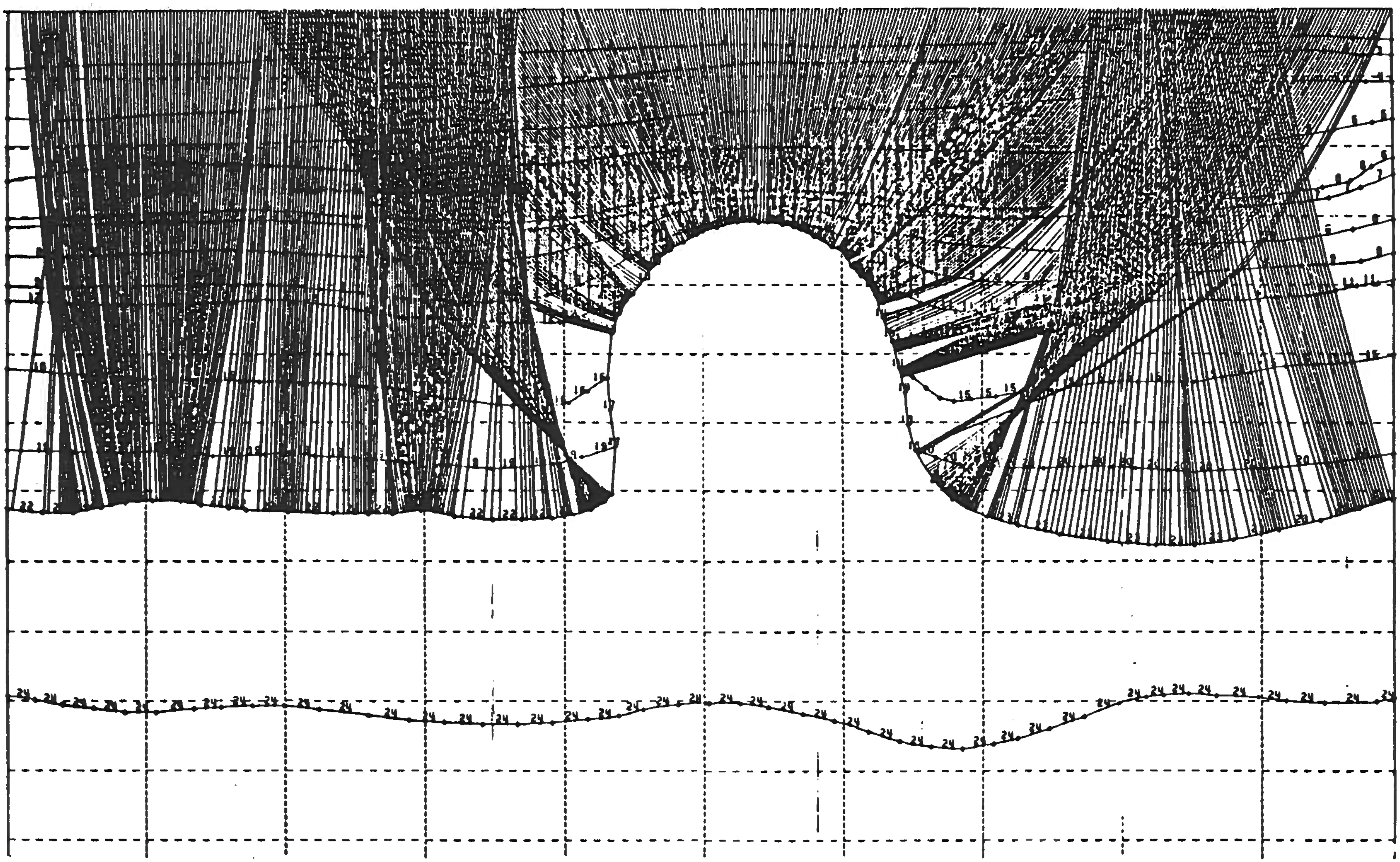


FIGURE 8A

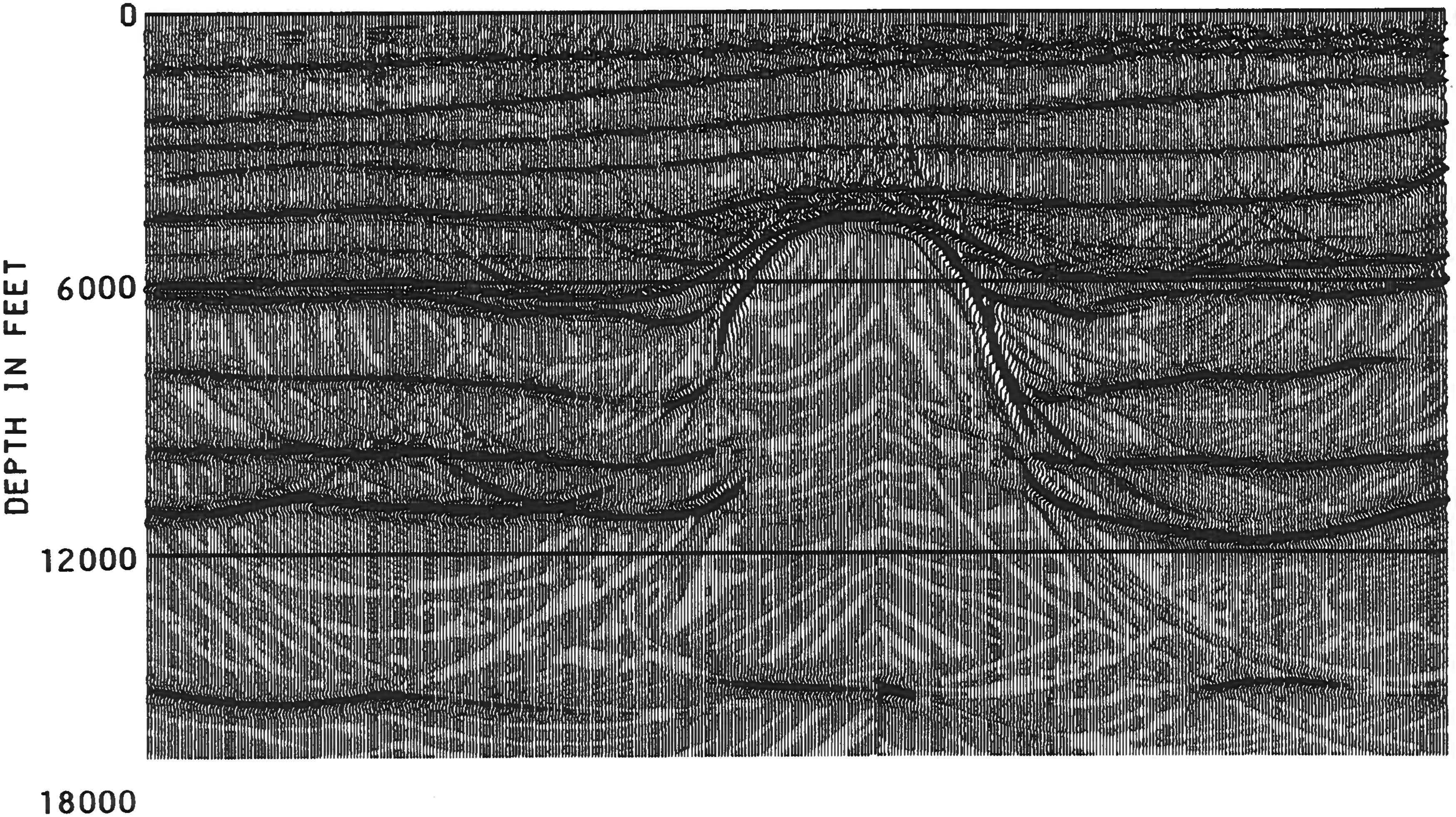


FIGURE 8B

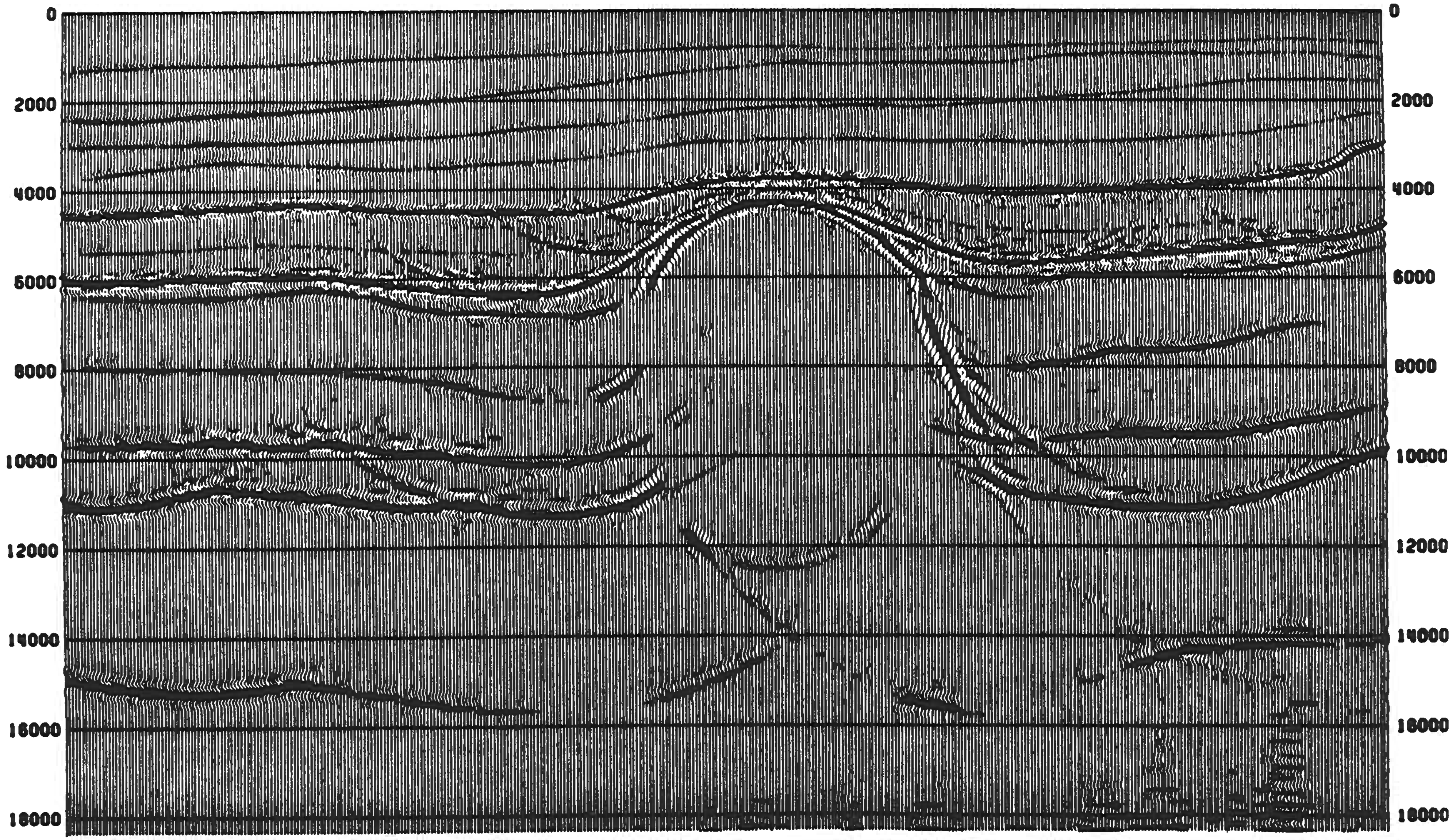


FIGURE 8C

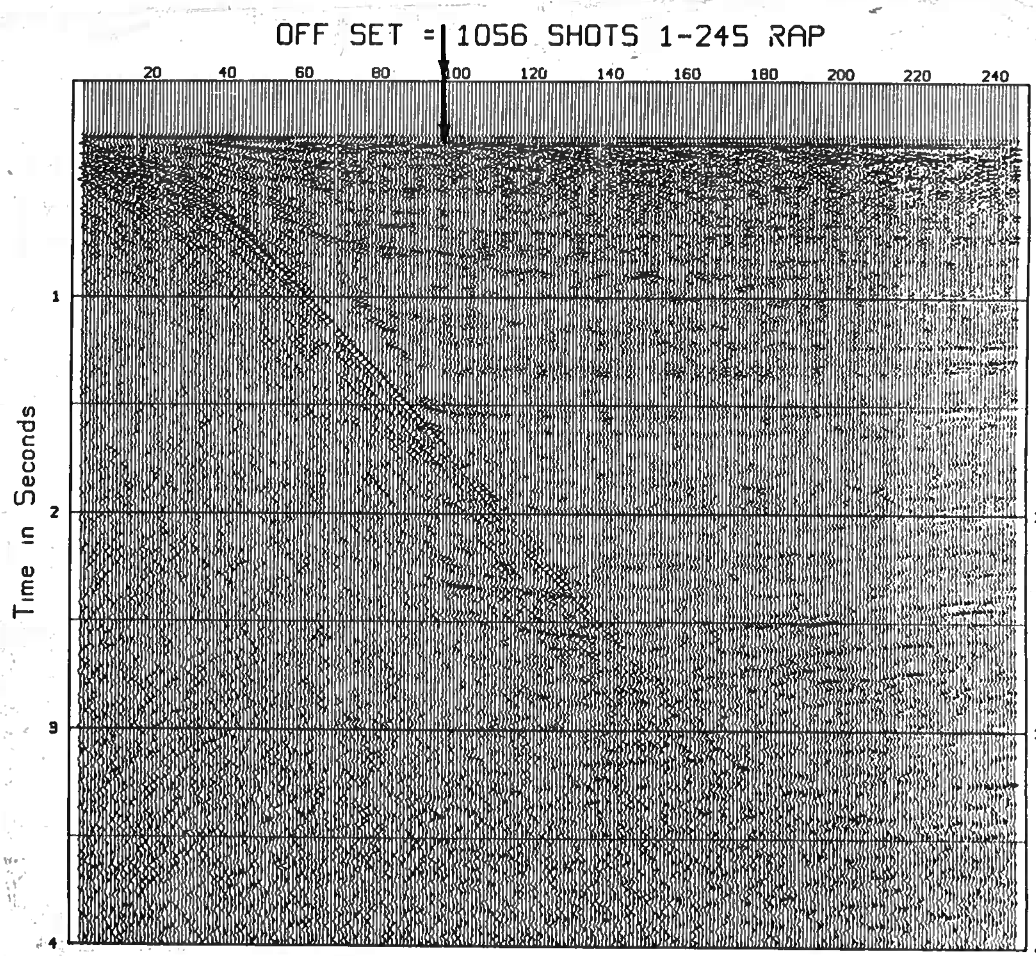

OFF SET = 1056 SHOTS 1-245 RAP
20
40
60
80
100
120
140
160
180
200
220
240
Time in Seconds
1
2
3
4

FIGURE 9A

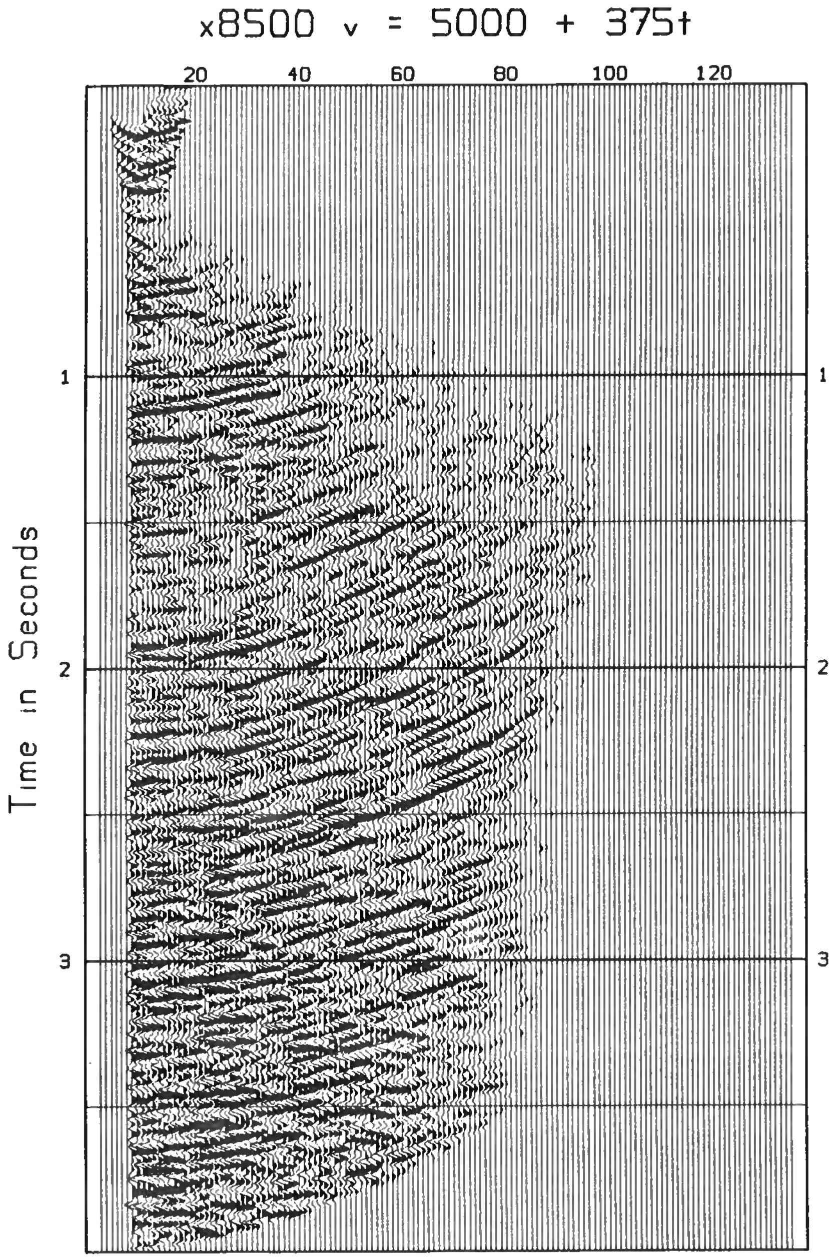
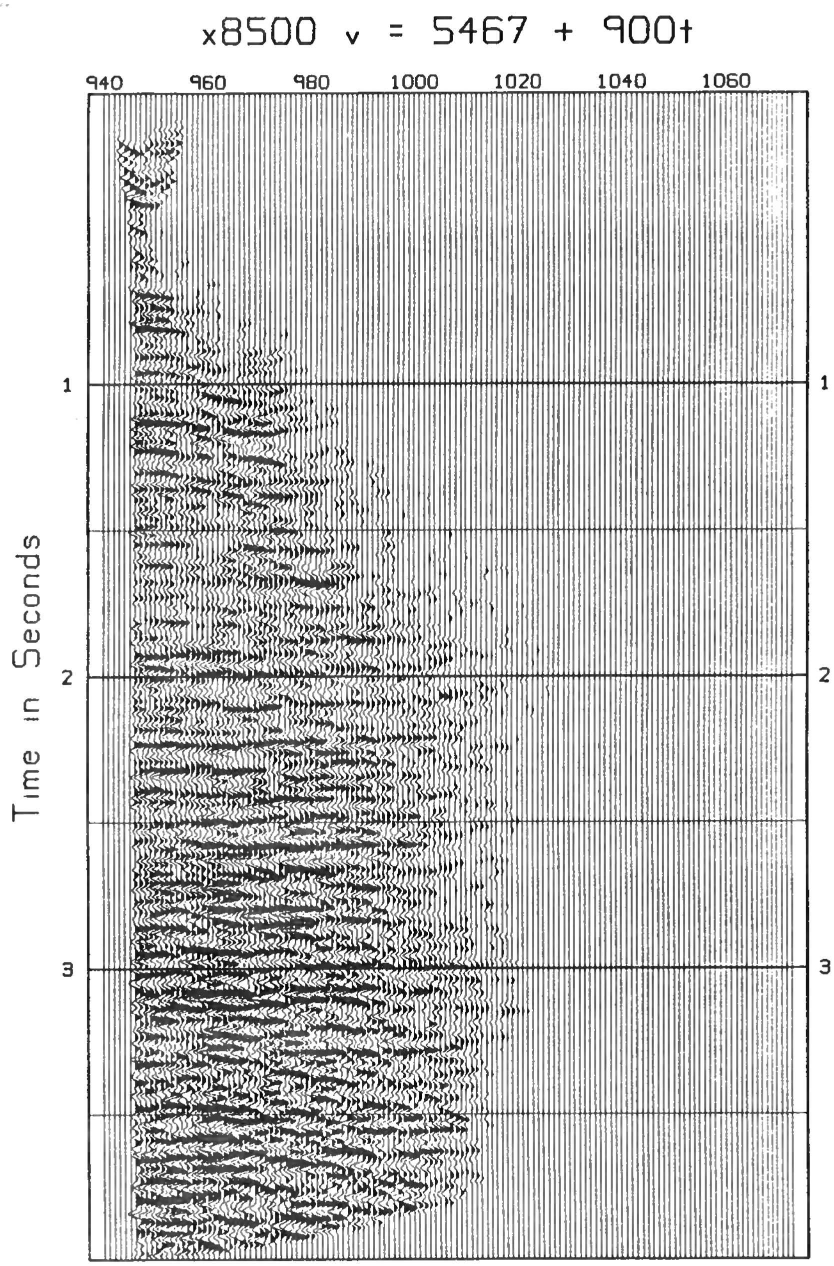


FIGURE 9B



x8500 v = 5467 + 900t
940
960
980
1000
1020
1040
1060
1
2
3
Time in Seconds

FIGURE 9c

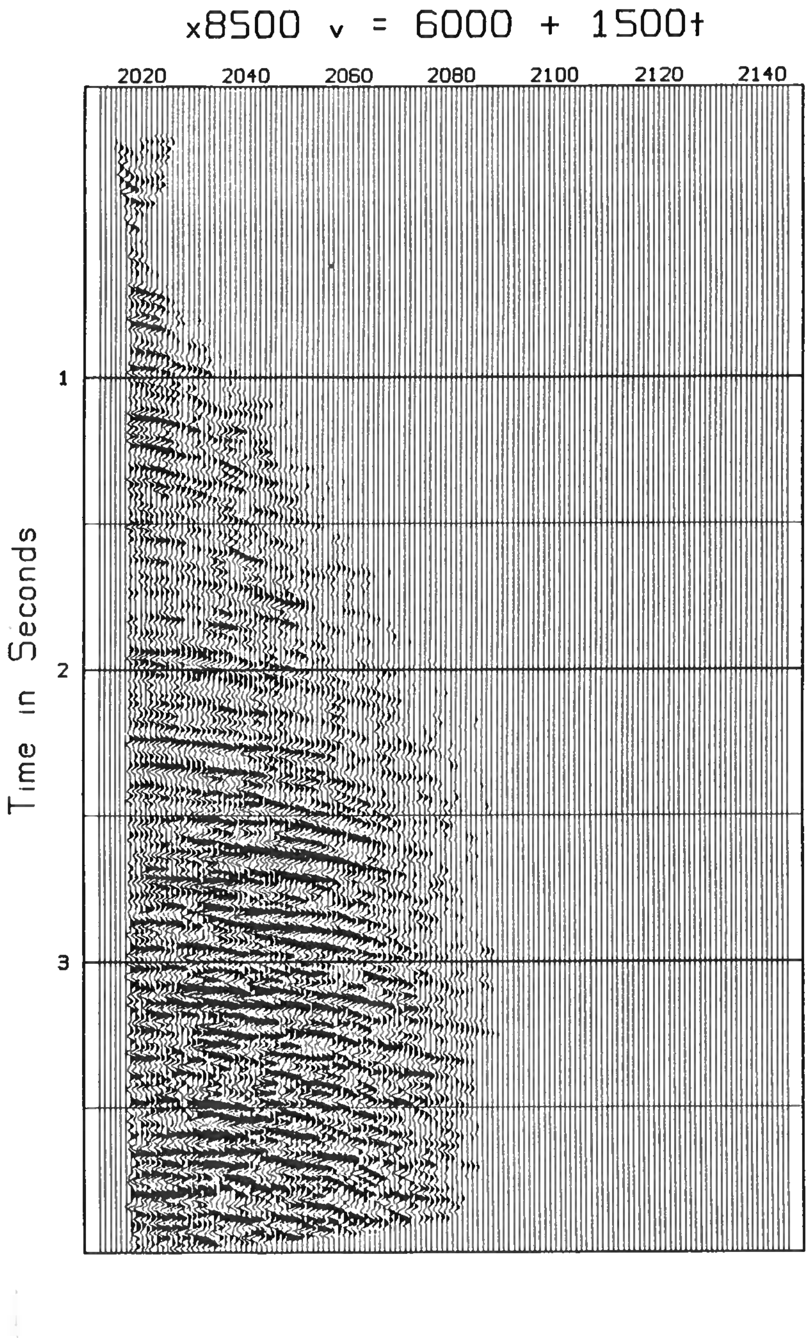

x8500 v = 6000 + 1500t
2020
2040
2060
2080
2100
2120
2140
Time in Seconds
1
2
3
1
2
3

FIGURE 9D

velocity at x =8500

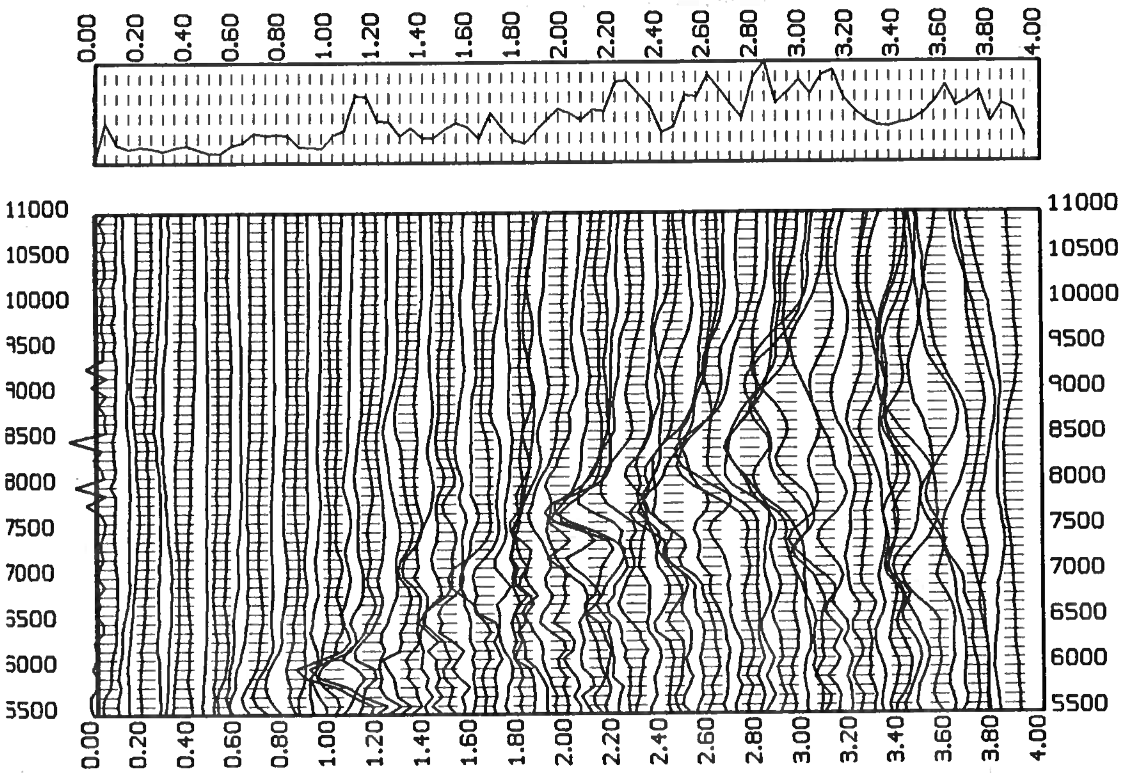
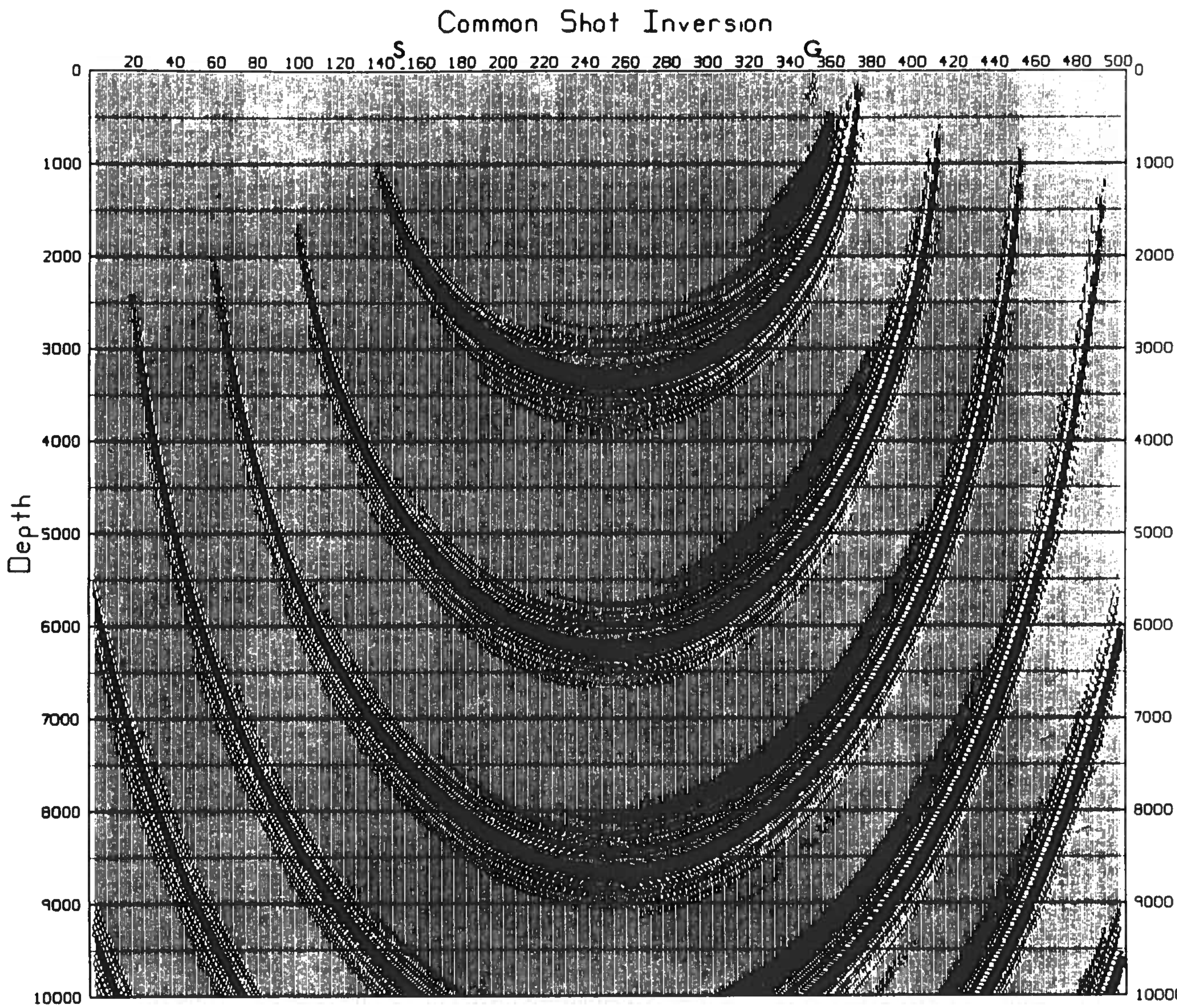
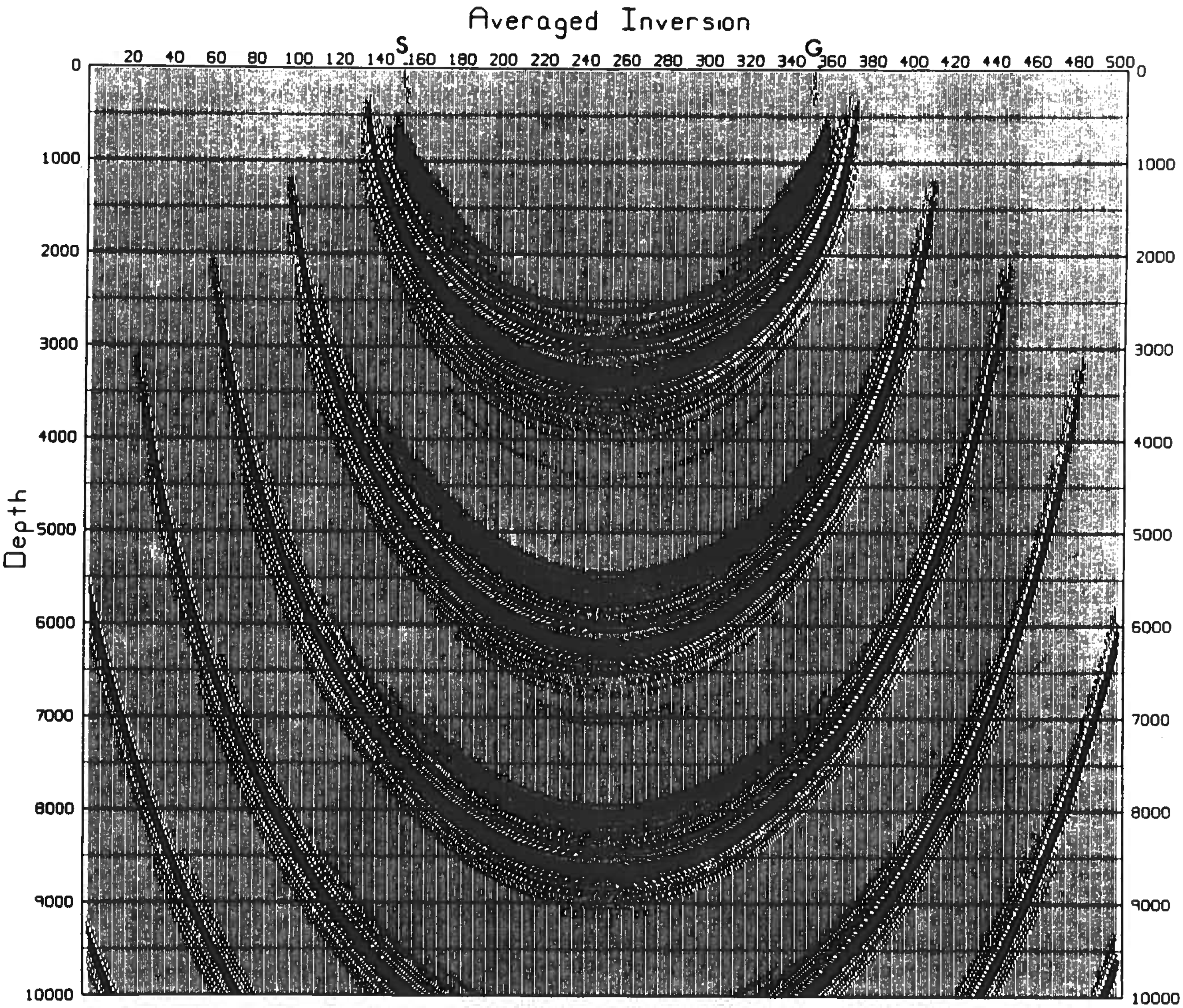


FIGURE 9E



Common Shot Inversion Operator. Source and geophone are at the locations marked. Trace spacing is 50 ft. Depth spacing is 20 ft. Velocity is a constant 10,000 ft/sec. The amplitudes in these figures have been compressed by taking the square root.

FIGURE 1ØA



Averaged Inversion Operator. Source and geophone are at the locations marked. Trace spacing is 50 ft. Depth spacing is 20 ft. Velocity is a constant 10,000 ft/sec.

FIGURE 1ØB

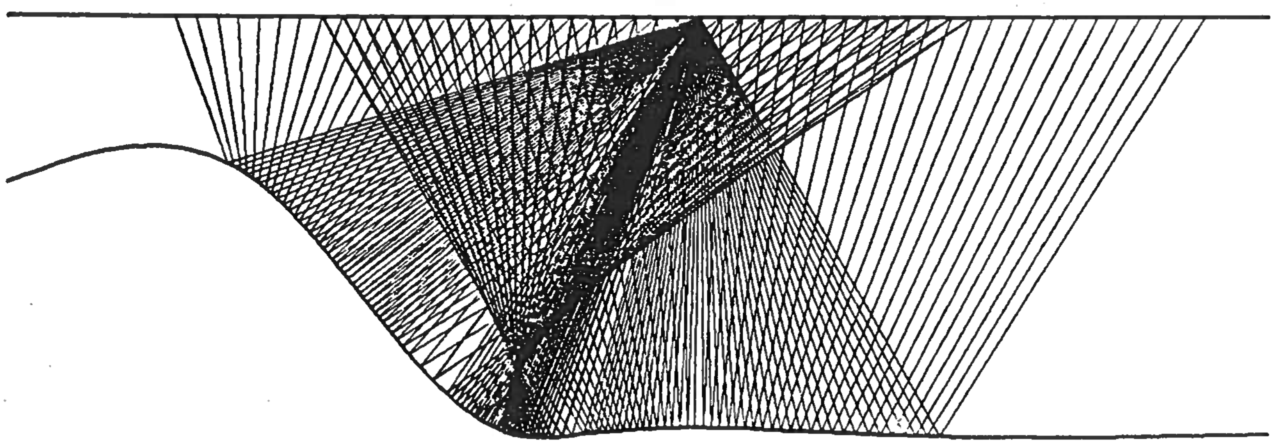


FIGURE 11A

File PARAM:

```
ftdata                              : name of time data file
n                                   : plot wavelet only ?
model1                              : name of model file
4                                   : # interfaces in model
n                                   : plot model only ?
y                                   : plot rays ?
cshot                               : name of output files
n                                   : shot record ?
n                                   : listing ?
0.        89.                       : min and max takeoff angles
1.                                  : takeoff angle increment
10.                                 : max angle between rays
2000.      0.                       : x, z coords. of shot
1                                   : # shots
0.         0.                       : x, z increments in shot coords.
4000.      0.                       : x, z coords. of first receiver
20                                  : # receivers per shot
200.                                : receiver spacing
0.                                  : receiver move-up between shots
4000.   6000.    9000.
10000.  13000.                      : velocities
4                                   : # primary reflections
0                                   : specific reflectors
0                                   : # extra events
```

Output:

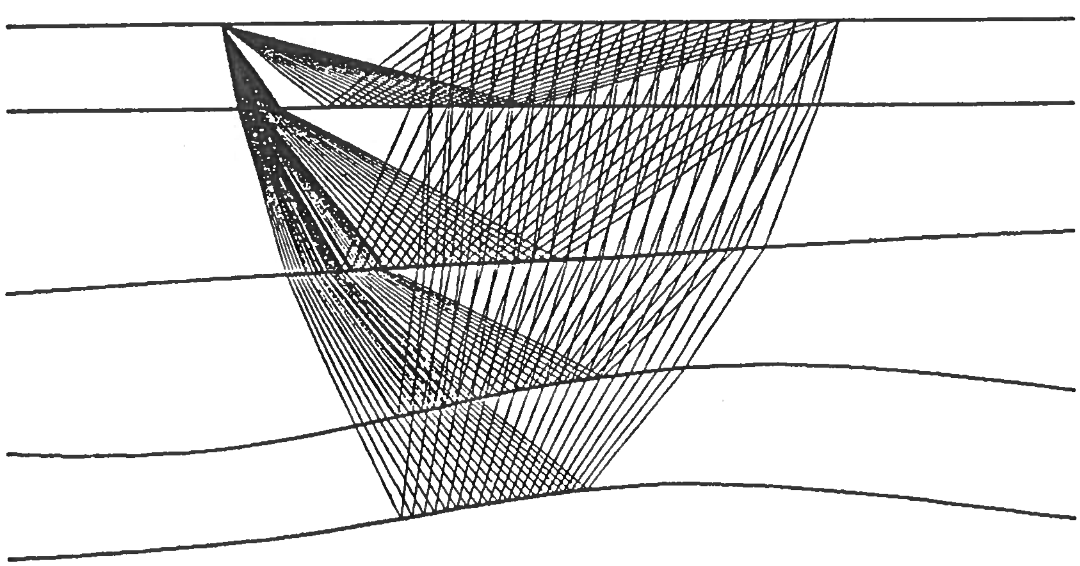


FIGURE 11B

File PARAM:

```
ftdata                                  : name of time data file
n                                       : plot wavelet only ?
model1                                  : name of model file
4                                       : # interfaces in model
n                                       : plot model only ?
y                                       : plot rays ?
cshot                                   : name of output files
n                                       : shot record ?
n                                       : listing ?
0.          89.                         : min and max takeoff angles
1.                                      : takeoff angle increment
10.                                     : max angle between rays
2000.        0.                         : x, z coords. of shot
1                                       : # shots
0.           0.                         : x, z increments in shot coords.
4500.        0.                         : x, z coords. of first receiver
20                                      : # receivers per shot
200.                                    : receiver spacing
0.                                      : receiver move-up between shots
4000.   6000.    9000.
10000. 13000.                           : velocities
0                                       : # primary reflections
0                                       : specific reflectors
1                                       : # extra events
9                                       : # intersections (next event)
1  2  1  2  3  4  3  2  1               : event specifier
```

Output:

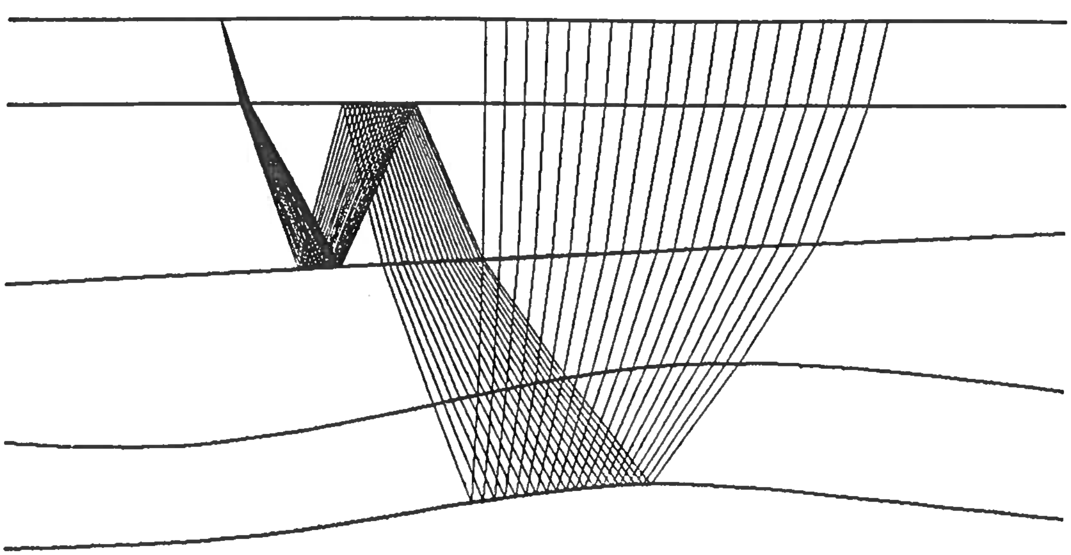


FIGURE 11c

Output:

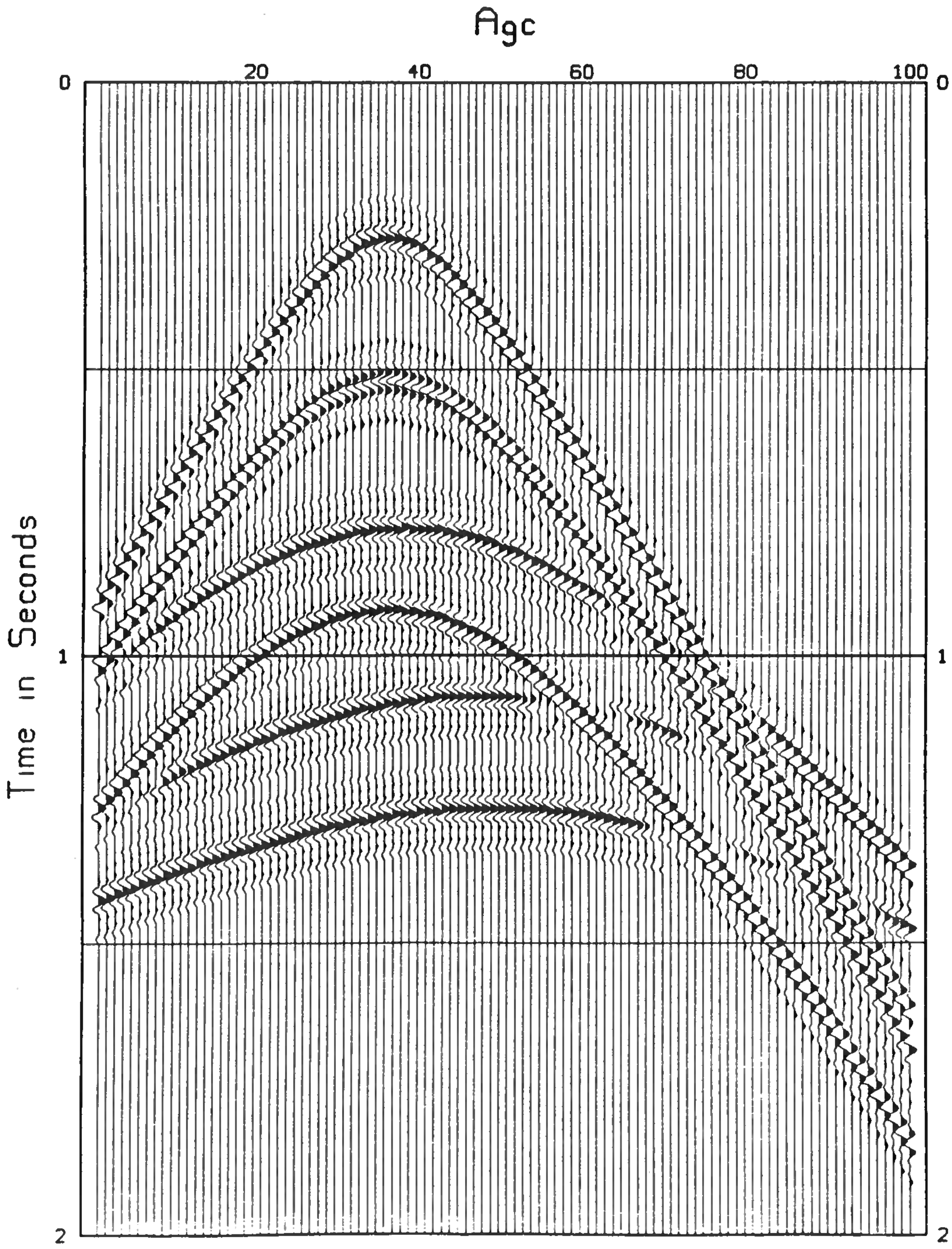


FIGURE 11D

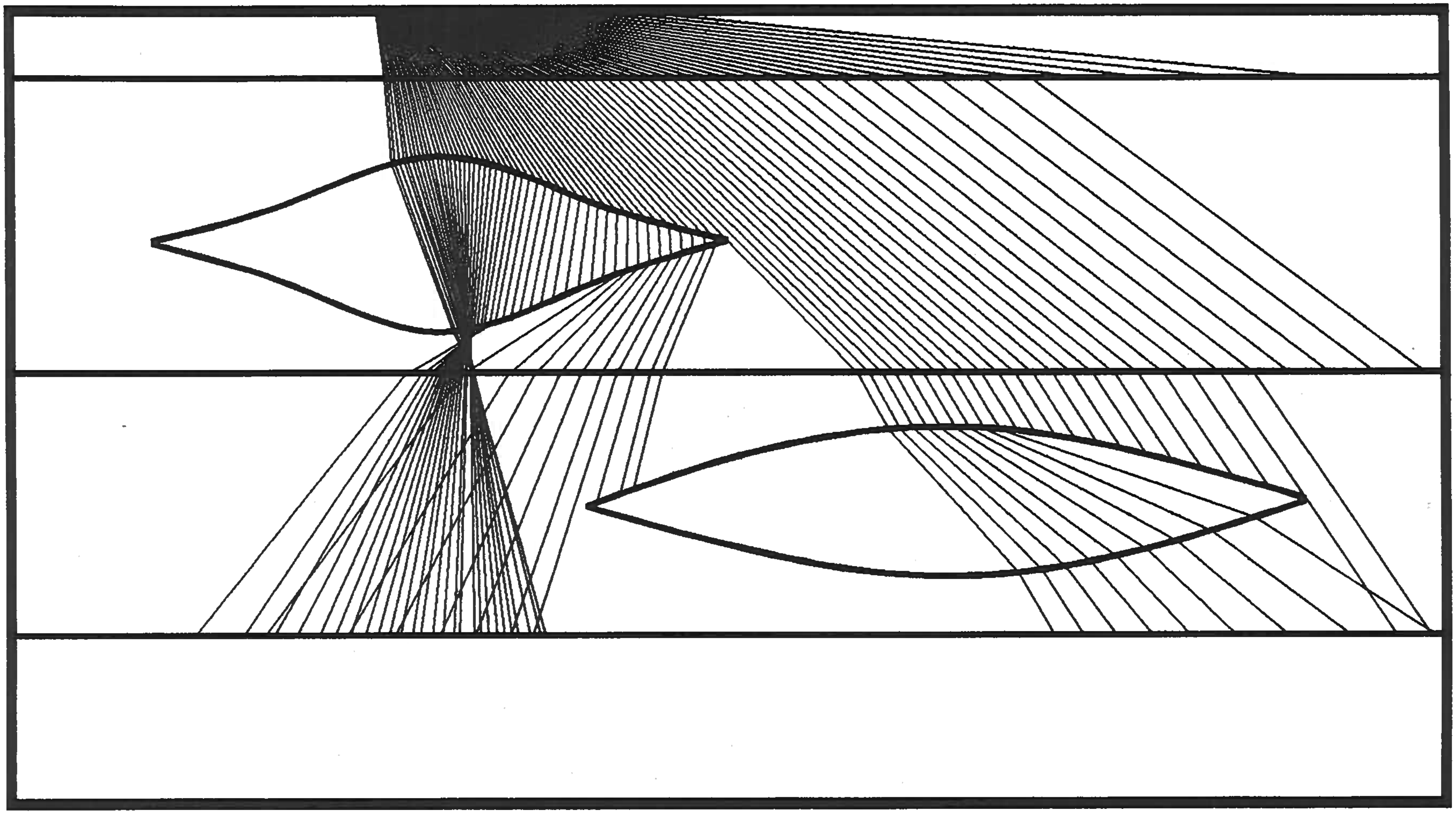


FIGURE 12A

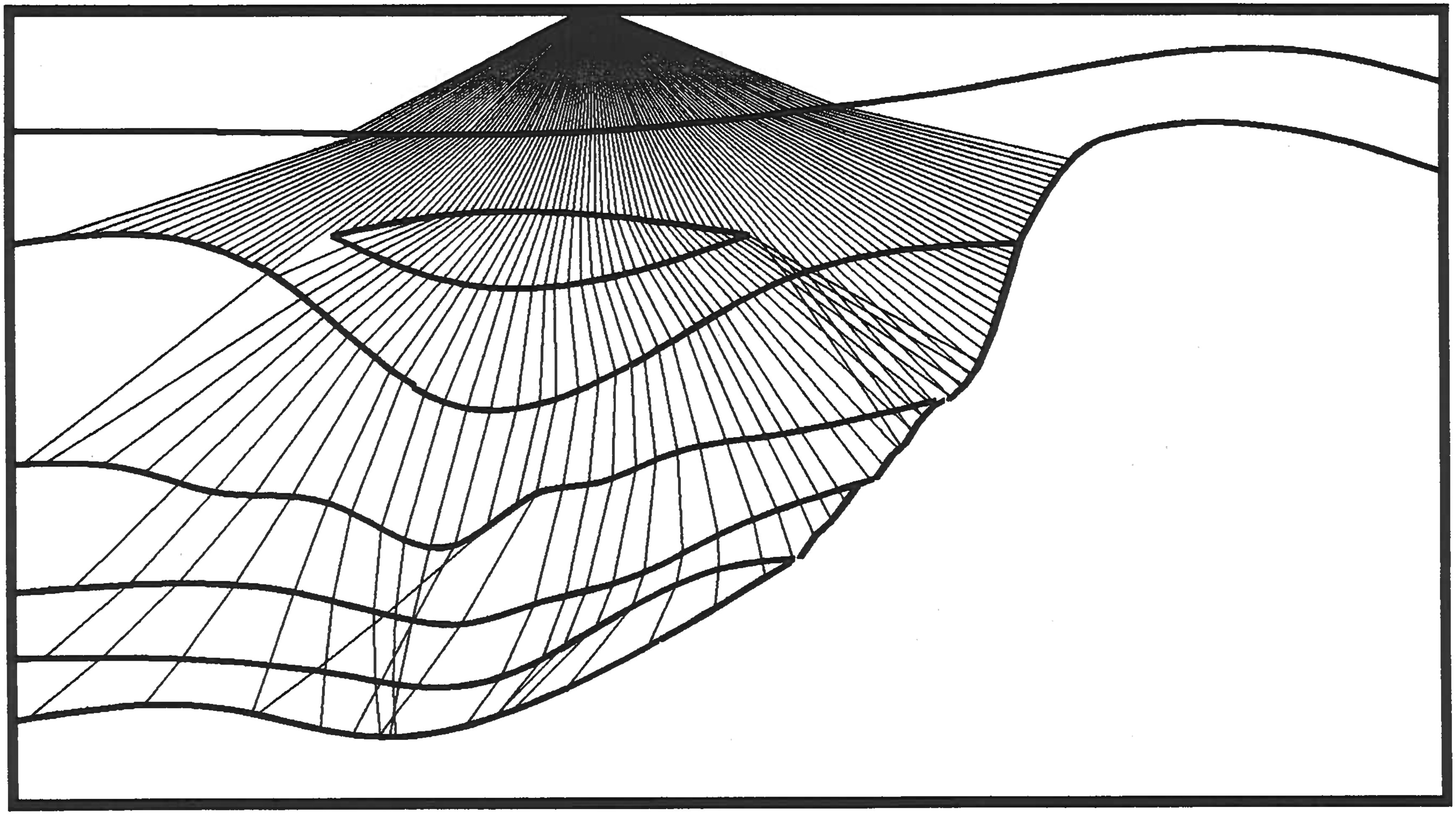


FIGURE 12B

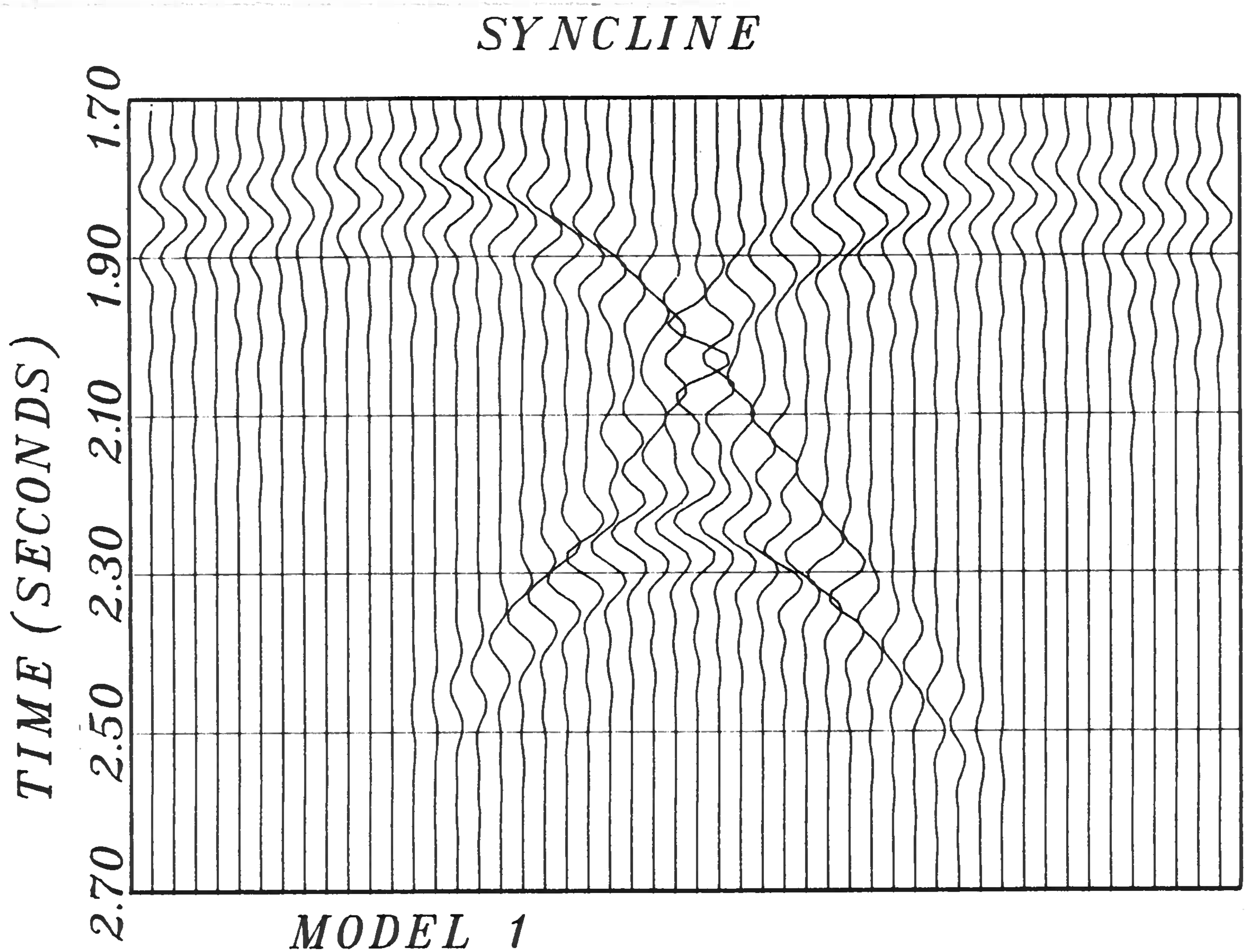


FIGURE 12c

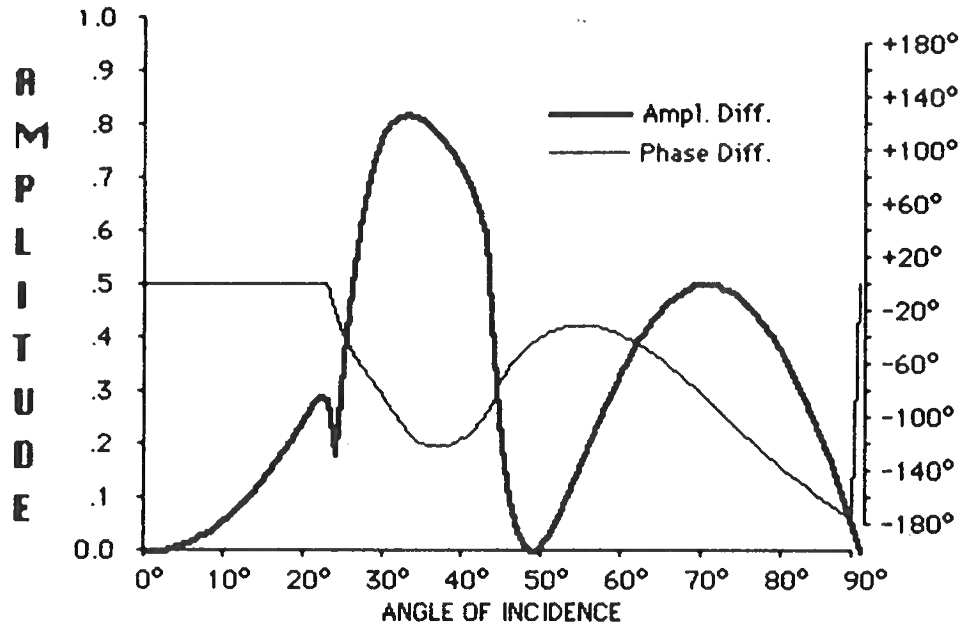


FIGURE 13A Non-acoustic behaviour of the elastic ARC; formed by subtraction of fig. 6 from fig. 5. Having two critical angles (23.6° and 43.8°), this is a 'prototype' for all cases where wave-speeds increase across the interface.

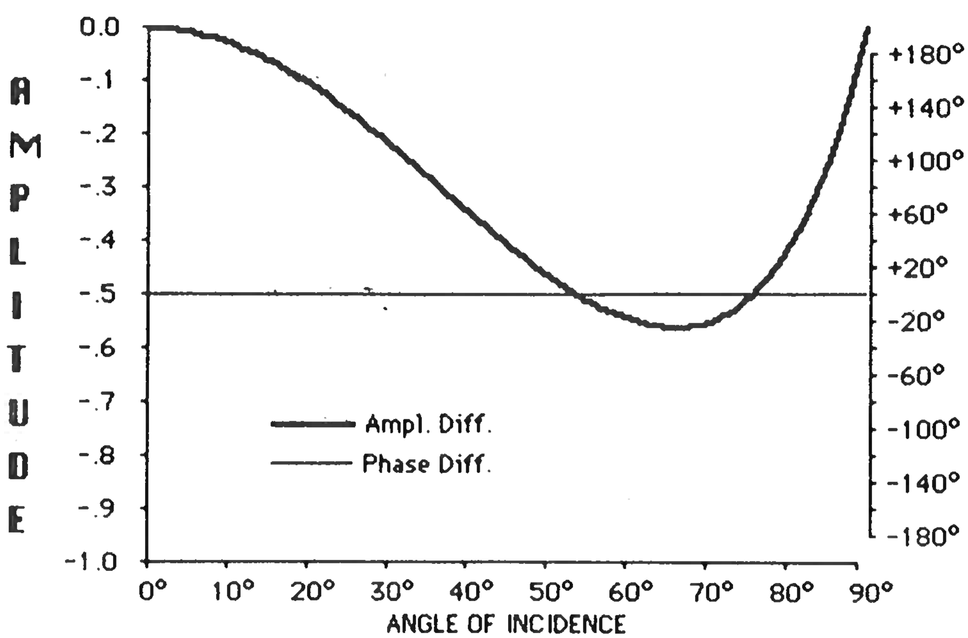


FIGURE 13B Non-acoustic behaviour of elastic ARC when layers are interchanged in model of fig. 3. This represents the exteme velocity-inversion case.

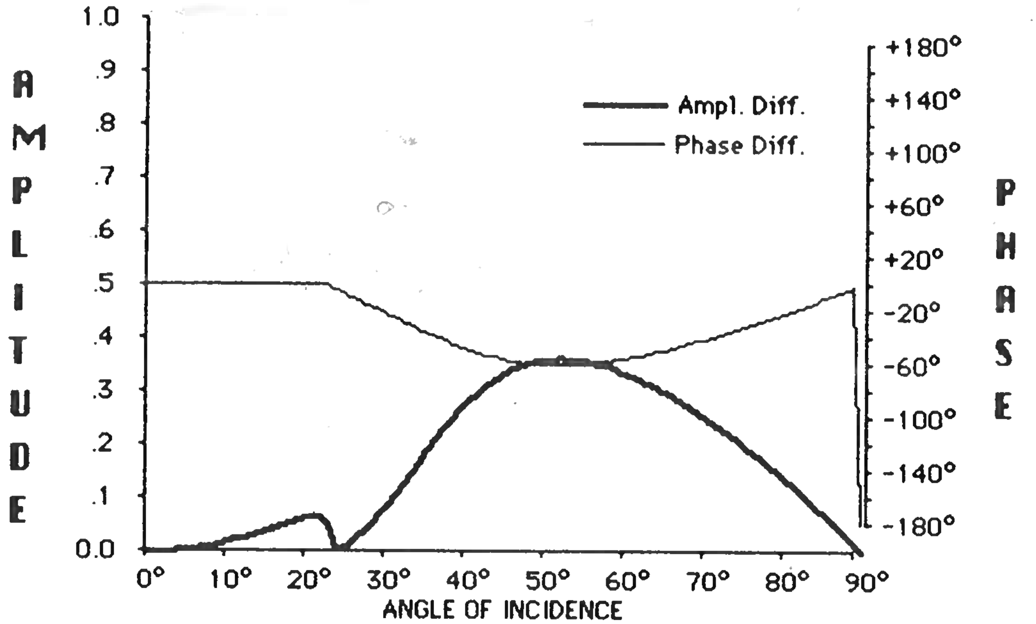


FIGURE 13c Non-acoustic behaviour for the model in fig. 3, except .45 = poisson ratio. This corresponds to a 'soft' solid. Compare figs. 7 and 10. No second crit. ang.

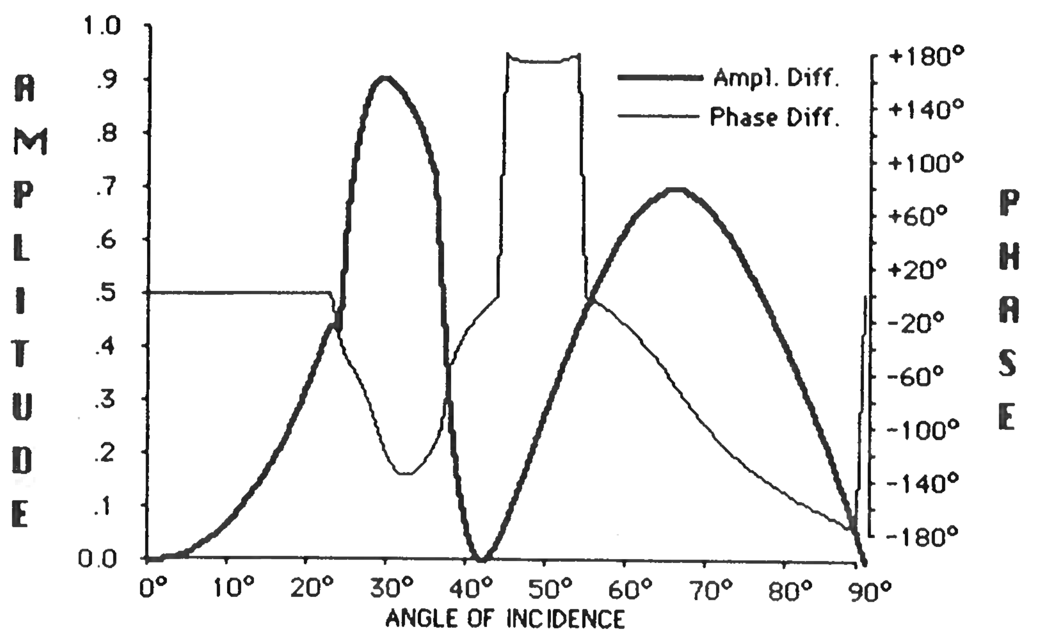


FIGURE 13D Non-acoustic behaviour for the model in fig. 3, except .10 = poisson ratio. This corresponds to a 'hard' solid. Compare figs. 7 and 9. Second crit. ang. = 36.9°

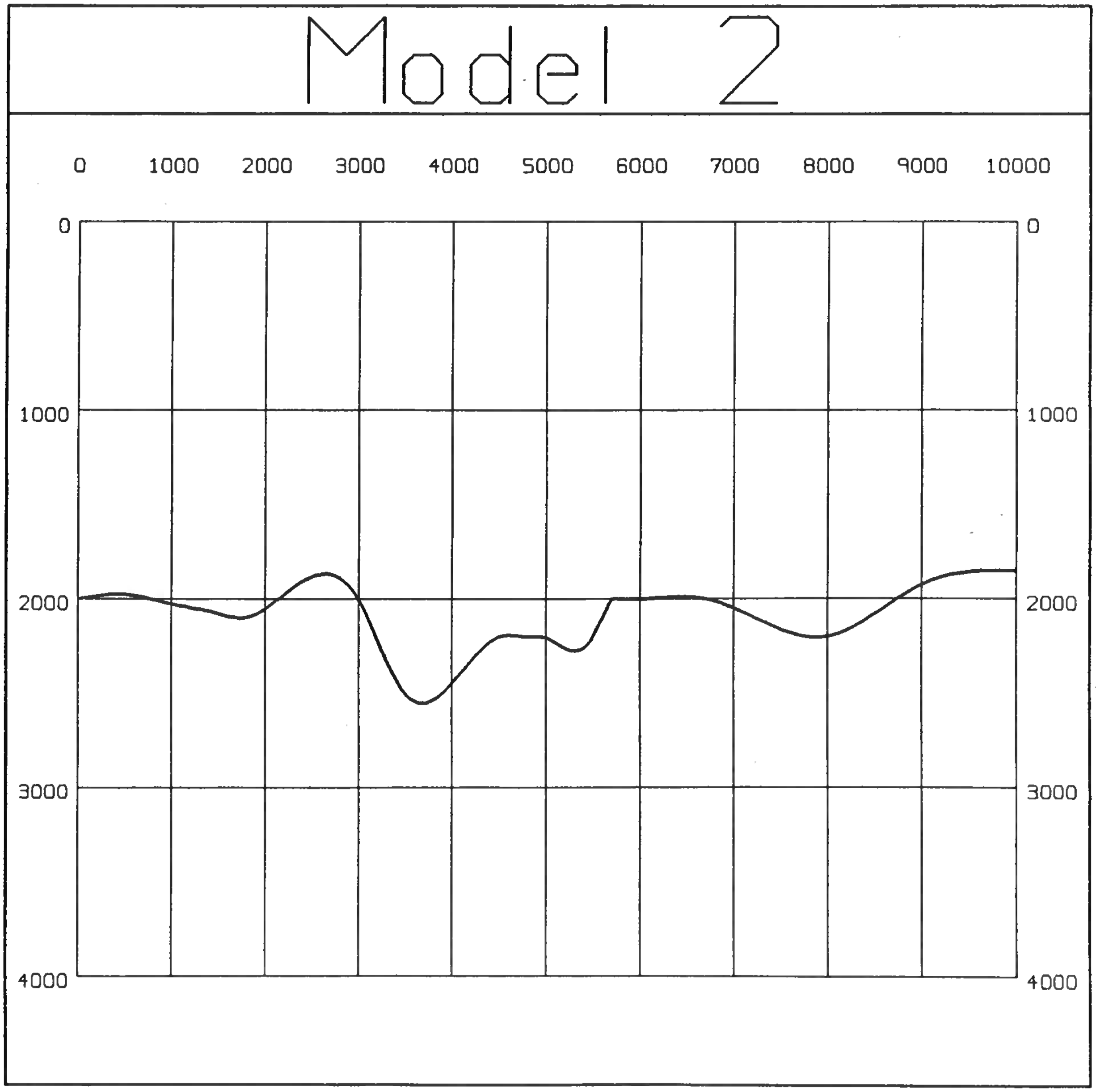

Model 2
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
0
1000
2000
3000
4000
0
1000
2000
3000
4000

FIGURE 14A

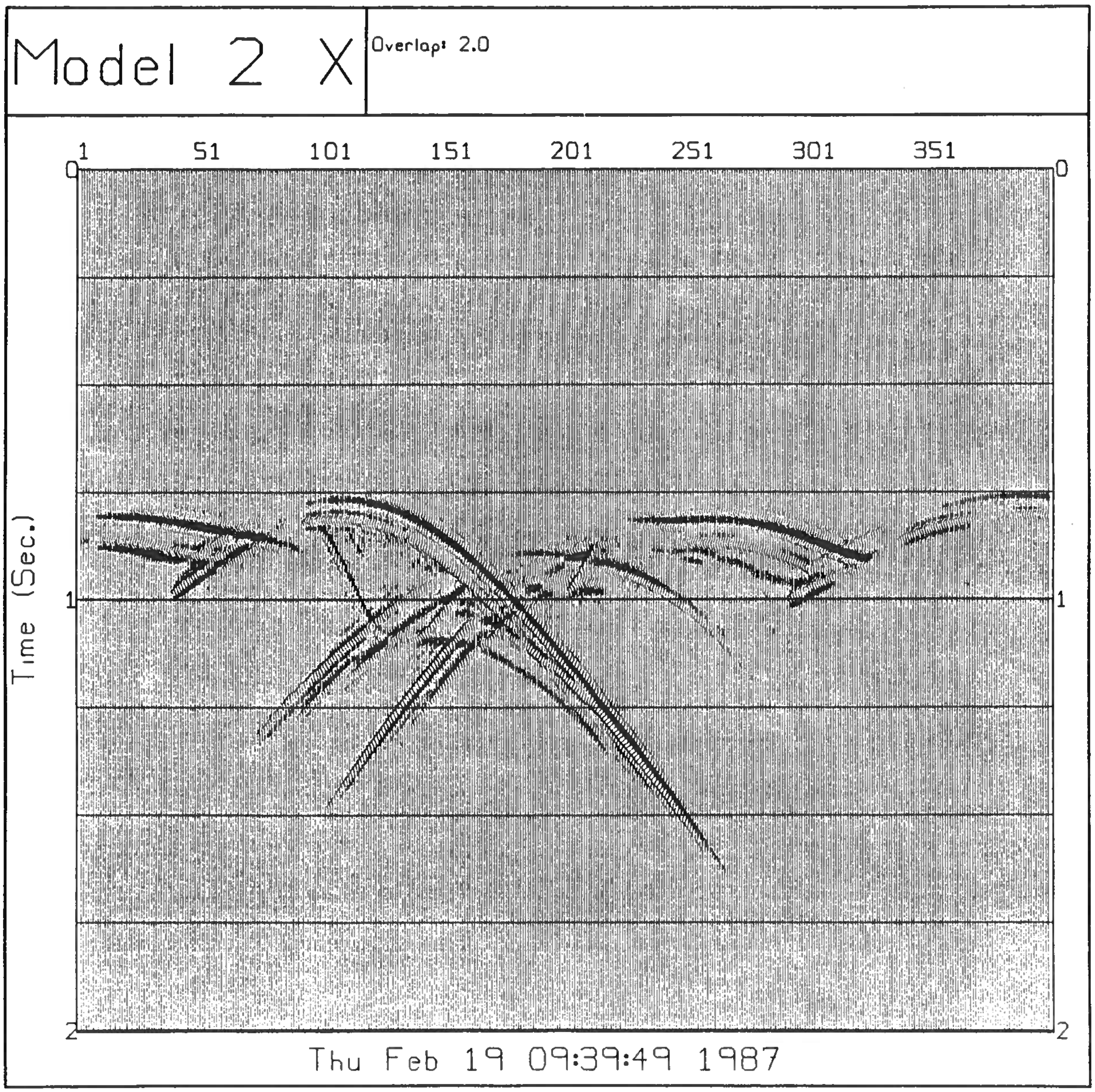
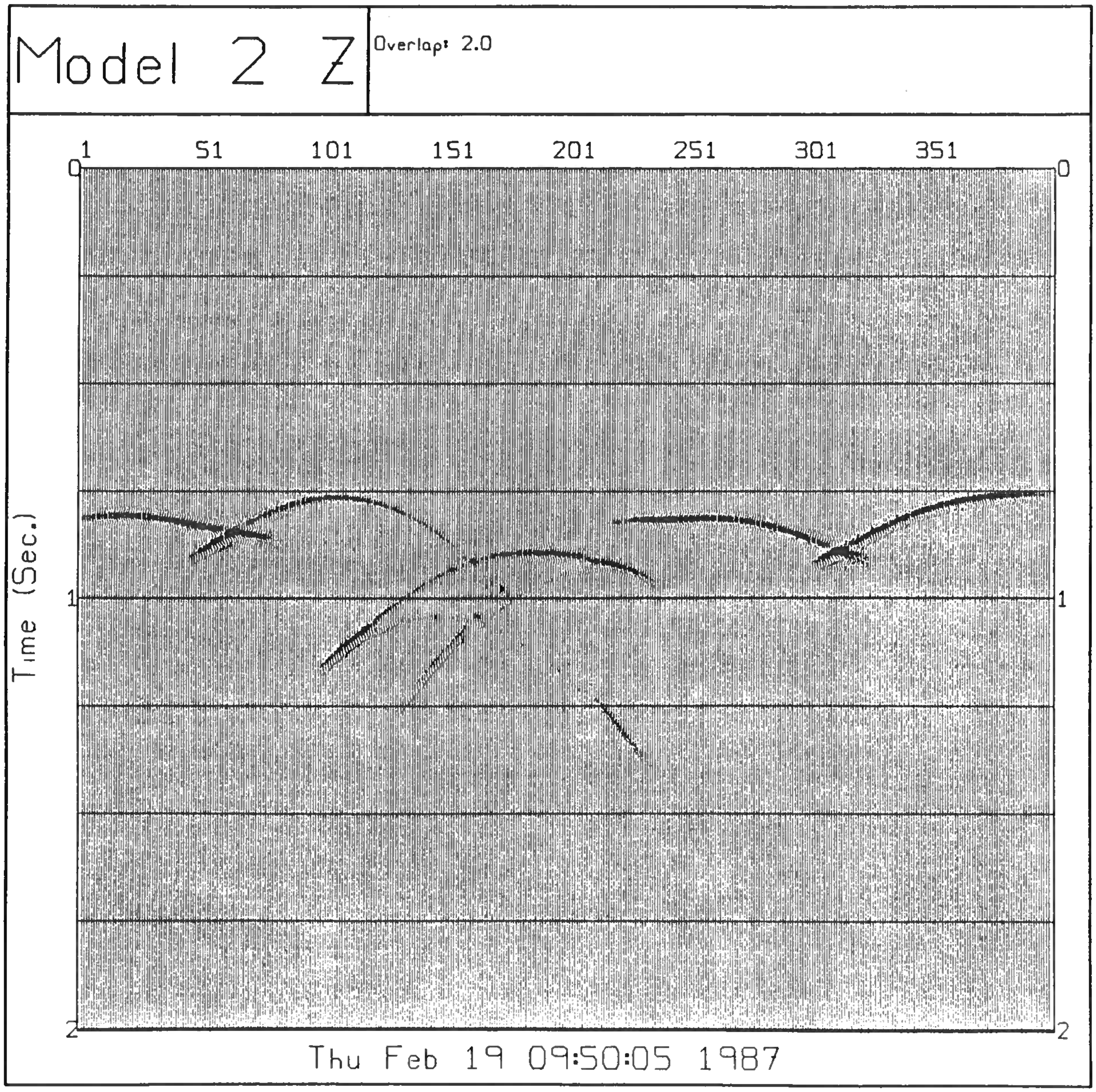
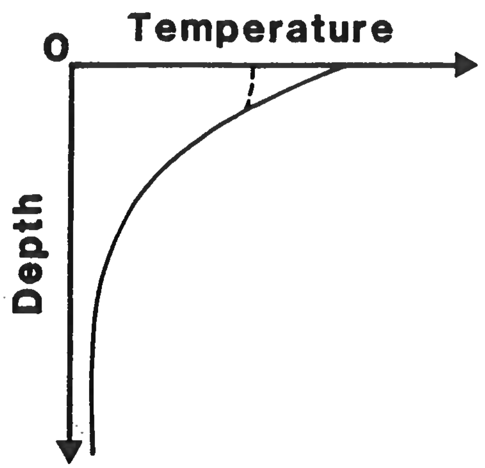

Model 2 X
Overlap: 2.0
1 51 101 151 201 251 301 351
0
1
2
Time (Sec.)
Thu Feb 19 09:39:49 1987

FIGURE 14B

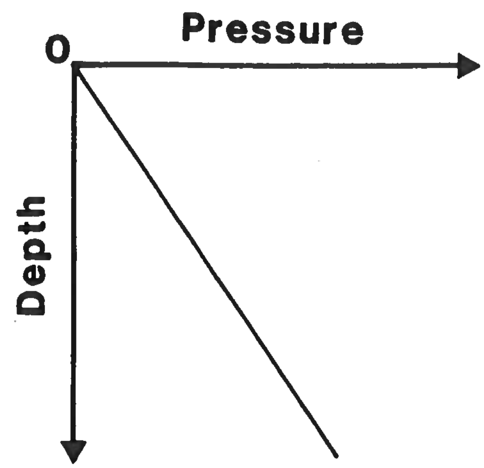


Model 2 Z
Overlap: 2.0
1
51
101
151
201
251
301
351
0
1
2
Time (Sec.)
Thu Feb 19 09:50:05 1987

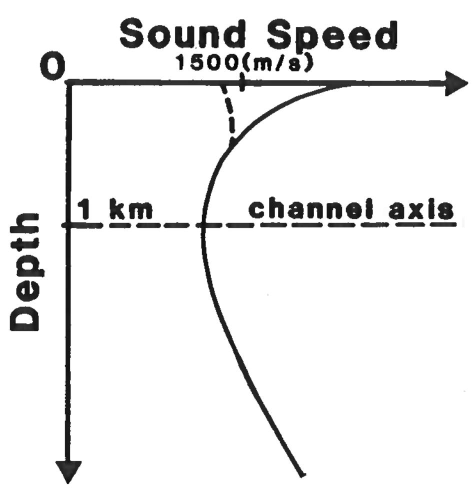
FIGURE 14c



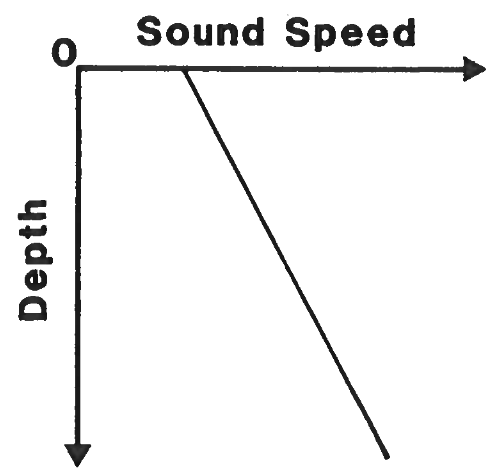
1a. Temperature profile in temperate waters.



1b. Hydrostatic pressure profile.



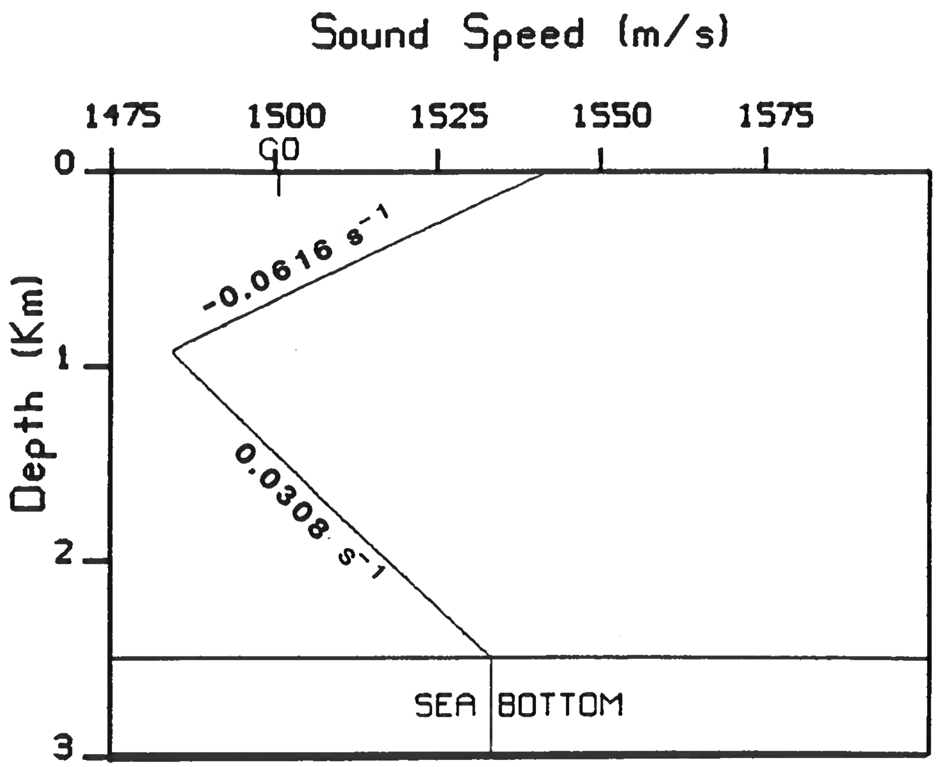
1c. Temperate Underwater Sound Channel sound speed profile.



1d. Arctic sound speed profile.

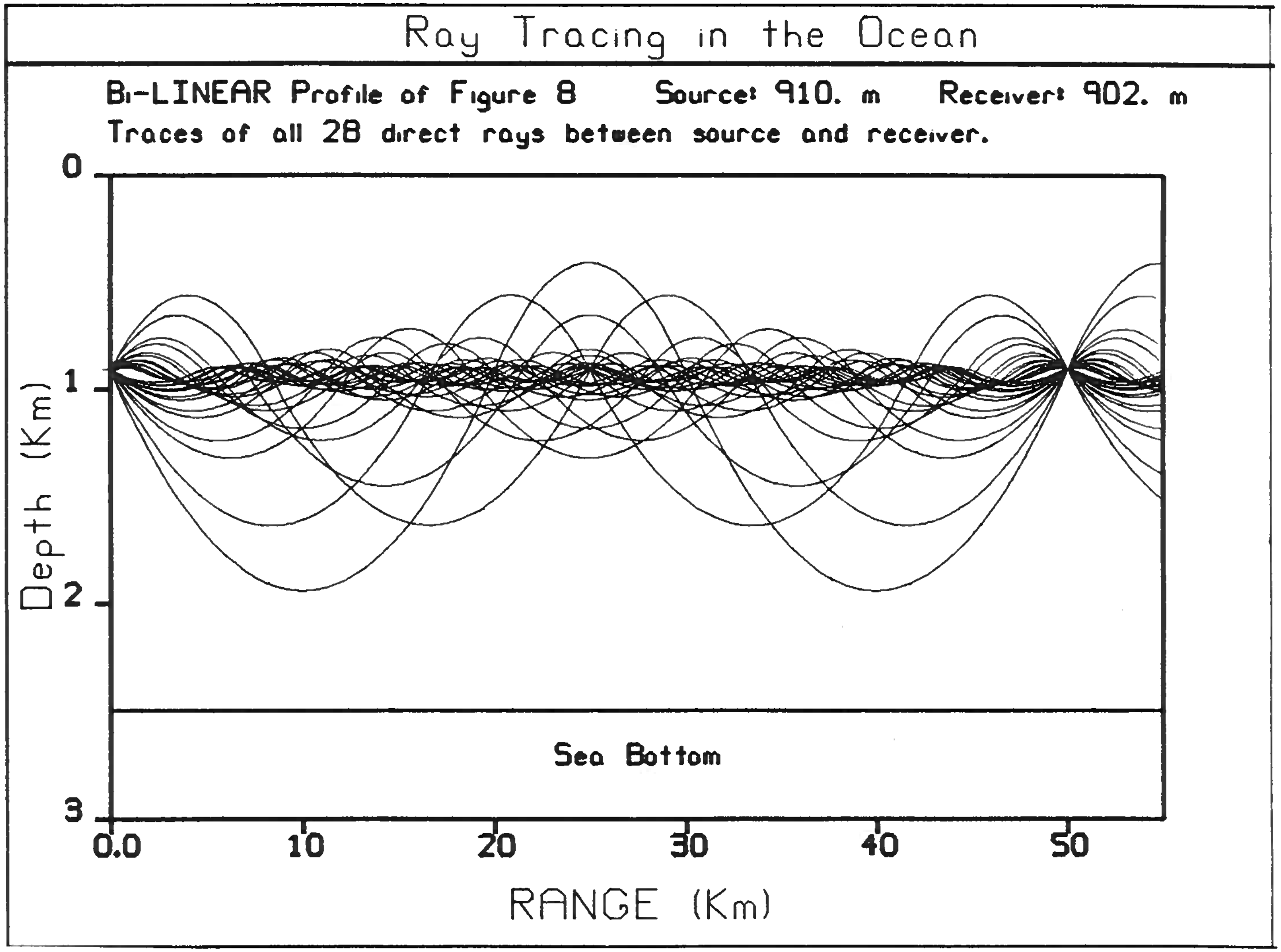
Typical sound speed profiles for Temperate and Arctic regions.

FIGURE 15



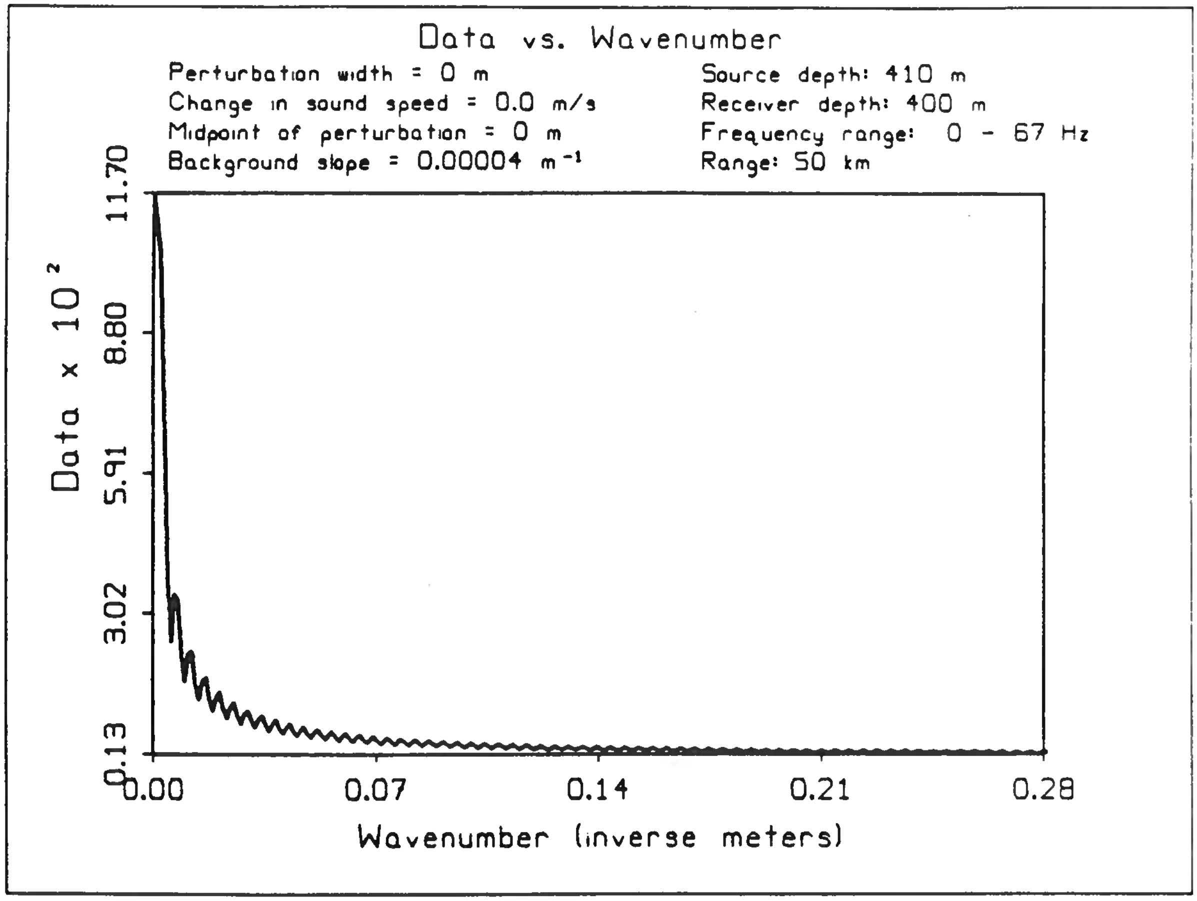
Example of a bi-linear sound speed profile.

FIGURE 16



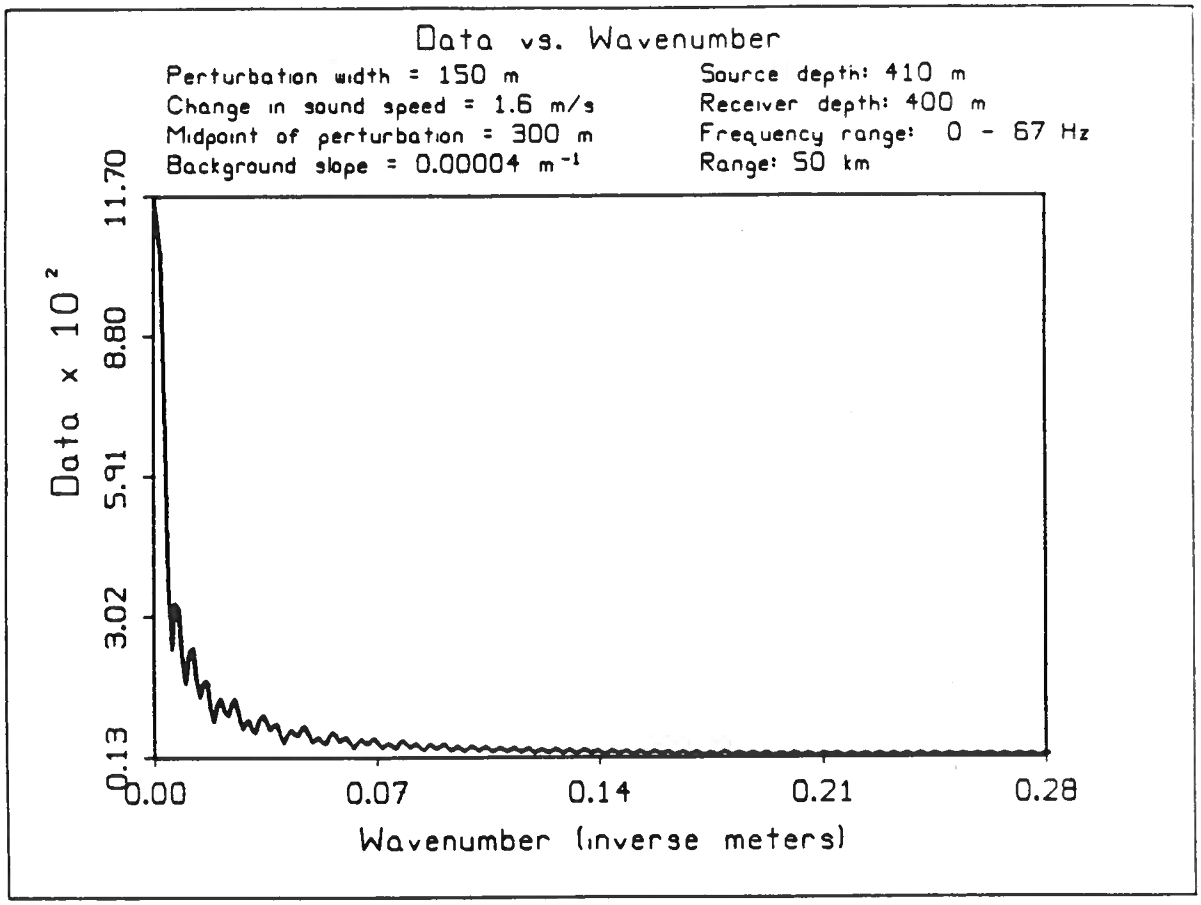
Raypaths of the 28 refracting rays joining source and receiver of Figure 15.

FIGURE 17



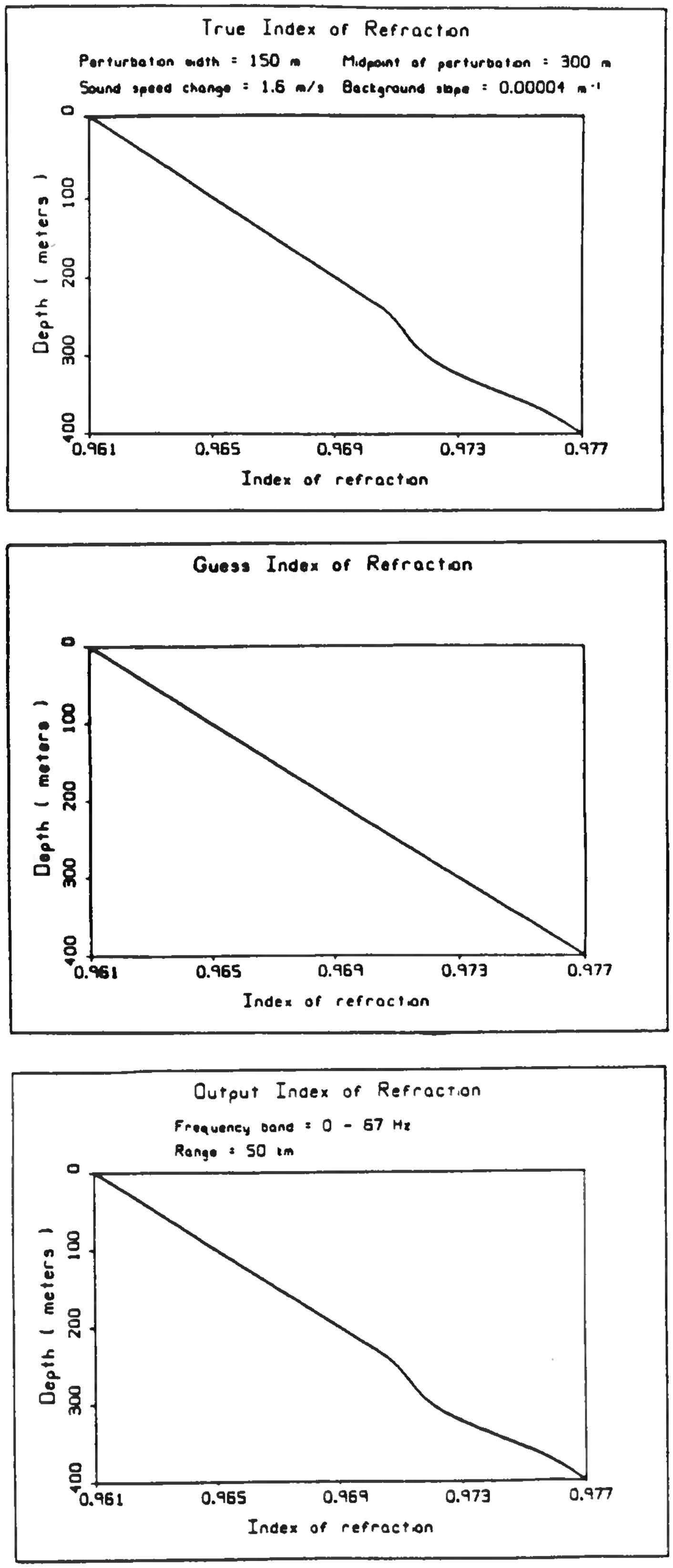
Magnitude of data vs. wavenumber.
No profile perturbation.

FIGURE 18



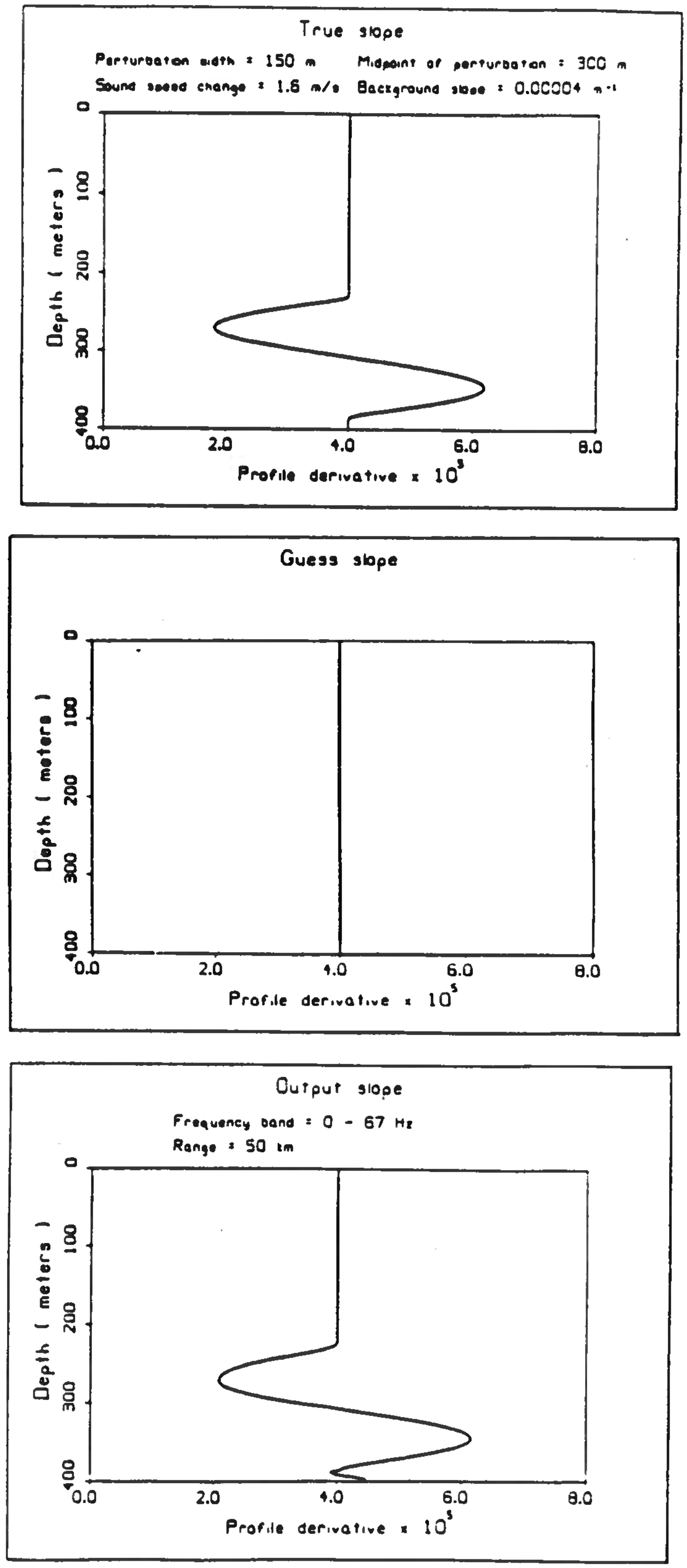
Magnitude of data vs. wavenumber.
Profile perturbation ~ 300 meters.

FIGURE 19



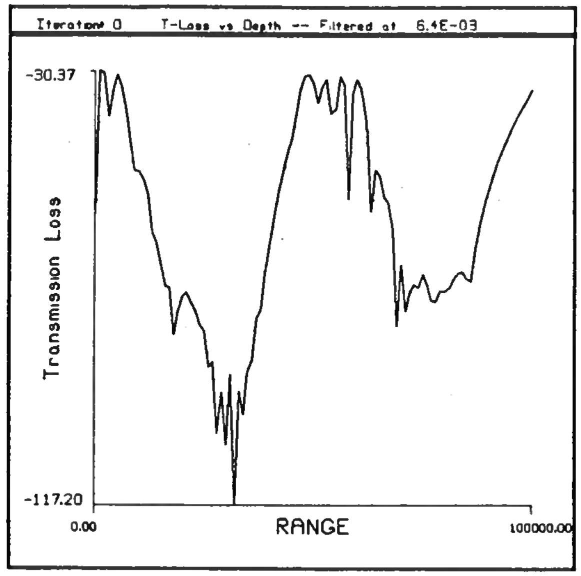
Ocean sound speed profile inversion: illustrated are the true index of refraction containing a small anomaly (upper figure), the guess value (center figure), and the predicted value (lower figure) from our profile inversion algorithm. Parameter values include the source at 410M, receiver at 400M, the range at 50KM, and the channel axis depth at 1KM.

FIGURE 20

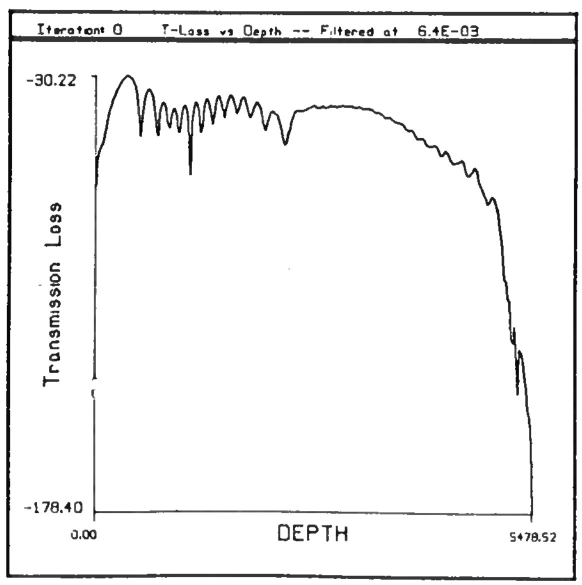


Ocean sound speed profile inversion: illustated are the true slope of the index of refraction (upper figure), the guess value (center figure), and the predicted slope (lower figure). Same parameters as Figure 20.

FIGURE 21



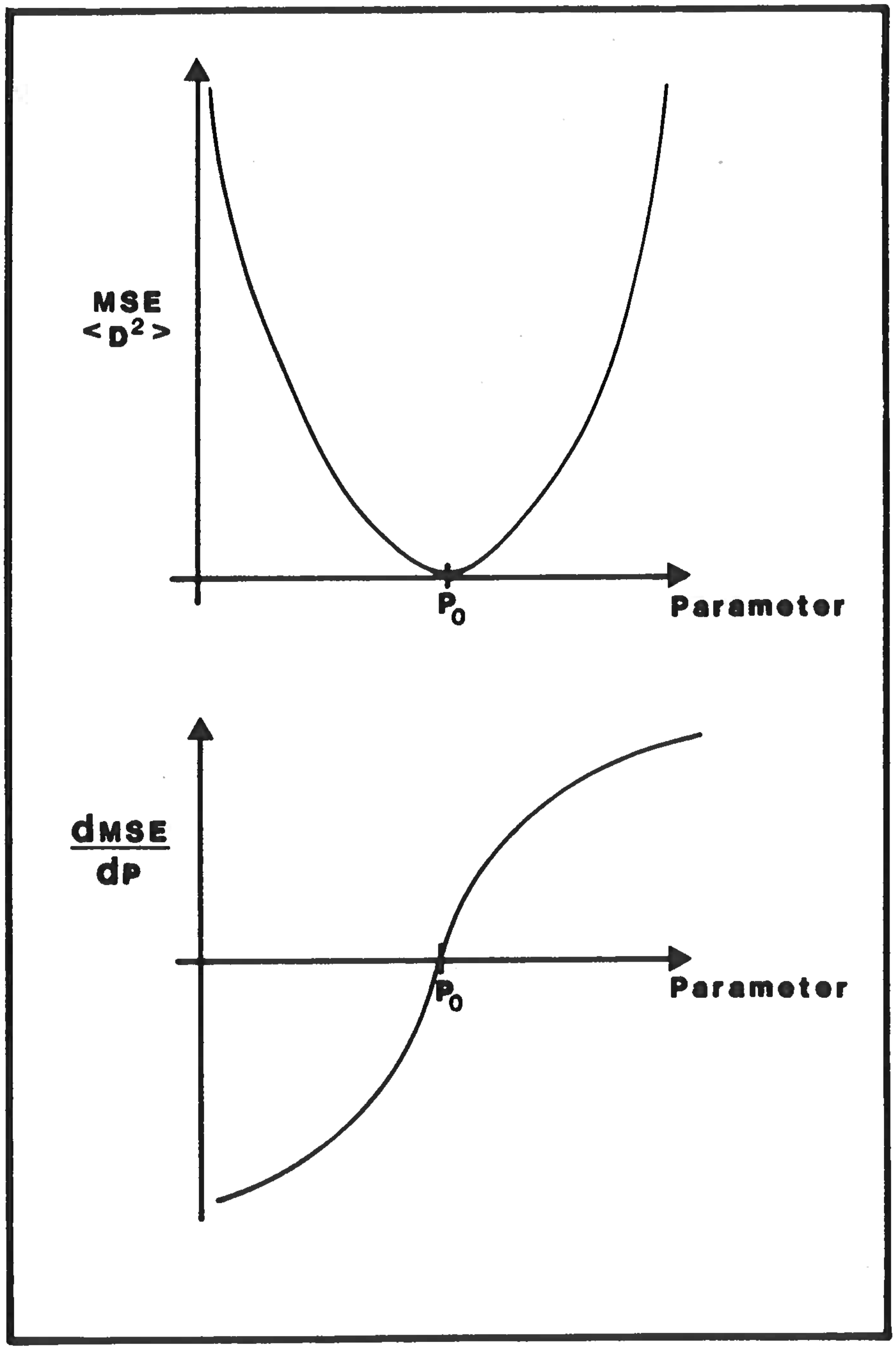
A). Tappert bi-linear range dependent transmission loss.



B). Tappert bi-linear depth dependent transmission loss.

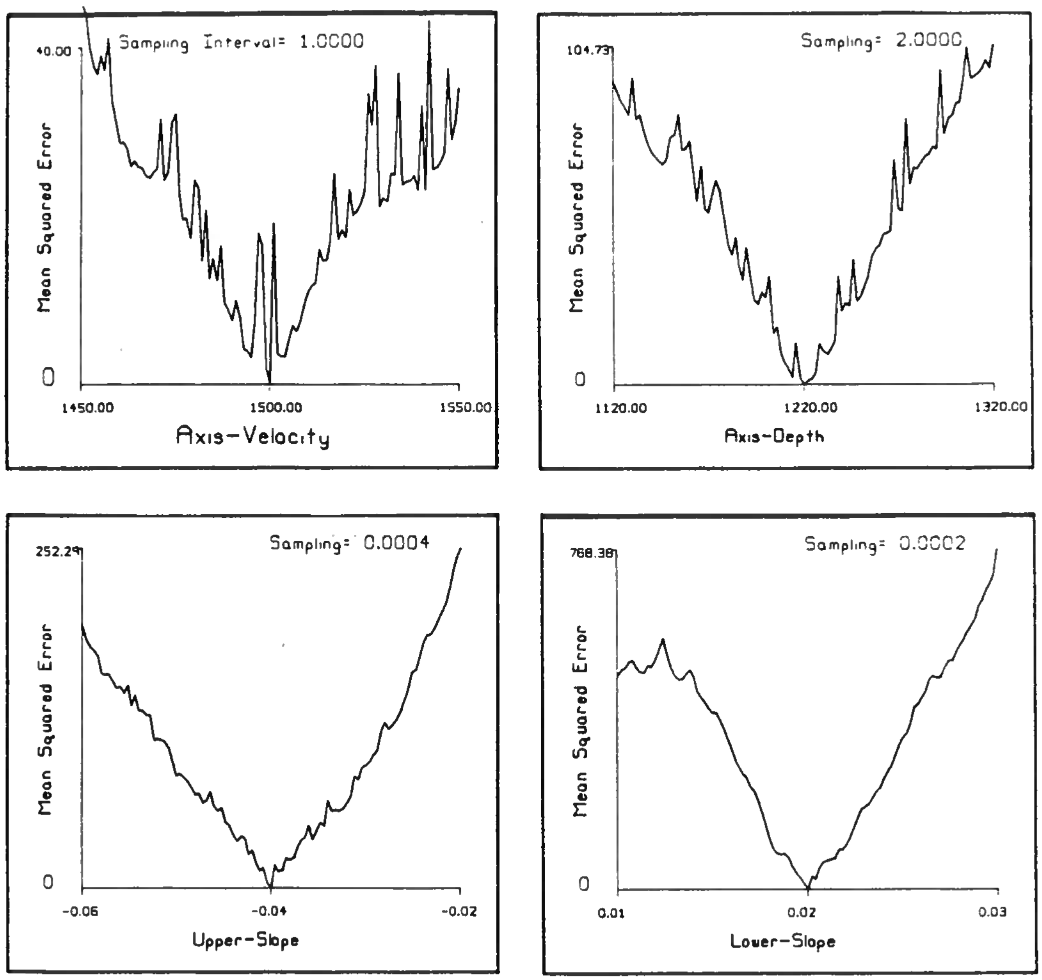
Synthetic data used in all inversions.

FIGURE 22



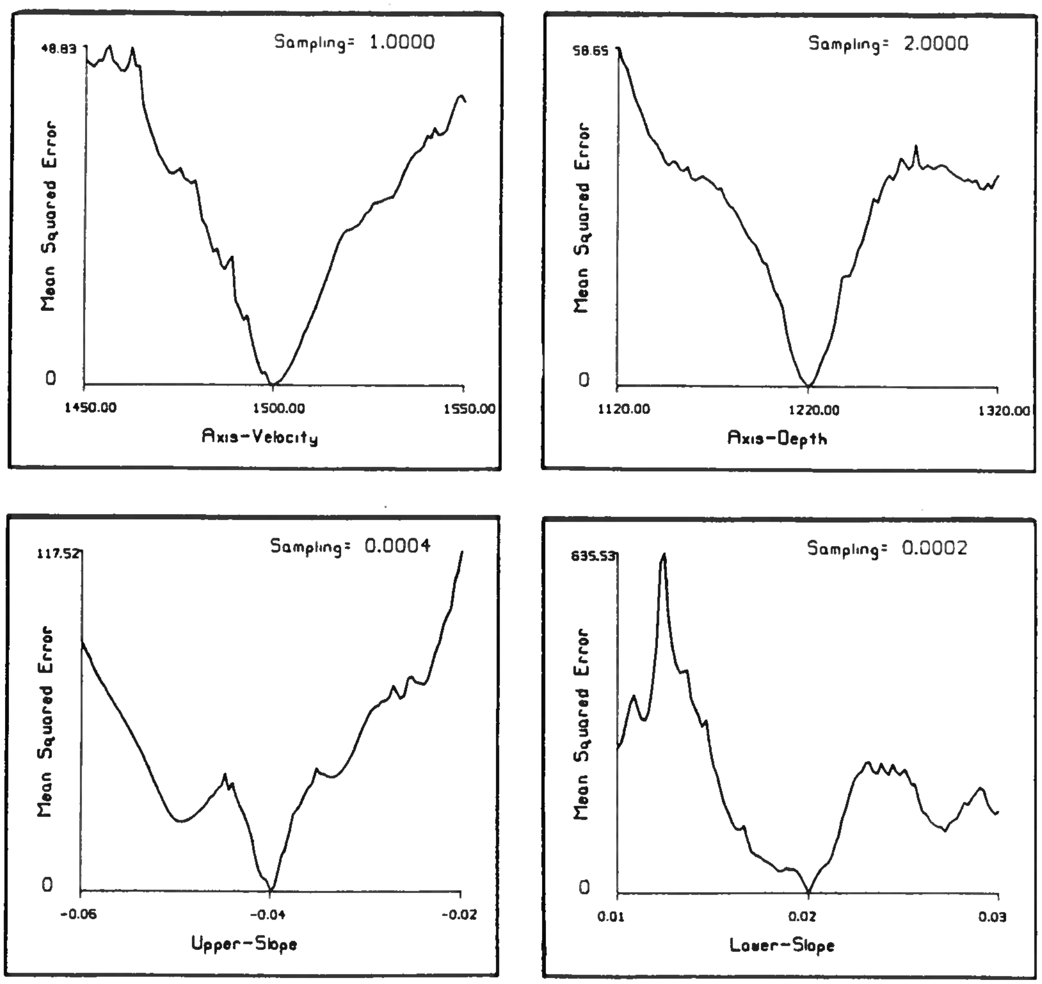
Ideal one-dimensional error curves.

FIGURE 23



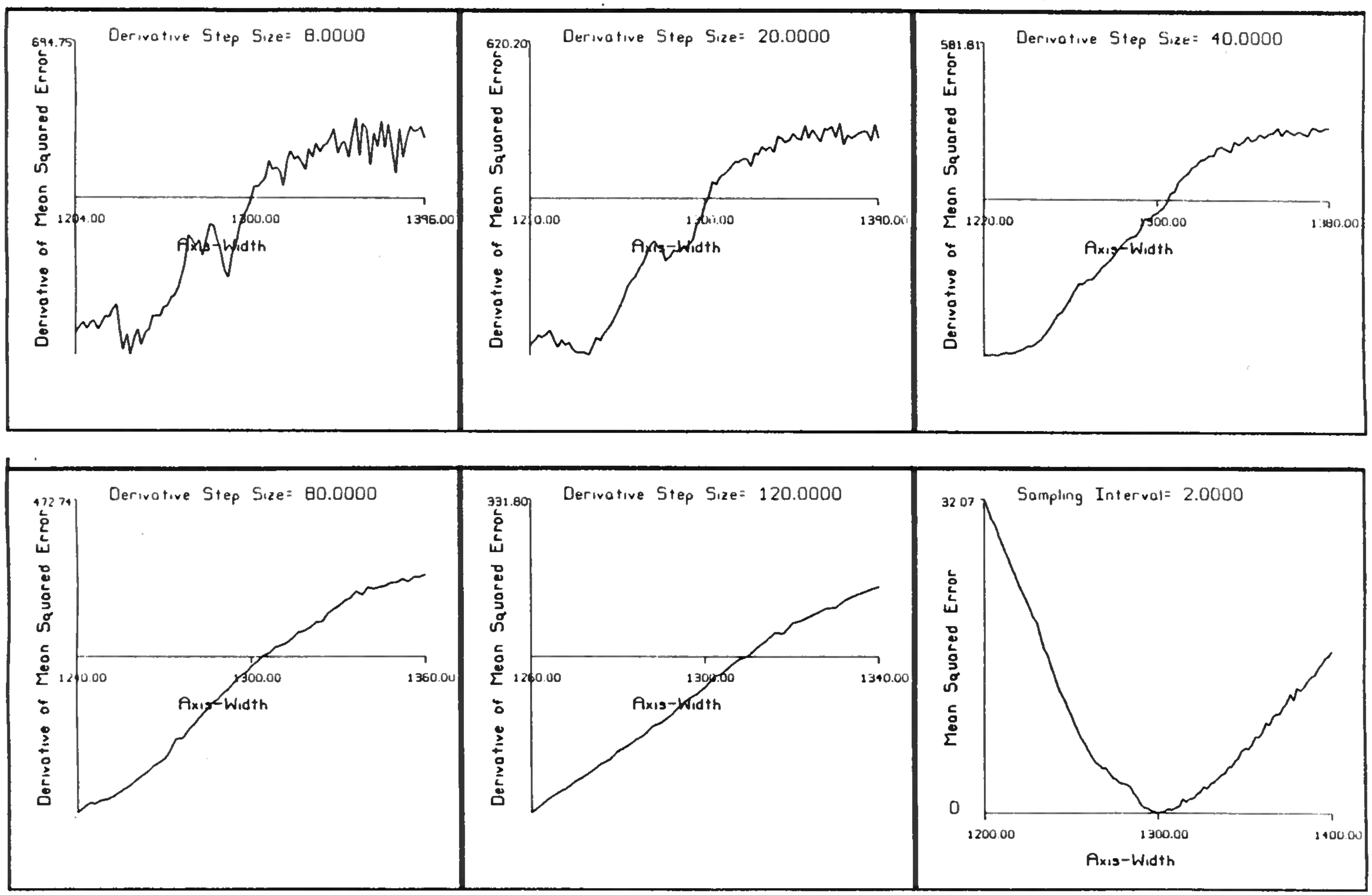
Error curves for the bi-linear parameters using range dependent transmission loss.

FIGURE 24



Error curves for the bi-linear parameters using depth dependent transmission loss.

FIGURE 25



Differentiated error curve for a well-behaved parameter.

FIGURE 26

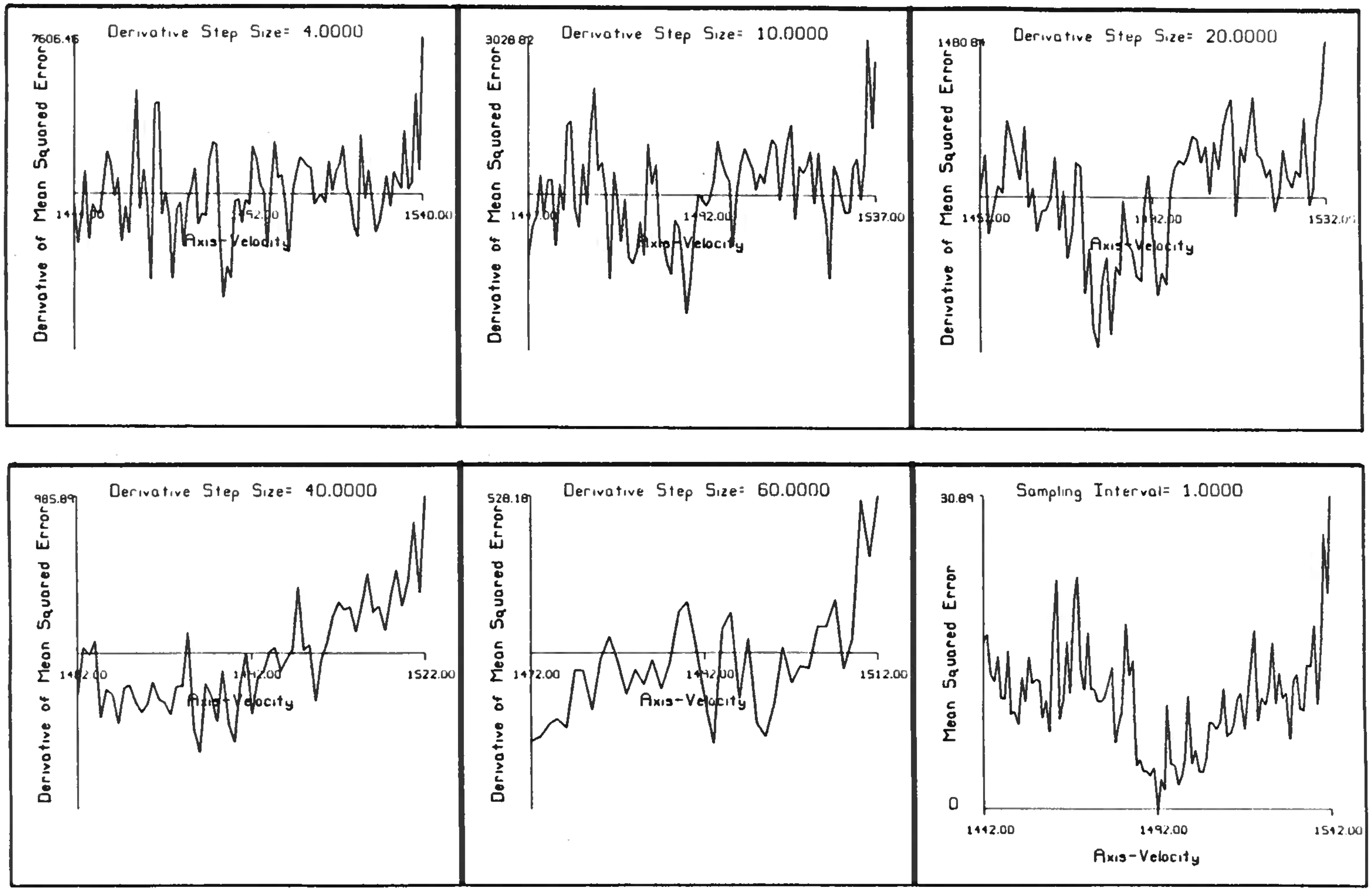
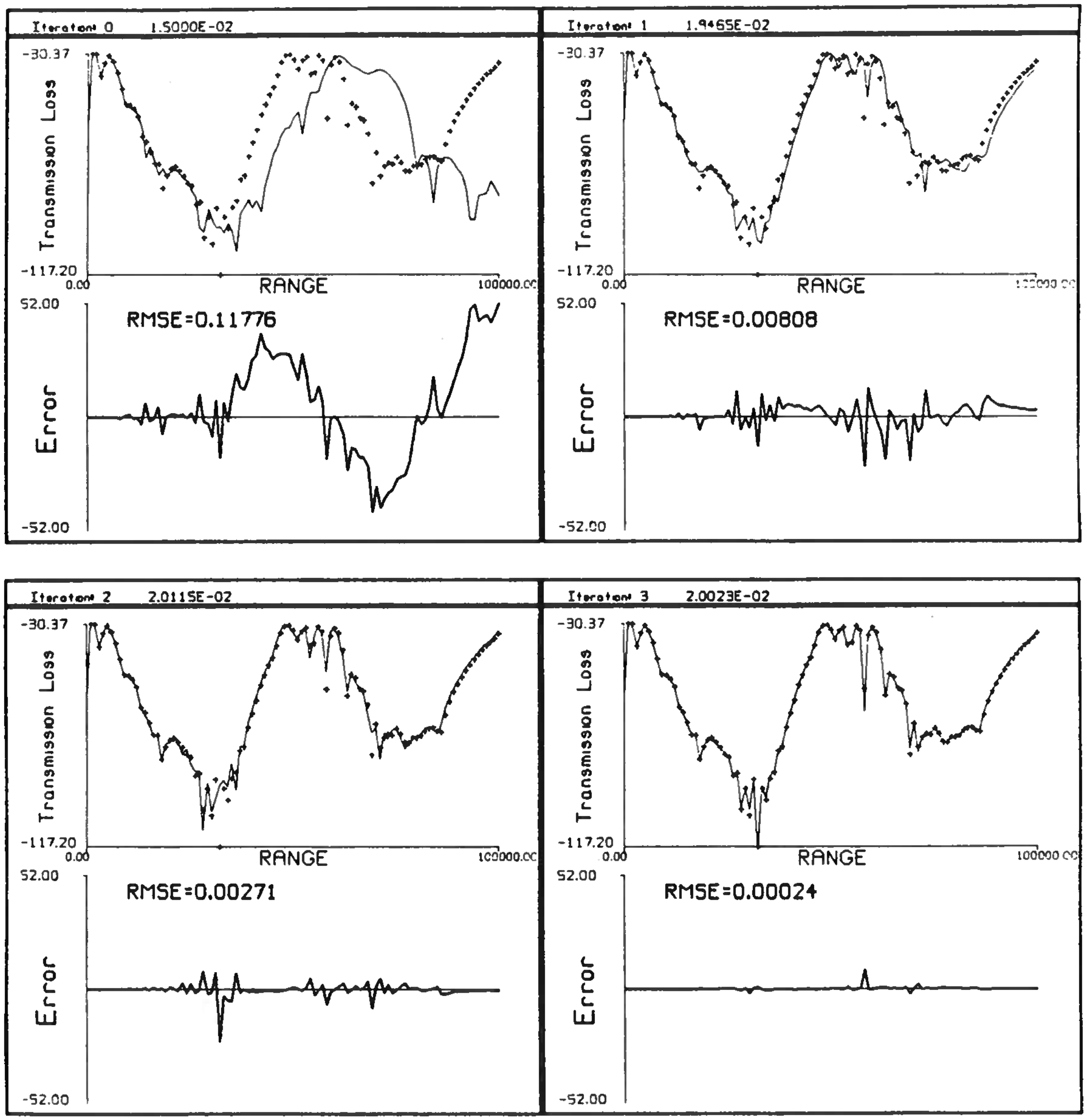


Figure 27. Differentiated error curve for a noisy parameter.

FIGURE 27



Example of a successful inversion for a single parameter using range-dependent data.

FIGURE 28

Appendix A: Graduate Students Supported by the SRO

A major strength of our program over the three years spanned by the SRO has been our graduate students. They have been an exceptionally talented and hardworking group, and a joy to work with. Moreover, they have substantially contributed to the quantity and quality of our results. Following is a list of these students and the degrees thus far earned and degrees projected. Below this list are the CWP reports and, in some cases, the resulting published articles.

Paul Docherty	(Geophysics) Master's Spring 1985; projected Ph.D. Summer 1987.
Tom Jorden	(Geophysics) Master's Spring 1987.
Peter Kaczkowski	(Geophysics) Master's Fall 1986.
Michael F. Sullivan	(Geophysics) Master's Spring 1983; (Geophysics) Ph.D. Spring 1987.
Brian L. Sumner	(Mathematics) projected Ph.D. Fall 1987.
Christopher Liner	(Geophysics) Ph. D. candidate
Donna Reeve	(Geophysics) Master's Spring 1986.
Shelby Worley	(Mathematics) Ph.D. candidate.

Student reports and papers

CWP-005 "Spatial-temporal Aliasing and the Wave Equation," Worley, S. C. and J. K. Cohen, 1984.

CWP-012 Seismic Tomography in Boreholes using an Algebraic Reconstruction Technique" by Kingsley Smith, Master's Thesis, 1984.

CWP-017 "Kirchhoff Modeling via Wave Equation Datuming", by Michael F. Sullivan, 1984

CWP-018 "A Fast Ray Tracing Routine for Laterally Inhomogeneous Media", by Paul Docherty, presented at the 1984 SEG Meeting, Atlanta, Ga., Master's Thesis, 1985.

CWP-027 "Pre-stack Kirchhoff Inversion of Common Offset Data", by Michael F. Sullivan and J. K. Cohen, Geophysics, to appear, June, 1987.

CWP-028 "Qualitative Analysis of Sign-bit Processing" by Isabelle Leroux, Master's Thesis, 1985.

CWP-029 "Analysis of Two-Parameter Constant Background Born Inversion for Acoustic Synthetic Data" by Paul B. Violette, Master's Thesis, 1985.

CWP-039 "Sound Speed Profile Inversion in the Ocean," by Linda Boden, Master's Thesis, 1985.

CWP-047 "Damped Least Squares Inversion Applied to Ducted Propagation of Acoustic Energy in the Ocean," by Peter Kaczkowski, Master's Thesis, 1986.

CWP-048 "Theory of Spatial Anti-aliasing of Log-stretched Multichannel Reflection Data," by Josua Ronen and Christopher Liner, 1987.

CWP-050 "Two-and-one-half Dimensional Common Shot Modeling," Paul Docherty, 1987.

CWP-052 "Transformation to Zero Offset," Thomas E. Jorden, Master's thesis, 1987.

CWP-054 "Pre-stack Kirchhoff Inversion and Modeling in 2.5 Dimensions", by Michael F. Sullivan, Ph. D. Dissertation, 1987.

Appendix B: Center for Wave Phenomena Visitors 1983-86

The following people visited during the indicated periods. In most cases the visitor presented one or more formal talks, and in all cases, shared research ideas. These visitors broadened our perspective and greatly enhanced our effectiveness.

Shimon Coen	University of California Berkeley	8/23/83
John Fawcett	Stanford University	3/20-21/84
Werner Güttinger	Institut fur Informationsverarbeitung, University of Tübingen, Cologne, Germany	5/15/84
Shimon Coen	University of California Berkeley	8/9/84
Joel S. Cohen	University of Denver	11/9/84
David Thomson	Defense Research Establishment, Canada	11/15/84
George V. Frisk	Woods Hole Oceanographic Institution	2/1-28/85
Jean Roberts	INRIA, France	2/4/85
Juan E. Santo	University of Buenos Aires, Argentina	2/27/85
Victor Barcilon	University of Chicago	3/22-4/10/85
Shalom Raz	The Technion, Israel Institute of Technology	11/10-21/85
Samuel Gray	Amoco	4/8-12/86
William Boyse	Stanford University	4/13-14/86
Thomas Roberts	Indiana University	4/15/86
T. M. Dunster	University of Dundee, Scotland	7/2-14/86
Malcolm Lightbody	Kings College, England	7/23-25/86
E. C. Shang	Institute of Acoustics, Academia Sinica, Beijing, China	8/10-13/86
Gregory Beylkin	Schlumberger/Doll Research Center	8/18/86
Robert Burridge	Schlumberger/Doll Research Center	8/18/86
Kees Wapenaar	Delft University, The Netherlands	11/19-30/87
Louis Fishman	Catholic University	11/16-21/86
Samuel Gray	Amoco	12/18-20/86

Appendix C: Published papers and research reports

DeSanto, J. A., "Oceanic sound speed profile inversion," IEEE J. Oceanic Eng., vol. OE-9, no. 1, pp. 12-17, 1984. Center for Wave Phenomena Research Report, CWP-001, 1983.

Hagin, F. G. and J. K. Cohen, "Refinements to the linear velocity inversion theory," Geophysics, vol. 49, no. 2, pp. 112-118, 1984. Center for Wave Phenomena Research Report, CWP-003, 1983.

Bleistein, N., J. K. Cohen, and F. G. Hagin, "Computational and asymptotic aspects of velocity inversion," Geophysics, vol. 50, no. 8, pp. 1253-1265, 1985. Center for Wave Phenomena Research Report, CWP-004, 1984.

Worley, S. C. and J. K. Cohen, "Spatial-temporal aliasing and the wave equation," Center for Wave Phenomena Research Report, CWP-005, 1984.

Bleistein, N., J. K. Cohen, J. A. DeSanto, and F. G. Hagin, "Project review on geophysical and ocean sound speed profile inversion," in Inverse problems of acoustic and elastic waves, ed. F. Santosa, Y. Pao, W. W. Symes, C. Holland, pp. 236-249, SIAM, Philadelphia, 1984. Center for Wave Phenomena Research Report, CWP-006, 1984.

Bleistein, N. and S. H. Gray, "An extension of the Born inversion method to a depth dependent reference profile," Geophys. Prosp., vol. 33, pp. 999-1022, 1985. Center for Wave Phenomena Research Report, CWP-007, 1984.

Sumner, B., "A Fortran 77 self-sorting mixed-radix fast Fourier transform package," Center for Wave Phenomena Research Report, CWP-009, 1984.

Gray, S. H. and N. Bleistein, "Seismic imaging and inversion," Proc. IEEE, vol. 74, no. 3, pp. 440-456, 1986. Center for Wave Phenomena Research Report, CWP-011, 1985.

Smith, K. L., "Acoustic tomography in boreholes using an algebraic reconstruction technique," Center for Wave Phenomena Research Report CWP-012, 1984. Master's thesis.

DeSanto, J. A., "Some computational problems in ocean acoustics," Proc. Computational Ocean Acoustics Workshop, Yale University, in Comp. and Maths. with Applics., vol. 11, pp. 755-763, 1985. Center for Wave Phenomena Research Report, CWP-013, 1984.

Bleistein, N., "Two-and-one-half dimensional in-plane wave propagation," Geophys. Prosp., vol. 34, no. 3, pp. 686-703, 1986. Center for Wave Phenomena Research Report, CWP-014, 1984.

Mager, R. D., "Asymptotic construction of a procedure for plane-wave synthesis and migration," Center for Wave Phenomena Research Report, CWP-015, 1984.

Bleistein, N., J. K. Cohen, F. G. Hagin, and J. A. DeSanto, "Progress Report: October 1, 1984 of the Selected Research Program of the Office of Naval Research at the Center for Wave Phenomena, Colorado School of Mines," Center for Wave Phenomena Research Report, CWP-016, 1984.

Sullivan, M. F., "Kirchhoff modeling via wave equation datuming," Center for Wave Phenomena Research Report, CWP-017, 1984.

Docherty, Paul, "A fast ray tracing routine for laterally inhomogeneous media," Center for Wave Phenomena Research Report, CWP-018, 1985. Presented at the 1984 SEG meeting.

Bleistein, N., J. K. Cohen, F. G. Hagin, J. A. DeSanto, and R. D. Mager, "Project Review, December 1, 1984, Consortium Project on Seismic Inverse Methods for Complex Structures," Center for Wave Phenomena Research Report, CWP-019, 1984.

Cohen, J. K. and F. G. Hagin, "Velocity inversion using a stratified reference," Geophysics, vol. 50, no. 11, pp. 1689-1700, 1985. Center for Wave Phenomena Research Report, CWP-021, 1984.

DeSanto, J. A., "Relation between the connected diagram and smoothing methods for rough surface scattering," J. Math. Phys., vol. 27, pp. 377-379, 1986. Center for Wave Phenomena Research Report, CWP-023, 1985.

DeSanto, J. A., "Ocean acoustics," in The encyclopedia of physics, third edition, ed. R. M. Besancon, pp. 836-840, Van Nostrand & Reinhold Inc, 1984. Center for Wave Phenomena Research Report, CWP-024, 1985.

Bleistein, N., J. K. Cohen, J. A. DeSanto, and F. G. Hagin, "Project Review, May 8, 1985, Consortium Project on Seismic Inverse Methods for Complex Structures," Center for Wave Phenomena Research Report, CWP-025, 1985.

DeSanto, J. A., "Exact spectral formalism for rough surface scattering," J. Opt. Soc. Am., Dec. 1985. Center for Wave Phenomena Research Report, CWP-026, 1985.

Sullivan, M. F. and J. K. Cohen, "Pre-stack Kirchhoff inversion of common offset data," Geophysics, vol. 52, no. 6, p. to appear, 1987. Center for Wave Phenomena Research Report, CWP-027, 1985.

Leroux, I., "Qualitative analysis of sign-bit processing," Center for Wave Phenomena Research Report, CWP- 028, 1985. Master's thesis.

Violette, Paul B., "Analysis of two-parameter constant background Born inversion for acoustic synthetic data," Center for Wave Phenomena Research Report, CWP-029, 1985. Master's thesis.

DeSanto, J. A. and G. S. Brown, "Analytical techniques for multiple scattering from rough surfaces," in Progress in optics, vol. 23, ed. E. Wolf, North-Holland Publishing Co. Inc, Amsterdam, 1986. Center for Wave Phenomena Research Report, CWP-030, 1985.

Cohen, J. K., F. G. Hagin, and N. Bleistein, "Three dimensional Born inversion with an arbitrary reference," Geophysics, vol. 51, no. 8, pp. 1552-1558, 1986. Center for Wave Phenomena Research Report, CWP-031, 1985.

Bleistein, N., J. K. Cohen, and F. G. Hagin, "Two-and-one-half dimensional Born inversion with an arbitrary reference," Geophysics, vol. 52, no. 1, pp. 26-36, 1987. Center for Wave Phenomena Research Report, CWP-032, 1985.

DeSanto, J. A. and G. S. Brown, "Some recent results in rough surface scattering," Center for Wave Phenomena Research Report, CWP-033, 1985. To be published in Proceedings of a Conference on "Multiple Scattering of Waves in Random Media and Random Rough Surfaces", Pennsylvania State University.

Bleistein, N., "An introduction to the mathematical theory of wave phenomena," in Encyclopedia of physical science and technology, Academic Press Inc, New York, 1986. Center for Wave Phenomena Research Report, CWP-034, 1985.

DeSanto, J. A., "Impedance at a rough waveguide boundary," Wave Motion, vol. 7, pp. 307-318, 1985. Center for Wave Phenomena Research Report, CWP-035, 1985.

Bleistein, N., "Progress on an inverse method for seabed mapping and seismic exploration," in Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling, January, 1984, SIAM, Philadelphia, 1986. Center for Wave Phenomena Research Report, CWP-036, 1985.

Bleistein, N., J. K. Cohen, F. G. Hagin, and J. A. DeSanto, "Progress Report: October 1, 1985 of the Selected Research Program of the Office of Naval Research at the Center for Wave Phenomena, Colorado School of Mines," Center for Wave Phenomena Research Report, CWP-037, 1985.

Bleistein, N., "On the imaging of reflectors in the earth," Geophysics, vol. 52, no. 7, p. to appear, 1987. Center for Wave Phenomena Research Report, CWP-038, 1985.

Boden, L., "Sound speed profile inversion in the ocean," in Proc. of the I.C.A. Congress Symposium on Underwater Acoustics, Halifax, N.S., Canada, 1986. Center for Wave Phenomena Research Report, CWP-039, 1985.

Bleistein, N., J. K. Cohen, and F. G. Hagin, "Project Review, May 16, 1986, Consortium Project on Seismic Inverse Methods for Complex Structures," Center for Wave Phenomena Research Report, CWP-040, 1986.

Bleistein, N., "Kirchhoff inversion for reflector imaging and sound speed and density variations," in Proceedings of EAEG/SEG Workshop on Deconvolution and Inversion, Rome, Italy, 1986, Center for Wave Phenomena Research Report, CWP-041, 1986.

Gilliam, D. S. and F. G. Hagin, "Some algebraic aspects of the Lamb problem," Center for Wave Phenomena Research Report, CWP-042, 1986.

Cohen, J. K. and N. Bleistein, "Asymptotic methods for seismic modeling and inversion," Center for Wave Phenomena Research Report, CWP-043, 1986. Short course, Norwegian Institute of Technology, Trondheim, Norway.

DeSanto, J. A., "Mathematics of seismology," Center for Wave Phenomena Research Report, CWP-044, 1986. Lecture notes.

Quist, P. W., "Computational aspects of three dimensional seismic inversion," Center for Wave Phenomena Research Report, CWP-045, 1986. Master's thesis.

Bleistein, N., F. G. Hagin, P. Docherty, T. E. Jorden, J. Ronen, M. F. Sullivan, and B. Sumner, "Project Review, November 2, 1986, Consortium Project on Seismic Inverse Methods for Complex Structures," Center for Wave Phenomena Research Report, CWP-046, 1986.

Kaczkowski, P., "Damped least squares inversion applied to ducted propagation of acoustic energy in the ocean," Center for Wave Phenomena Research Report, CWP-047, 1986. Master's thesis.

Appendix D: Presentations at National and International Meetings

This appendix contains a list of contributions and invited lectures by the principal investigators at national and international meetings through the period covered by the Selected Research Opportunities Porject.

Norman Bleistein

"Wave equation migration deduced as an inversion method from the wave equation", Society of Exploration Geophysicists, 53rd Annual International Meeting, Las Vegas, October, 1983.

"Highly accurate inversion methods for three dimensional stratified media", with M. Lahlou and J. K. Cohen, International meeting to honor J. B. Keller on his 60th birthday, Northwestern University, September, 1983.

"Inverse methods for seismic exploration", SOHIO Petroleum, Dallas, TX, March, 1984.

"Inverse methods for reflector imaging and parameter estimation", Colloquium, California Institute of Technology, Pasadena, CA, March, 1984.

"Multi-valued functions and their application in applied analysis", one week short course, Naval Undersea Systems Center, New London, CT, May, 1984.

"Progress report of ONR research program on geophysical and ocean sound speed inversion", with J. A. DeSanto, Conference on Inverse Problems of Acoustic and Elastic Waves, Cornell University, Ithace, NY, June, 1984.

"Seismic inverse methods for reflector imaging", Invited Plenary Lecture, International Symposium on Nonlinear Differential Equations, University of Dundee, Scotland, June 1984.

"Born inversion for depth-dependent background velocity", co-authored with S. H. Gray, International Meeting of the European Association of Exploration Geophysicists, London, June 1984.

"Inversion of CMP stacked data with a depth-dependent background velocity", with S. H. Gray, Society of Exploration Geophysicists, 54th Annual International Meeting, Atlanta, December, 1984.

"Computational and asymptotic aspects of velocity inversion", co-authored with J. K. Cohen and F. G. Hagin, Society of Exploration Geophysicists, 54th Annual International Meeting, Atlanta, December, 1984.

"A fast ray tracing routine for laterally inhomogeneous media", with P1 Docherty, Society of Exploration Geophysicists, 54th Annual International Meeting, Atlanta, December, 1984.

"Project review: seismic inverse methods for complex structures", Consortium Project Review Meeting, Society of Exploration Geophysicists, 54th Annual International Meeting, Atlanta, December, 1984.

"Progress on inverse methods for seabed mapping and seismic exploration", Invited Plenary Lecture, Conference on Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling, Houston, TX, January, 1985. Published in Conference Proceedings, Ed., W. E. Fitzgibbon, SIAM, Philadelphia, 1986.

"Inverse methods for seismic exploration", with J. K. Cohen, Mobil Research and Development Corp., Dallas, TX, January, 1985.

Visiting Scholar, Undergraduate Honors Program, University of Tulsa, February, 1985.

"Inverse methods for seismic exploration", with J. K. Cohen, Union Oil Company, Brea, CA, March, 1985.

"Overview, inversion research at CSM", Consortium Project Review Meeting, CSM, May, 1985

"Mathematical methods for wave phenomena", one week short course, Naval Undersea Systems Center, New London, CT, May, 1985.

"Two-and-one-half dimensional in-plane wave propagation", European Association of Exploration Geophysicists, International Meeting, Budapest, June, 1985.

"Asymptotic two-and-one-half dimensional modeling from two-dimensional computations", Society of Exploration Geophysicists, 55th Annual International Meeting, Washington, D. C., October, 1985.

"Project review: seismic inverse methods for complex structures", Consortium Project Review Meeting, Society of Exploration Geophysicists, 55th Annual International Meeting, Washington, D. C., October, 1985.

"Research on inverse methods at the Center for Wave Phenomena", one week seminar at the University of Kassel, West Germany, February, 1986.

"Seismic imaging and inversion", with S. H. Gray, invited paper, Special Issue on Inversion, Proceedings, IEEE, 440-456, March 1986.

"Project review: seismic inverse methods for complex structures", Consortium Meeting, CSM, May, 1986.

"Kirchhoff inversion for reflector mapping and soundspeed and density variations", invited lecture, Bay Area Conference on Inverse Problems, Stanford University, July, 1986.

"Kirchhoff inversion for reflector mapping and soundspeed and density variations", invited lecture, First Joint EAEG/SEG Workshop on Deconvolution and Inversion, Rome, September, 1986.

"Kirchhoff inversion for reflector mapping and soundspeed and density variations", invited lecture, First Joint EAEG/SEG Workshop on Deconvolution and Inversion, Rome, September, 1986.

"Multi-dimensional seismic inversion", two week short course presented at the Technological Institute of Norway, Trondheim, with J. K. Cohen, October, 1986.

"Project overview: seismic inverse methods for complex structures", STATOIL of Norway, Stavanger, October, 1986.

"Project overview: seismic inverse methods for complex structures", GECO of Norway, Stavanger, October, 1986.

"Project review: seismic inverse methods for complex structures", Consortium Project Review Meeting, Society of Exploration Geophysicists, 56th Annual International Meeting, Houston, November, 1986.

Jack K. Cohen

"Born inversion for a stratified media", with F. G. Hagin, Conference on Inverse Problems of Acoustic and Elastic Waves, Cornell University, Ithaca, NY, June 5, 1984.

"Algorithm for Born inversion in a stratified medium: poster talk", with F. G. Hagin, Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, Dec. 1984, Atlanta.

"Born inversion with a stratified reference", invited workshop lecture at the Joint SIAM-SEG-SPE Meeting, Jan. 1985, Houston.

"Overview of research supported by the Consortium Project", with N. Bleistein, Mobil Oil Company, Jan. 1985, Dallas.

"Born inversion with a stratified reference", Mobil Oil Company, Jan. 1985, Dallas.

"Overview of research supported by the Consortium Project", with N. Bleistein, Union Oil Company, Mar. 1985.

"Born inversion with a stratified reference", Union Oil Company, Mar. 1985.

"Overview of research supported by the Consortium Project", Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, May 1985, Golden.

"Born inversion with a stratified reference", Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, May 1985, Golden.

"Inversion with a curved datum", Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, May 1985, Golden.

"A new inversion methodology", Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, May 1985, Golden.

"Recent advances in algorithms for the seismic inverse problem", S-Cubed Company, July 1985, La Jolla.

"Recent work at the Center for Wave Phenomena", Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, Sep. 1985, Washington.

"Imaging of Flaws in Solids by Velocity Inversion", with N. Bleistein and F. G. Hagin, Annual Review of Progress in Quantitative NDE at the University of California, La Jolla, August 3-8, 1986.

"Kirchhoff Migration with Amplitude Preservation," lecture series at the Texaco Briarpark Research Center, Houston, Texas, Fall, 1986.

"Asymptotic Methods for Seismic Modeling and Inversion", with N. Bleistein, a two week short course at The Norwegian Institute of Technology, Trondheim, Norway, October 6-17, 1986.

John A. DeSanto

"Scattering from Periodic Surfaces", Mathematics Department Colloquium, University of Denver, February 2, 1983.

"Scattering from Random Surfaces", J. Acoust. Soc. Am. Suppl. 1, 73, S10, 1983, invited paper.

"Sound Speed Profile Inversion Using Modes", J. Acoust. Soc. Am. Suppl. 1, 73, S38, 1983, invited paper.

"Scattering from a Rough Interface", SIAM 1983 National Meeting, Denver, CO, June 6, 1983.

"Single Integral Equation Formalism for Scattering from a Rough Interface", Army Research Office Conference on Propagation in Random Media and Scattering from Rough Surfaces, Quail Roost, NC, August 10, 1983.

"Integral Equation Solution for the Coherent Wave for Scattering from a Rough Surface", Army Research Office Conference on Propagation in Random Media and Scattering from Rough Surfaces, Quail Roost, NC August 10, 1983.

"Oceanic Sound Speed Profile Inversion", IEEE IAGARS Symposium, San Francisco, September 1, 1983, invited paper.

"Scattering from Rough Surfaces", Office of Naval Research Workshop on the Arctic, San Diego, CA, November 4, 1983.

"Progress Report of ONR Research Group on Geophysical and Ocean Sound Speed Profile Inversion", Conference on Inverse Problems of Acoustic and

Elastic Waves, Cornell University, Ithaca, NY, June 5, 1984, with N. Bleistein, invited paper.

"Scattering from Rough Ocean Surfaces", Woods Hole Oceanographic Institution, Woods Hole, MA, July 10, 1984, invited paper.

"Reconstructing the Ocean", Woods Hole Oceanographic Institution, Woods Hole, MA, July 11, 1984, invited paper.

"Sound Speed Profile Inversion", Woods Hole Oceanographic Institution, Woods Hole, MA, July 13, 1984, invited paper.

"Some Computational Problems in Ocean Acoustics", Workshop on Computational Ocean Acoustics, Yale University, New Haven, CT, August 1, 1984, invited paper.

"A Critique of Theoretical Approaches and a Discussion of Outstanding Problems Associated with Scattering from Randomly Rough Surfaces", XXIst General Assembly of URSI, Florence, Italy, August 31, 1984, with G. S. Brown, invited paper.

"Sound Speed Profile Inversion in the Ocean", Consortium Project Review, Colorado School of Mines, Golden, CO, May 9, 1985.

"Impedance at a Rough Waveguide Boundary", URSI Meeting, Vancouver, British Columbia, June 17, 1985.

"Some Recent Results in Rough Surface Scattering", Workshop on Multiple Scattering of Waves in Random Media and Random Rough Surfaces, Penn State University, University Park, PA, August 1, 1985, invited paper.

"Sound Speed Profile Inversion in the Ocean", Society of Exploration Geophysicists (SEG) Meeting, Washington, DC, October 6-11, 1985, with L. Boden.

"Review of Rough Surface Scattering", John Wright Memorial Lecture, Naval Research Laboratory, Washington, DC, October 10, 1985, invited lecture.

"Multiple Scattering at Rough Ocean Boundaries", Symposium on Underwater Acoustics, Halifax, Nova Scotia, Canada, July, 1986.

"Sound Speed Profile Inversion, Symposium on Underwater Acoustics, Halifax, Nova Scotia, Canada, July, 1986, with L. Boden.

An Exact Modal Solution in Two Dimensions", Acoustical Society of America, 112th Meeting, Anaheim, CA, December 9, 1986.

"Soundspeed Profile Inversion in the Ocean", Acoustical Society of America, 112th Meeting, Anaheim, CA, December 11, 1986 (with L. Boden).

"Application of Discrete Linearized Inversion to the Sofar Inverse Problem", Acoustical Society of America, 112th Meeting, Anaheim, CA, December 11, 1986.

Frank G. Hagin

"Born inversion for a stratified media", with J. K. Cohen, Conference on Inverse Problems of Acoustic and Elastic Waves, Cornell University, Ithaca, NY, June 5, 1984.

"Algorithm for Born inversion in a stratified medium: poster talk", with J. K. Cohen, Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, Dec. 1984, Atlanta.

"Some analytical treatment of ill-posed problems", invited talk at Amoco Production Company, 1985, Tulsa, Ok.

"Velocity inversion using a variable reference", with J. K. Cohen, Annual Meeting of the Society of Exploration Geophysicists, 1985, Washington.

"Some experiences with 3-dimensional velocity inversion using the Cray", Consortium Project on Seismic Inverse Methods for Complex Structure Meeting, May, 1986, Golden.

Graduate Students

Graduate students have made presentations at each of our consortium project reviews. In addition, the following students made presentations at the ONR project review on Inverse Methods for Seabed Mapping and Ocean Sound Speed Profile Inversion, Cornell University, Ithaca, New York, June, 1984. Linda Boden, Paul Docherty, Michael J. Sullivan, Brian Sumner, Shelbey Worley.

Other talks by students follow.

Linda Boden: Sound Speed Profile Inversion in the Ocean: Woods Hole Oceanographic Institute, Woods Hole, MA, August 21, 1985.

Linda Boden: Sound Speed Profile Inversion in the Ocean: Society of Exploration Geophysicists 55th Annual Internation Meeting, October, 1985.

Linda Boden: Sound Speed Profile Inversion in the Ocean: Naval Research Laboratory, Washington, D. C., October 9, 1985.

Linda Boden: Sound Speed Profile Inversion in the Ocean: Symposium on Underwater Acoustics, Halifax, N. S., Canada, July 17, 1986.

Paul Docherty: A fast ray tracing routine for laterally inhomogeneous media: Presented at 54th Annual SEG Meeting, Atlanta, November, 1984.

Paul Docherty: Accurate migration of laterally inhomogeneous media: Presented at 55th Annual SEG Meeting, Washington D. C., October, 1985.

Michael F. Sullivan: Design Considerations in Finite Element Program

Development: Society of Exploration Geophysicists 53rd Annual Internation Meeting, Las Vegas, October, 1983.

Michael F. Sullivan: An Efficient 2.5-D Kirchhoff Modeling Routine: Society of Exploration Geophysicists 55th Annual Internation Meeting, Washington, D.C., 1985.

Michael F. Sullivan: Kirchhoff Modeling and Wave Equation Datuming Mobil Research and Development, Dallas, Texas, 1984

Thomas E. Jorden: Transformation to Zero Offset: Society of Exploration Geophysicists 56th Annual Internation Meeting, Houston, November, 1986.

Thomas E. Jorden: An Inversion-Based Integral Dip-Moveout: 40th Annual Meeting of the Midwest SEG, Dallas, December, 1986.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Abstract

This is a final report for the project, The Application of Inverse Methods to the Ocean Environment, supported by the Selected Opportunities Research Program of the Office of Naval Research at the Colorado School of Mines. The research has been carried out in the Center for Wave Phenomena in the Mathematics Department. The SRO has partially supported four faculty members: Norman Bleistein, Jack K. Cohen, John A. DeSanto and Frank G. Hagin. Over the course of the program, seven students were also partially supported by this program.

The SRO program has had an extremely positive effect on graduate education and scholarly activity at the Colorado School of Mines. It has been a catalyst for generating support for related research, so that, in total, we have supported between five and seven graduate students through each year of the SRO program. Five graduate courses directly related to our research program have been introduced. Six students have completed -- or are about to complete -- graduate degrees including three PhDs. In addition, many distinguished scholars have visited the Center in support of our research activities.

During the three years spanned by the SRO, we have made major progress in the theory and application of inversion methods. At the start of the SRO, our inversion techniques were restricted to idealized data and physical assumptions. At the conclusion of the SRO, the theory and techniques had been extended to encompass most of the standard data collection procedures and were based on much more realistic physical assumptions. Our research efforts have been reported in technical reports, journal articles and proceedings and have been presented at national and international meetings including invited talks and invited articles.

The SRO program at the Colorado School of Mines consisted of two major projects in inverse scattering. The first of these is reflector imaging for seabed mapping and seismic exploration. The second is ocean profile inversion. The progress achieved in both of these projects under the support of this contract is described in this report. The work continues.