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Fully Coupled Thermal-Hydraulic-Mechanical Reservoir Simulation with Non-Isothermal Multiphase Compositional Modeling

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Abstract

We present the development and application of a multi-physical simulator for evaluating the combined thermal-hydraulic-mechanical behaviors of petroleum reservoirs. The simulator combines non-isothermal multiphase compositional modeling with coupled geomechanical simulation module. The simulator consists of two major modules, namely, the fluid and heat flow module and the geomechanical module. An isenthalpic flash calculation approach is implemented in the fluid and heat flow module. In the flash calculation module, a nested approach is adopted, in which PT flash calculations are conducted in the inner loop and temperature is updated in the outer loop. The iteration is continued until both the fugacity and energy stopping criteria are satisfied. An improved version of the Beltrami-Michell equation, called extended Beltrami-Michell equation, has been derived and implemented in the geomechanical simulation module to simulate heterogeneous and plastic behavior of formation rocks. The three normal stress components inside the stress tensor are solved simultaneously with the pressure and enthalpy in the fluid/ heat module, ensuring the mass/energy conservation. The newly-derived extension of the Beltrami-Michell equation is capable of handling materials with changing mechanical properties. This way, the simulator is able to capture the phase change as well as the poro-mechanical effects on rock deformation induced by fluid injection/extraction. The multi-physics simulator is built on an object-oriented parallel simulation framework, with a speedup factor up to hundreds.

Introduction

The recovery of oil/gas and thermal energy from petroleum/geothermal reservoirs typically involves complex thermal-hydraulic-mechanical (THM) processes. For instance, problems with rock failure in the vicinity of an injection well induced by cold water injection and permeability-porosity change during production require the simulator to be able to accurately predict the transient pressure, temperature, and stress fields of the reservoir.

Recently, coupled simulation of thermal-hydraulic-mechanical processes in reservoirs has become an appealing subject in reservoir simulation (Fung et al., 1994; Wan et al., 2003). Rutqvist et al. (2002)

developed TOUGH2-FLAC3D, which couples TOUGH2 (Pruess and Oldenburg, 1999) reservoir simulator with FLAC3D (Itasca Consulting Group Inc., 1997). (Fu, et al., 2013) and (Ghassemi et al., 2015) brought out approaches to simulate discrete fracture networks. (Liu and Bodvarsson, 2003; Li, et al., 2014) proposed to use a constitute model to simulate the fractures by a 'two parts' Hook's law.

The above-mentioned mechanical coupling frameworks are either sequentially or iteratively coupled, where the pressure-temperature filed and the stress filed are solved separately. Kim at el. (2009) analyzed that for linear problems, the sequentially coupled framework could be unconditionally stable. However, for highly nonlinear problems, the stability and accuracy of the sequentially coupled framework is still questionable. In this sense, fully coupled THM frameworks that solve all physical field simultaneously are needed in industrial application, where problems are usually highly nonlinear. For fully coupled frameworks, 'mean stress' method, which solves the volumetric stress of the system, is a simple yet accurate approach. The sequentially-coupled and fully-coupled mean stress method has presented in (Zhang et al., 2015) and (Zhang et al., 2012), respectively.

Modern reservoir simulation should be able to simulate giant reservoirs with a giant reservoir simulator. Meanwhile, as mentioned above, multi-physical problems introduce more equations to be solved than before. Therefore, such fast increasing computational demand in reservoir simulation is pushing the industry to improve the speed of current reservoir simulators. As a result, parallel computing has been developed and widely applied. Based on the granularity, parallel computing can be classified into several categories, such as bit-level, thread-level, core-level, task-level, and so on. Based on the memory usage and data storage approach, parallel computing can be classified into two forms: shared memory computing and distributed computing. The very early trails on parallel reservoir simulation (Dongarra et al., 1989) were mostly based on share memory computing, in which all threads/processes have equal access to the memory. As the development of computing clusters as well as parallel communication platforms, parallel reservoir simulation has switched to distributed computing (Wheeler and Smith, 1990; Killough, J. E., & Bhogeswara, R. (1989, July).), in which data is stored and distributed among processors and all processors work together to conduct the reservoir simulation, which enables it to handle a large scale problem with billions of grid blocks (Dogru, et al, 2009).

In this work, we present a novel reservoir simulator THM-EGS. THM-EGS is a massively-parallel multiphysical reservoir simulator. Initially designed for the simulation of enhanced geothermal reservoirs, (Wang et al., 2016). THM-EGS has been modified and applied to oil/gas reservoirs. THM-EGS has been equipped with an extensible framework to include multiple physical processes. Based on its power parallel framework, THM-EGS has the capability to simulate cases with millions of grid blocks. The current version of THM-EGS, as presented in this paper, is able to simulate the three normal components of the stress tensor as well as to conduct non-isothermal composition simulation.

Fluid and Heat Flow Formulation

The governing equations of the thermal-hydraulic simulation module of THM-EGS are based on mass/ energy conservation. The mass/energy conservation equation for component k is as follows

$$\frac{\partial M^k}{\partial t} = \nabla \cdot \vec{F}^k + q^k \tag{1}$$

In the above equation, the *M* is the accumulated term, \overline{F}^k is the flux term, and q is the sink/source term. In THM-EGS, k = 1 refers to water component. k=2,...,nc+1 refer to hydrocarbon component (*nc* is the number of components). k=nc+2 refers to thermal energy. In the current study, water and hydrocarbons are assumed to be not mutual soluble, meaning that there is neither water component in hydrocarbon phases nor hydrocarbon components in water phases.

For water components,

$$M^{1} = \phi S_{W} \rho_{W} \tag{2}$$

For hydrocarbon components

$$M^{k} = \phi S_{L} \rho_{L} L^{k} + \phi S_{G} \rho_{G} G^{k}, \quad k = 2, .., nc + 1$$
(3)

In the above equation, G refers to gas hydrocarbon phase while L refers to liquid hydrocarbon phases.

$$M^{nc+2} = (1-\phi)C_r\rho_r T + \phi \sum_l S_l\rho_l U_l$$
(4)

The flux term \vec{F}^k is calculated as

$$\vec{F}^{k} = \sum_{\beta} \vec{F}_{\beta} X^{k}_{\beta}$$
⁽⁵⁾

In the above equations, X_l^k is the mass fraction of component k in phase l. In our case, $X_L^1 = X_G^1 = 0$, $L^k = X_L^k$, k = 2,...,nc+1, $G^k = X_G^k$, k = 2,...,nc+1 and $X_W^k = 0, k = 2,...,nc+1$.

The phase flux \vec{F}_{β} is calculated by the multiphase Darcy's law:

$$\vec{F}_{\beta} = -K \frac{K_{r\beta} \rho_{\beta}}{\mu_{\beta}} \Big(\nabla P + \nabla P_{c,\beta} - \rho_{\beta} \mathbf{g} \Big)$$
(6)

where *K* is absolute permeability, $K_{r\beta}$ is relative permeability of phase β , μ is phase viscosity, P_c is phase capillary pressure, and *g* is the gravity vector. The energy flux has heat conduction and convection as

$$\mathbf{F}^{N+1} = -k_t \nabla T + \sum_l h_l \mathbf{F}_l \tag{7}$$

where k_t is thermal conductivity and h is specific enthalpy.

Mechanical Simulation Formulation

The geomechanical simulation module of the previous version of THM-EGS is based on mean stress method, as discussed in (Wang et al., 2016). In the current improved version of THM-EGS, the three normal components of the stress tensor are all solved simultaneously with the fluid and heat flow equations, making the geomechanical simulation a 'true' stress simulation module. The governing stress equations for the three normal stresses are as follows

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2}{\partial x^2} \left[\frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left(\sigma_{xx} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) + \frac{\partial}{\partial x} F_x = 0$$
(8)

$$\frac{\partial^2 h}{\partial y^2} + \frac{\partial^2}{\partial y^2} \left[\frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left(\sigma_{yy} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) + \frac{\partial}{\partial y} F_y = 0$$
(9)

$$\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2}{\partial z^2} \left[\frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left(\sigma_{zz} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) + \frac{\partial}{\partial z} F_z = 0$$
(10)

In the above equations, the mean stress term σ_m is defined as $\sigma_m = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$ and *h* is the pressureheat term as $h = \alpha P + 3\beta K (T - T_{reference})$. The geomechanical equations are essentially 'stress' equations, meaning that the primary variables of them are stresses-strain rather than force-displacement. The governing geomechanical equations are an extended version of the Beltrami-Michell equation. This extended Beltrami-Michell equation is able to solve heterogeneous materials. Meanwhile, the extension of the Beltrami-Michell equations to multi-porosity system is presented in (Winterfeld and Wu, 2015). The detailed derivation of the extended Beltrami-Michell equation used in this work is shown in the appendix.

Non-isothermal Flash Calculation

We adopt a nested approach to conduct the isenthalpic flash calculation, as used (Agarwal et al., 1991). In an isenthalpic flash calculation, mass component distribution among all phases is calculated with given pressure (P) and enthalpy (H). In this way, isenthalpic flash calculation is also called PH-flash. Compared to isothermal flash calculation (PT-flash), isenthalpic flash calculation is believed to be more robust in capturing the 'narrow-boiling' behavior of fluids (Zhu and Okuno, 2014).

In the nested PH-flash, the 'target' variable is the enthalpy of the system and the target function is shown below.

$$g_m = h_m - h_{given} = \left[(1 - \phi) \rho_R C_R T + \phi \sum_\beta S_\beta \rho_\beta h_\beta \right] - h_{given}$$
(11)

The algorithm starts with an initial enthalpy (usually the one used for the last time step). From the given enthalpy, the temperature of the system can be solved based on the 'current' component distribution.

The temperature is then sent to PT-flash. Therefore, for each given temperature, a PT-flash is conducted to determine the corresponding component distribution.

The temperature is updated as follows

$$T_{m+1} = T_m - g_m \frac{T_m - T_{m-1}}{g_m - g_{m-1}}$$
(12)

As for the VLE PT-flash, we use negative flash algorithm (Whitson and Michelsen, 1989) with the object function, Rachford-Rice (R-R) equation, as shown in the equation below

$$\sum_{i=1}^{N_c} \frac{Z_i \left(K_i - 1 \right)}{1 + v_\beta \left(K_i - 1 \right)} = 0 \tag{13}$$

During the PT-flash iteration, the R-R equation is solved with the Newton-Raphson method and stability test approach (Michelsen, 1982) is adopted to determine phase transition. Peng-Robinson equation of state (Peng and Robinson, 1976) is used to calculate the pressure-volume-temperature (PVT) properties of the mixture system. Once the Newton-Raphson method gets converged, the saturation of each phase is calculated from the solved phase densities and phase ratios as

$$S_{\beta} = \frac{v_{\beta} / \rho_{\beta}}{\sum_{j=1}^{N_{p}} v_{j} / \rho_{j}}$$
(14)

The equilibrium ratio used in the flash calculation is initialized by Wilson's equation (Wilson, 1969)

$$K_{i} = (P_{ci}/P) \exp[5.37(1+\omega_{i})(1-T_{ci}/T)]$$
(15)

The flowchart of the PH-flash is shown in Figure 1. In THM-EGS, the flash calculation is conducted 'outside' the time loop. Therefore, this part is separately parallelized as shown in Figure 2. The flash calculation loops through the entire (T, P, X) domain to construct the phase behavior contour. Inside the time loop, the secondary variables for each set of primary variables are interpolated from the contour. In this way, the computing load is significantly reduced.



Figure 2—Flowchart of the parallelization of PH-flash.

Numerical Methods

THM-EGS adopts the Integral Finite Difference (IFD) method (Narasimhan and Witherspoon, 1976; Celia et al., 1990; Winterfeld and Wu, 2015) to solve the above highly nonlinear governing equation. In the IFD method, all the governing equations are discretized as follows (Winterfeld and Wu, 2015)

$$\frac{V^{i+1}M^{k,i+1} - V^{i}M^{k,i}}{\Delta t} = \sum_{m} A_{m}F_{m}^{k} + q^{k}$$
(16)

Where *i* denotes the 'current' time step and *i*+1 denotes the 'next' time step. *m* denotes the connection between two neighboring grid blocks. *V* is the volume of the grid block. F^k is the flux term for equation *k* between two neighboring grid blocks. *A* is the area of the connection. q^k is the sink/source term of equation *k*.

Compared with the conventional Finite Difference method, the Integral Finite Difference method is more flexible in handling unstructured grid blocks. THM-EGS is based on a connection list. Within each iteration step, the simulator loops through all of the neighing grid blocks of a grid block and constructs the corresponding portion that is related to the grid block. The constructed Jacobian matrix and the associated residual vector are then sent to the linear solver. The detailed structure is shown in (Wang, et al., 2014). As for the nonlinear solver, THM-EGS has been implemented with both Newton-Raphson method (Wang et al., 2016) and the inexact Newton nonlinearity elimination method (Wang et al., 2015)

Simulator Framework

THM-EGS is a massively parallel simulator with a master-slave parallel communication framework. The parallel framework of its previous versions has been presented in (Wang et al., 2014). In this current version, we use the same parallel framework with slight modifications. The modifications are mostly distributing and communicating newly added data among processes.

In THM-EGS, the reservoir is viewed as a computational domain. The domain is firstly discretized and then divided into several subdomains via certain graph partitioning techniques. The subdomains are distributed to processes and the related data, including the reservoir properties and numerical parameters of the subdomain, is stored in memory that is only allowed to be accessed by certain processes. During the simulation process, all processes exchange data via parallel communication. Such storage and communication scheme is a typical distributed computing scheme, as shown in Figure 3.



Figure 3—Conceptual model of domain decomposition and communication among processors in parallel reservoir simulation.

Each subdomain has *Inner* and *Border* grid blocks, surrounded by solid and dash red line, respectively. For one subdomain, its *Border* grid blocks are the boundary of it. It is obvious that some *Inner* grid blocks of one subdomain will be the *Border* grid blocks of its neighboring submains. In Figure 3, such overlapped grid blocks are marked by green color, in contrast to the non-overlapped grid blocks in yellow. During the simulation process, neighboring processes the data of the overlapped grid blocks. The communication between processes is realized by MPICH2 (Gropp, et al. 1999) and PVM (Geist, 1994).

Case Study

We present the application of THM-EGS to two cases to show the capability and accuracy of our simulator. The geometry we use is shown in Figure 4



Figure 4—Conceptual model of Case 1 and Case 2.

Case 1 Stress-sensitive permeability problem

In this problem, we use THM-EGS to simulate a reservoir whose permeability is sensitive to stress changes.

A well is injecting water at the center of a reservoir, as shown in Figure 4. The reservoir zone (2m thick) is below a caprock zone (1000m thick), but only 20m of the caprock zone is taken into consideration. According to the simulation results, the pressure variation only propagates 5m into the caprock zone, due to its low permeability. The initial temperature of the reservoir is 140 °C.

The permeability of the reservoir is subject to the following correlation

$$K = K_0 \exp\left(\frac{c \cdot dP}{\sigma}\right) \tag{17}$$

In the above equation, K_0 is the initial permeability of the reservoir. dP is the pressure change. The mean stress is defined as

$$\sigma = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} \tag{18}$$

C is a constant. The input parameters of Case 1 are listed in Table 1.

Properties	Values	Units
Initial permeability of the reservoir	1.0*10 ⁻¹³	m^2
Initial permeability of the caprock	$1.0*10^{-18}$	m^2
Porosity of the reservoir zone	0.0001	dimensionless
Porosity of the caprock	0.01	dimensionless
Young's modulus of the caprock	600	GPa
Young's modulus of the reservoir	180	GPa
Poisson's ratio of the caprock	0.1	dimensionless
Biot's coefficient of the caprock	0.7	dimensionless
Poisson's ratio of the reservoir zone	0.1	dimensionless
Biot's coefficient of the caprock	0.7	dimensionless
Thermal conductivity of the caprock	4.0	W/(m K)
Thermal conductivity of the reservoir rock	4.0	W/(m K)
Constant c	10	dimensionless
σ	45	MPa
Injection rate	80	kg/s
Injection temperature	140	°C
Initial pressure	20	MPa

Table 1—Input parameters used in Case 1.

The transient pressure curves at (1m,1m,2000m), (2.5m,2.5m,2000m), (5m,5m,2000m), (10m, 10m, 2000m), (50m, 50m, 2000m), (500m, 500m, 2000m) are shown in Figure 5



Figure 5—Transient pressure curves at different points of the reservoir.

Along the diagonal of the reservoir, the permeability profiles at 100s, 1000s, 10000s, 50000s and 100000s are shown in Figure 6.



Figure 6—Permeability profiles along the diagonal of the reservoir at 100s, 1000s, 1000s, 50000s and 100000s.

Case 2 Rock failure induced by thermal uploading

In this case, we present the application of THM-EGS to a problem that has a cold water injection well, from which cold water is injected to a hot reservoir. The geometry of the problem is the same as that used in Case 1, shown in Figure 4. Because of the temperature difference between the injected fluid and the insitu reservoir, the local effective stress around the injector will get reduced (thermal unloading) and rock may fails. Such fracturing phenomena have been found in both geothermal reservoirs as well as deep oil/ gas reservoirs. The input parameters for Case 2 are shown in Table 2.

Table 2—input parameters used in Case 2		
Properties	Values	Units
Initial permeability of the reservoir	2.6*10 ⁻¹⁵	m ²
Porosity	0.1	dimensionless
Young's modulus	25	GPa
Poisson's ratio	0.2	dimensionless
Biot's coefficient	0.7	dimensionless
Thermal expansion coefficient	3.5*10 ⁻⁵	1/K
Thermal conductivity	2.2	W/(m K)
Injection pressure	15	MPa
Injection temperature	100	°C
Initial pressure	5.8	MPa
Initial temperature	190	°C
μ_{s}	0.5	dimensionless
S_0	3.0	MPa

Table 2—Input parameters used in Case 2

In this problem, the absolute permeability of reservoir rock is correlated with the in-situ stress. Mohr-Coulomb failure is used to determine the rock condition as well as the according rock permeability. The Mohr-Coulomb stress is calculated based on the following equation

$$MC = 0.5(\sigma_1 - \sigma_3)(\mu_s^2 + 1)^{0.5} - 0.5\mu_s(\sigma_1 + \sigma_3) + \mu_s P - S_0$$
(19)

Basically, as Mohr-Coulomb stress increases, rock permeability increases accordingly. We assume the existence of a 'maximum deformation' point, beyond which the rock is supposed to 'completely' fail and its permeability does not increase any more. Before that point, the rock permeability is assumed to be linearly dependent on the Mohr-Coulomb stress. For convenience, we use a 'dimensionless' permeability that is the ratio between the permeability as a function of Mohr-Coulomb stress and the initial permeability of the rock.

$$K_D = K(MC)/K_0 \tag{20}$$

The relationship between the dimensionless permeability and the Mohr-Coulomb stress is shown in Figure 7. The pressure, temperature and permeability at the injector is shown in Figure 8 and Figure 9 respectively. From the two figures, we can see that THM-EGS is capable of capturing the complex THM behaviors of the reservoir.



Figure 7—Relationship between the dimensionless permeability and Mohr-Coulomb stress.



Figure 8—Pressure and temperature variation at the injection well.



Figure 9—Permeability and temperature variation at the injection well.

Summary and Conclusions

In summary, we have presented in this paper the development and application of a multiphysical reservoir simulator, THM-EGS. We have presented the mathematical model and numerical methods of THM-EGS. We have equipped THM-EGS with an extended version of the Beltrami-Michell equations, which are able to simulate the full stress tensor and can be fully coupled with the fluid (thermal-hydraulic) simulation module.

THM-EGS, with its flexible framework, has been proven to be accurate and efficient in the simulation of complex thermal-hydraulic-mechanical behavior of petroleum/geothermal reservoirs. The simulator can be also readily tailored and extended to IOR/EOR problems.

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Nomenclature

- A = area
- C =specific heat
- F = mass flux
- F_{adv} = advective mass flux
 - $F_{\rm b}$ = body force, mL/t², kg-m/s²
 - g = gravity vector
 - G = shear modulus
 - h =specific enthalpy
 - I = identity matrix
 - K = permeability
 - K_0 = initial permeability
 - K_r = relative permeability
 - k_t = thermal conductivity
 - M =component mass/energy per unit volume
 - N_C =Number of components
 - P = pressure
 - P_c = capillary pressure, m/Lt²
 - q = mass source/sink per unit volume
 - S =saturation
 - S_0 = cohesion
 - t = time
 - T =temperature
- T_{ref} = reference temperature
 - \boldsymbol{u} = displacement vector
- U = specific internal energy
- V =bulk volume
- V_s = solid volume
- X = mass fraction
- Z =compressibility

Greek

- α = Biot's coefficient
- β = linear thermal expansion coefficient
- ε = strain tensor
- ε_v = volumetric strain
- λ = Lame parameter
- μ = viscosity
- μ_s = static friction
- v = Poisson's ratio
- ρ = density
- τ = stress tensor
- τ_m = mean stress
- $\tau' = \text{effective stress}$
- ϕ = porosity

Subscripts

- l = phase
- $r = \operatorname{rock}$
- 0 = unstrained

Superscripts

- i = time step
- k = mass component

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Appendix A

DERIVATION OF EXTENDED BELTRAMI - MICHELL EQUATIONS

In this section, we revisit the mean stress and extend it to heterogeneous and damage mechanics problems, where rock properties are not constant but spatial/time variables. We start from the definition of stress tensor

$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$
(A.1)

The force balance equation is

$$\nabla \bullet \bar{\bar{\sigma}} + \bar{F} = 0 \tag{A.2}$$

The above equation can be express with stress components as

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + F_x = 0$$
(A.3)

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + F_y = 0 \tag{A.4}$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + F_z = 0$$
(A.5)

The normal stress components are related with normal strain by the generalized Hook's law as

$$\sigma_{kk} - \left[\alpha P + 3\beta K(T - T_{ref})\right] = 2G\varepsilon_{kk} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}), k = x, y, z$$
(A.6)

The shear stress components are related with shear strain as

$$\sigma_{ij} = 2G\varepsilon_{ij}, i \neq j \tag{A.7}$$

The strain tensor is

$$\bar{\varepsilon} = \frac{1}{2} \left[\nabla \bar{u} + (\nabla \bar{u})^T \right]$$
(A.8)

Note, here all rock properties, including $\alpha K G$ and λ are all functions of the primary variables, therefore their partial derivatives with respect to *x*, *y*, *z* are not zero any more. Considering the above facts, the governing force balance equations in three dimensions are

$$\alpha \frac{\partial P}{\partial x} + 3\beta K \frac{\partial T}{\partial x} + 2G \frac{\partial \varepsilon_{xx}}{\partial x} + \lambda \frac{\partial (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})}{\partial x} + 2G \frac{\partial \varepsilon_{yx}}{\partial y} + 2G \frac{\partial \varepsilon_{zx}}{\partial z} + F_{x}$$

$$+ P \frac{\partial \alpha}{\partial x} + 3\beta \frac{\partial K}{\partial x} T + \frac{\partial (2G)}{\partial x} \varepsilon_{xx} + \frac{\partial (\lambda)}{\partial x} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \frac{\partial (2G)}{\partial y} \varepsilon_{yx} + \frac{\partial (2G)}{\partial z} \varepsilon_{zx} = 0$$

$$\alpha \frac{\partial P}{\partial y} + 3\beta K \frac{\partial T}{\partial y} + 2G \frac{\partial \varepsilon_{yy}}{\partial y} + \lambda \frac{\partial (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})}{\partial y} + 2G \frac{\partial \varepsilon_{yx}}{\partial x} + 2G \frac{\partial \varepsilon_{yz}}{\partial z} + F_{y}$$

$$+ P \frac{\partial \alpha}{\partial y} + 3\beta \frac{\partial K}{\partial y} T + \frac{\partial (2G)}{\partial y} \varepsilon_{yy} + \frac{\partial (\lambda)}{\partial y} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \frac{\partial (2G)}{\partial x} \varepsilon_{yx} + \frac{\partial (2G)}{\partial z} \varepsilon_{yz} = 0$$
(A.9)
$$(A.9)$$

$$\alpha \frac{\partial P}{\partial z} + 3\beta K \frac{\partial T}{\partial z} + 2G \frac{\partial \varepsilon_{zz}}{\partial z} + \lambda \frac{\partial (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})}{\partial z} + 2G \frac{\partial \varepsilon_{yz}}{\partial y} + 2G \frac{\partial \varepsilon_{xz}}{\partial x} + F_{z}$$

$$+ P \frac{\partial \alpha}{\partial z} + 3\beta \frac{\partial K}{\partial z} T + \frac{\partial (2G)}{\partial z} \varepsilon_{zz} + \frac{\partial (\lambda)}{\partial z} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \frac{\partial (2G)}{\partial y} \varepsilon_{yz} + \frac{\partial (2G)}{\partial x} \varepsilon_{xz} = 0$$
(A.11)

Where terms in blue are the newly added terms. Replacing strain with displacement gives

$$\begin{aligned} \alpha \frac{\partial P}{\partial x} + 3\beta K \frac{\partial T}{\partial x} + (2G + \lambda) \frac{\partial^2 u_x}{\partial x^2} + \lambda \frac{\partial^2 u_y}{\partial x \partial y} + \lambda \frac{\partial^2 u_z}{\partial x \partial z} + G(\frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_y}{\partial x \partial y}) + G(\frac{\partial^2 u_x}{\partial z^2} + \frac{\partial^2 u_z}{\partial x \partial z}) + F_x \\ + P \frac{\partial \alpha}{\partial x} + 3\beta \frac{\partial K}{\partial x} T + \frac{\partial (2G)}{\partial x} \frac{\partial u_x}{\partial x} + \frac{\partial (\lambda)}{\partial x} (\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}) \\ + \frac{\partial (2G)}{\partial y} \left(\frac{1}{2} \frac{\partial u_x}{\partial y} + \frac{1}{2} \frac{\partial u_y}{\partial x} \right) + \frac{\partial (2G)}{\partial z} \left(\frac{1}{2} \frac{\partial u_x}{\partial z} + \frac{1}{2} \frac{\partial u_z}{\partial x} \right) = 0 \end{aligned}$$

$$\begin{aligned} \alpha \frac{\partial P}{\partial y} + 3\beta K \frac{\partial T}{\partial y} + (2G + \lambda) \frac{\partial^2 u_y}{\partial y^2} + \lambda \frac{\partial^2 u_x}{\partial x \partial y} + \lambda \frac{\partial^2 u_z}{\partial y \partial z} + G(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial x \partial y}) + G(\frac{\partial^2 u_y}{\partial z^2} + \frac{\partial^2 u_z}{\partial y \partial z}) + F_y \\ + P \frac{\partial \alpha}{\partial y} + 3\beta \frac{\partial K}{\partial y} T + \frac{\partial (2G)}{\partial y} \frac{\partial u_y}{\partial y} + \frac{\partial (\lambda)}{\partial y} (\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}) \\ + \frac{\partial (2G)}{\partial y} \left(\frac{1}{2} \frac{\partial u_x}{\partial y} + \frac{1}{2} \frac{\partial u_y}{\partial y} \right) + \frac{\partial (2G)}{\partial z} \left(\frac{1}{2} \frac{\partial u_y}{\partial z} + \frac{1}{2} \frac{\partial u_z}{\partial y} \right) = 0 \end{aligned}$$

$$\begin{aligned} \alpha \frac{\partial P}{\partial z} + 3\beta K \frac{\partial T}{\partial z} + (2G + \lambda) \frac{\partial^2 u_z}{\partial z^2} + \lambda \frac{\partial^2 u_y}{\partial z} + \lambda \frac{\partial^2 u_x}{\partial x^2} + G(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial x \partial y}) + G(\frac{\partial^2 u_z}{\partial z^2} + \frac{\partial^2 u_z}{\partial y \partial z}) + F_z \\ + \frac{\partial (2G)}{\partial y} \left(\frac{1}{2} \frac{\partial u_x}{\partial y} + \frac{1}{2} \frac{\partial u_y}{\partial z} \right) + \frac{\partial (2G)}{\partial z} \left(\frac{1}{2} \frac{\partial u_y}{\partial z} + \frac{1}{2} \frac{\partial u_z}{\partial y} \right) = 0 \end{aligned}$$

$$\begin{aligned} A.13) \\ + \frac{\partial (2G)}{\partial z} \left(\frac{1}{2} \frac{\partial u_x}{\partial x} + \frac{\partial (\lambda)}{\partial z} (\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}) \right) = 0 \end{aligned}$$

$$\begin{aligned} A.14) \\ + \frac{\partial (2G)}{\partial y} \left(\frac{1}{2} \frac{\partial u_x}{\partial y} + \frac{1}{2} \frac{\partial u_y}{\partial z} \right) + \frac{\partial (2G)}{\partial z} \left(\frac{1}{2} \frac{\partial u_x}{\partial x} + \frac{1}{2} \frac{\partial u_y}{\partial y} \right) = 0 \end{aligned}$$

Take the divergence to the above equation and adding the three equations up and replace the displacement derivatives with strain and rearrange the resulted equation as

$$\nabla^{2}(\alpha P) + 3\beta \nabla^{2}(KT) + \nabla \cdot \left(\nabla ((\lambda + 2G)\varepsilon_{\nu})\right) + \nabla \cdot \overline{F} = 0$$
(A.15)

Set

$$h = \alpha P + 3\beta KT \tag{A.16}$$

Equation 20 can be rewrite as

$$\nabla^2 \left(\frac{3(1-\nu)}{1+\nu} \sigma_m \right) - \nabla^2 \left(\frac{2(1-2\nu)}{1+\nu} h \right) + \nabla \cdot \overline{F} = 0$$
(A.17)

The above equation is similar to the previous mean stress equation. The governing equations three normal stress components can be derived as

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2}{\partial x^2} \left[\frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left(\sigma_{xx} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) + \frac{\partial}{\partial x} F_x = 0$$
(A.18)

$$\frac{\partial^2 h}{\partial y^2} + \frac{\partial^2}{\partial y^2} \left[\frac{3}{2(1+\upsilon)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left(\sigma_{yy} - h - \frac{3\upsilon}{1+\upsilon} (\sigma_m - h) \right) + \frac{\partial}{\partial y} F_y = 0$$
(A.19)

$$\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2}{\partial z^2} \left[\frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left(\sigma_{zz} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) + \frac{\partial}{\partial z} F_z = 0$$
(A.20)