# SPE-SAS-984



# An Alternative Approach to Modeling Non-Darcy Flow for Pressure Transient Analysis in Porous and Fractured Reservoirs

Ajab Al-Otaibi, College of Technological Studies, Yu-Shu Wu, Colorado School of Mines

Copyright 2011, Society of Petroleum Engineers

This paper was prepared for presentation at the 2011 SPE Saudi Arabia Section Technical Symposium and Exhibition held in AlKhobar, Saudi Arabia, 15-18 May 2011.

This paper was selected for presentation by an SPE program committee following review of information contained in an abstract submitted by the author(s). Contents of the paper have not been reviewed by the Society of Petroleum Engineers and are subject to correction by the author(s). The material, as presented, does not necessarily reflect any position of the Society of Petroleum Engineers, its officers, or members. Papers presented at the SPE meetings are subject to publication review by Editorial Committee of Society of Petroleum Engineers. Electronic reproduction, distribution, or storage of any part of this paper without the written consent of the Society of Petroleum Engineers is prohibited. Permission to reproduce in print is restricted to an abstract of not more than 300 words; illustrations may not be copied. The abstract must contain conspicuous acknowledgment of where and whom the paper was presented. Write Liberian, SPE, P.O. Box 833836, Richardson, TX 75083-3836, U.S.A., fax 01-972-952-9435.

## Abstract

Non-Darcy flow is an important factor affecting productivity of gas and high production rate oil wells and the analysis of pressure-transient well tests in porous media. The motivation for this study to investigate, validate and extend the Barree-Conway model (BCM) for pressure-transient analysis of wells involving hydraulic fractures and naturally fractured reservoirs. Numerical models based on finite control-volume method were developed according to not only the Forchheimer equation and, for the first time in the literature, but also the Barree-Conway model specifically for pressure-transient analysis of single-phase fluid flow in porous and fractured reservoirs. The developed numerical models are capable of simulating all near wellbore effects coupled with the non-Darcy flow behavior in porous and fractured reservoirs.

This study shows that the BCM can be applied, similar to the Forchheimer model, to analyzing and interpreting pressuretransient responses of non-Darcy flow in porous and fractured reservoirs. The BCM predicts similar pressure-transient responses of non-Darcy flow as predicted by the Forchheimer model. The parameter of characteristic length among the BCM model parameters is more sensitive than the minimum permeability. The minimum permeability is only sensitive at low values of characteristic length or at extremely high flow rates. Numerical simulation results indicate that the value of characteristic length must be low for significant effect of non-Darcy flow and it is related to small effective radius. The non-Darcy flow parameters of the BCM may not be all estimated from single-rate tests in single-porosity reservoirs using conventional analysis approaches. However, they can be estimated by a fitting process based on non-linear optimization algorithm incorporated into the numerical model. Type curves generated by the BCM are provided to demonstrate a methodology for analyzing the effect of non-Darcy flow on pressure-transient tests in porous and fractured reservoirs. As application examples, numerical models of non-Darcy flow are used to model and interpret actual field pressure-transient data from high production rate oil wells in Kuwait.

## Introduction

In studies of non-Darcy flow through porous median, the Forchheimer equation is generally used to describe single-phase non-Darcy flow (Forchheimer, 1901). Forchheimer observed that at high flow velocities the relationship between pressure gradient and fluid velocity is no longer linear, as described by linear Darcy's flow. In an attempt to describe this nonlinear relationship, Forchheimer added an additional quadratic flow term to the Darcy's linear form. Forchheimer's equation is given by

$$-\nabla p = \mu [\mathbf{k}]^{-1} \vec{\mathbf{v}} + \rho \beta | \vec{\mathbf{v}} | \vec{\mathbf{v}}$$
<sup>(1)</sup>

where  $\beta$  is the non-Darcy flow coefficient. The non-Darcy flow coefficient is estimated by analysis of laboratory experimental data and multi-rate pressure transient tests, but such data are not always available. The usual practice to estimate values of the  $\beta$  factor is to use empirical correlations or theoretical equations obtained from the literature.

In hydraulically fractured wells, pressure transient tests are run to evaluate both fracture and formation properties. Unfortunately, most analysts use conventional well-testing techniques that do not account for effects of non-Darcy flow on the pressure distribution. In hydraulically fractured wells, the pressure drop in the hydraulic fracture under high flow rates is dominated by non-Darcy flow. The resulting pressure distribution due to non-Darcy flow effects in the fracture affects the pressure distribution in the entire reservoir. Numerous efforts have been made to study non-Darcy flow effects and to explain how they affect estimates of the fracture half-length and fracture conductivity (McGuire and Sikora, 1960; Wattenbarger and

Ramey, 1969; Holditch and Morse, 1976; Lee and Holditch, 1981; Guppy et al., 1982; Gidley, 1991; Umnuayponwiwat et al., 2000). From these efforts we can conclude that depending on flow rate non-Darcy flow is more important inside the fracture than within the reservoir. A numerical model that incorporates non-Darcy flow effects is the best technique to correctly analyze pressure-transient tests in hydraulically fractured wells. The effect of non-Darcy flow on transient pressure responses of hydraulically fractured wells of finite-conductivity fractures has been investigated in the literature (Wattenbarger and Ramey, 1969; Holditch and Morse, 1976; Guppy et al., 1982). Holditch and Morse (1976) developed a numerical model to analyze the effect of non-Darcy flow in the reservoir and fractures. They studied non-Darcy flow effects on various flow patterns and concluded that non-Darcy flow effects, mainly in the fracture, reduced the apparent conductivity. Guppy et al. (1982) showed that the changes in the flux distribution along the fracture under non-Darcy flow that caused reduction in the apparent conductivity of the fracture. Umnuayponwiwat et al. (2000) studied non-Darcy flow effects on the interpretation of pressure-transient response from wells in low-permeability reservoirs. They concluded that the results from well tests conflict with those expected from the original design, if non-Darcy flow effect is not considered.

Significant progress has been made in understanding and modeling fracture flow phenomena in porous media (Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985). Kazemi (1969) presented a new idealized conceptual model for naturally fractured reservoirs, where he considered unsteady-state flow in both matrix and fractures. Kazemi presented solutions for both pressure drawdown and buildup using his new model. He compared his results with those obtained using other models previously published, in particular the Warren and Root model. The generalized dual-continuum method (Pruess and Narasimhan, 1985), such as the MINC (Multiple Interacting Continua) concept and the multiporosity model (Wu and Pruess, 1988), can describe flow in a fracture/matrix system with any size and shape of matrix blocks and with fully transient handling of fracture/matrix interactions. Studies on modeling non-Darcy flow in naturally fractured reservoirs are limited. Villalobos-L et al. (1989) derived an expression of a rate-dependent pseudo-skin term similar to that in single porosity reservoirs based on numerical results of a radial liquid flow model using the Forchheimer equation. Wu (2002) modeled non-Darcy flow in a naturally fractured system using the MINC method. He applied the results of Darcy flow to approximate the characteristic distance of non-Darcy flow between fractures and the matrix crossing the interface for the dual porosity formation. Wu (2002) observed that semi-log plots of pressure drop versus time are extremely sensitive to the values of the non-Darcy flow coefficient. In particular, effects of non-Darcy flow on early transient pressure responses are very strong, such that the first semi-log straight lines may not develop when non-Darcy flow is involved. Tavares et al. (2004) used a 2D finite difference model to study combined effects of non-Darcy flow and skin damage in gas well performance. They found that skin damage may accentuate the non-Darcy flow effect and drastically influence pressure transient characteristics of low pressure, naturally fractured reservoirs; in high pressure reservoirs, this effect is significant only at high flow rates.

Many researchers have observed the limitations of the Forchheimer's equation to describe all ranges of fluid velocities (Carman, 1937; Fand et al., 1987; Montillet, 2004; Barree and Conway, 2004; Lai et al., 2009). Barree and Conway (2004) developed a new flow model (BCM) that uses a new concept of an apparent permeability to describe Darcy's (linear) and non-Darcy or Forchheimer (nonlinear) flow in porous media. The complete nonlinear flow model, BCM, for non-Darcy flow extended to multidimensional fluid flow is given by

$$-\nabla p = \left(\mu \,\vec{\mathbf{v}}\right) \left[ k_d \left( k_{mr} + \frac{(1 - k_{mr})\mu\tau}{\mu\tau + \rho |\,\vec{\mathbf{v}}|} \right) \right]$$
(2)

where  $k_{mr}$  is the minimum permeability ( $k_{min}$ ) relative to Darcy's permeability ( $k_d$ ). The minimum permeability is defined as the permeability at high flow rate when non-Darcy flow effects exist. According to Barree and Conway (2004), the value of the characteristic length,  $\tau$ , indicates the magnitude of ( $\rho v/\mu$ ) at which the apparent non-Darcy permeability is one-half the lowrate Darcy apparent permeability. Therefore, smaller values of  $\tau$  indicate an earlier onset of non-Darcy flow effects. They suggest that  $\tau$  is related to the particle size or size distribution of the porous medium and may be determinable from an accurate description of the sieve distribution using the reciprocal of twice the mean diameter.

Lai et al. (2009) recently presented laboratory data conducted in packed tubes, at 4000 psig and they used a single-phase nitrogen non-Darcy flow apparatus developed by Lopez-H. (2007) and a large number of experiments with proppant packs have been carried out. Proppant samples tested to date include a wide range of commonly used sizes and types, including ceramics and natural sands. Lai et al. (2009) showed that the experimental data and results support the BCM. Experimental data are analyzed using a regression method for the Forchheimer and BCM models. An example of the results using Forchheimer and BCM models is shown in Fig 1, where the x-axis is normalized for mass flow rate instead of fluid velocity. The data plotted in Fig 1 are taken under a confining stress of 27.5 MPa (4000 psig). As shown in Fig 1, the experimental data agree extremely well with the BCM across the entire flow velocity range from low to high gas flow rates. The Forchheimer correlation overestimates the associated pressure drop at high gas flow rates. According to Lai et al. (2009), all sample data taken to date show similar agreement with the BCM equation across the observed flow spectrum.

In this study, we investigate, validate and extend the BCM for pressure-transient analysis of wells involving hydraulic fractures and in naturally fractured reservoirs.

### Numerical Models of Non-Darcy Flow

The numerical approach to simulate the non-Darcy flow consists of spatial discretization of the mass conservation equation, time discretization; and iterative approaches to solve the resulting nonlinear, discrete algebraic equations. A mass-conserving discretization scheme, based on control-volume or integral finite-difference, IFD (Pruess, 1991) is used. The control-volume approach provides a general spatial discretization scheme that can represent a one-, two- or three-dimensional domain using a set of discrete meshes. Time discretization is carried out using a backward, first-order, fully implicit finite-difference scheme. The general continuity equation for single-phase fluid flow is given by

$$\frac{\partial}{\partial t} \left( \phi \, \rho_{\beta} \right) = -\nabla \, \bullet \left( \rho_{\beta} \, \vec{v}_{\beta} \right) + q_{\beta} \tag{3}$$

Equation (3) is discretized in space using an integral finite-difference or control-volume scheme for a porous non-fractured and/or fractured medium with an unstructured grid as shown in Fig 2. The time discretization is carried out with a backward, first-order finite difference method. Then the discrete non-linear equations of element *i* are as follows:

$$\{ (\phi \ \rho_{\beta})_{i}^{n+1} - (\phi \ \rho_{\beta})_{i}^{n} \} \frac{V_{i}}{\Delta t} = \sum_{j \in \eta_{i}} F_{ij}^{n+1} + Q_{\beta i}^{n+1}$$
(4)

where *n* denotes the previous time level, n+1 is the current time level,  $V_i$  is the volume of element *i* (porous non-fractured or fractured block),  $\Delta t$  is the time step size,  $\eta_i$  contains the set of neighboring elements (*j*) (non-fractured or fractured) to which element *i* is directly connected,  $Q_i$  is the mass sink/source term at element *i*, for the fluid and  $F_{ij}^{n+1}$  is the mass "flow" term for the fluid between elements *i* and *j*.

#### **Forchheimer Flow Model**

If the Forchheimer's flow model is used to simulate the pressure transient responses of non-Darcy flow then the mass flow term for the fluid between elements i and j is defined (Wu, 2002) as

$$F_{ij} = \frac{A_{ij}}{2(k\beta)_{ij+1/2}} \left\{ -\frac{1}{\lambda_{\beta}} + \left[ \left( \frac{1}{\lambda_{\beta}} \right)^2 - \gamma_{ij} \left( \Phi_{\beta j} - \Phi_{\beta i} \right) \right]^{1/2} \right\}$$
(5)

where  $\beta$  is the non-Darcy flow coefficient, subscript ij+1/2 denotes a proper averaging of properties at the interface between the two elements,  $A_{ij}$  is the common interface area between connected elements i and j,  $\lambda$  is the mobility of phase  $\beta$ ,  $\gamma$  is the transmissivity of flow between elements *i* and *j*, and  $\Phi$  is the flow potential gradient of phase  $\beta$ .

#### **Barree-Conway Flow Model (BCM)**

If the BCM is used to simulate the pressure transient responses of non-Darcy flow effects, the mass flow term for the fluid between elements i and j is defined (Al-Otaibi and Wu, 2010) as

$$F_{ij} = \frac{A_{ij}}{2\mu} \Big[ f_{ij} + h_{ij} \Big] \tag{6}$$

where

$$f_{ij} = -\mu^2 \tau - \left(k_d k_{mr} \rho\right)_{ij+1/2} \left[\frac{\left(\Phi_j - \Phi_i\right)}{D_i + D_j}\right]$$
(7)

and

$$h_{ij} = \sqrt{\left(\mu^{2}\tau - \left(k_{d}k_{mr}\rho\right)_{ij+1/2} \left[\frac{\left(\Phi_{j} - \Phi_{i}\right)}{D_{i} + D_{j}}\right]\right)^{2} + 4\mu^{2}\left(\rho k_{d}\right)_{ij+1/2}\tau\left[\frac{\left(\Phi_{j} - \Phi_{i}\right)}{D_{i} + D_{j}}\right]}$$
(8)

The numerical solution method used is based on solving Equation (4) fully implicitly using a Newton-Raphson iteration method for any flow model, i.e. Forchheimer's or BCM, and the numerical model formulation is implemented into a general-purpose reservoir simulator of MSFLOW (Wu, 1998). Equation (4) can be written in a residual form as follow:

$$R_{i} = \left\{ \left( \phi \ \rho \right)_{i}^{n+1} - \left( \phi \ \rho \right)_{i}^{n} \right\}_{\Delta t}^{V_{i}} - \sum_{j \in \eta_{i}} F_{ij}^{n+1} - Q_{i}^{n+1} \qquad \left\{ i = 1, 2, 3, \dots, N \right\}$$
(9)

where N is the total number of nodes, elements or grid blocks of the grid system. Equation (9) defines a set of (N) coupled nonlinear mass balance equations that need to be solved simultaneously. One primary variable per node is needed to use in the Newton-Raphson iteration for solving one equation per node. In the numerical simulator, for single-phase fluid flow, the fluid pressure is selected as the primary variable.

In a previous work, a steady-state analytical solution for non-Darcy radial flow has been derived according to the BCM and was used to verify the numerical model (Al-Otaibi and Wu, 2010). Excellent results are obtained from the numerical simulation, as compared to the analytical solution.

## **Apparent Permeability of Non-Darcy Flow**

The common factor between non-Darcy flow models considered in this study (Forchheimer's and BCM) is the apparent permeability of non-Darcy flow. This can be used to compare between the two nonlinear flow models under specific flow conditions in single- or double-porosity (fractured) reservoirs for a well producing at a constant production rate. The apparent permeability of non-Darcy flow according to BCM and Forchheimer models for 1-D flow are given by Equations (10) and (11), respectively.

$$k_{app} = k_{\min} + \left(k - k_{\min}\right) \left/ \left(1 + \frac{\rho |v|}{\tau \mu}\right)$$
(10)

$$k_{app} = k \left/ \left( 1 + \frac{\beta k \rho |v|}{\mu} \right) \right.$$
(11)

The apparent permeability of non-Darcy flow estimated by the two models, i.e. Equations (10) and (11) should be the same. Thus solving for  $k_{min}$  and  $\beta$  parameters of the two flow models gives:

$$k_{\min} = \frac{\mu \left(k - \beta k^2 \tau\right)}{\mu + \beta k \rho \left|\nu\right|} \tag{12}$$

$$\beta = \frac{\mu(k - k_{\min})}{k(k\tau\mu + k_{\min}\rho|v|)}$$
(13)

Equation (13) is very useful and can be used to compare between Forchheimer and BCM models. The fluid velocity, v, in Equations (12) and (13), is still a function of coordinates and time and may be approximated using the constant production rate and cross-sectional area of flow at the well.

In the first example, we show a comparison of non-Darcy flow models for a vertical well in a single-porosity reservoir. We used the developed numerical models to generate synthetic pressure transient responses for two flow cases (Darcy and non-Darcy flow). The porous reservoir system is assumed uniform, radially finite of 100 ft thick (total thickness) and is represented by a one-dimensional radial grid of 2,000 volume elements with a  $\Delta r$  size that increases logarithmically away from the wellbore ( $r_w = 0.33$  ft). The perforated thickness is same as the formation thickness (fully penetrating well,  $h_p = h$ ). The formation is initially at a constant pressure of 5,000 psi and is subjected to a constant production rate of 10,000 STB/day at the producing well, starting at time of zero. Input data used for simulating the pressure transient responses in numerical models are listed in Table 1. We first run the BCM using the input data shown in Table 1 for Darcy flow case ( $k_{min} = k_d$ ) and non-Darcy flow case ( $k_{min} = 1$  md,  $\tau = 100$  ft<sup>-1</sup>). Then we compute the equivalent  $\beta$  factor for Forchheimer model using Equation (13) and run the numerical model of Forchheimer with the same input data for Darcy flow ( $\beta = 0$  ft<sup>-1</sup>) and non-Darcy flow ( $\beta = 9.064E+09$  ft<sup>-1</sup>, calculated using Equation (13)). Fig 3 shows semi-log plot of wellbore pressure versus time for all simulation cases. Also, Fig 3 shows the difference in percentage of pressure transient responses predicted by BCM compared to the same responses predicted by Forchheimer model for the case of non-Darcy flow. Results shown in Fig 3 indicate that both models predict approximately the same pressure transient responses of non-Darcy flow with a maximum computational difference of 0.42% for this example.

Interporosity flow is the fluid exchange between the matrix and fractures constituting a dual-porosity system. The interporosity flow coefficient is a measure of how easily fluid flows from the matrix to the fractures. Warren and Root (1963)

defined the interporosity flow coefficient,  $\lambda$ , as

$$\lambda = \frac{4n(n+2)}{L^2} \frac{k_m}{k_f} r_w^2$$
(14)

where  $k_m$  is the permeability of the matrix,  $k_f$  is the permeability of the natural fractures, *L* is a characteristic dimension of a matrix block and *n* is the number of normal sets of planes limiting the less-permeable medium (n = 1, 2 or 3). The storativity ratio,  $\omega$ , is a measure of the relative fracture-storage capacity in the reservoir and is given by Equation (15) (Warren and Root, 1963) where the subscripts *f* and *m* refer to the fracture and matrix, respectively.

$$\omega = \frac{\left(\phi c_t\right)_f}{\left(\phi c_t\right)_f + \left(\phi c_t\right)_m} \tag{15}$$

In the following examples, the naturally fractured reservoir is represented by a 2D radial grid of 2,051 volume elements with a  $\Delta r$  size that increases logarithmically away from the wellbore ( $r_w = 0.33$  ft). We used the MINC approach (Pruess, 1983) to generate fracture-matrix (double-porosity system) elements and connections for the radial reservoir. In creating the mesh for the double-porosity reservoir using the MINC approach, three sets of plane parallel infinite fractures at right angles is used. We used a fracture spacing of 9.843 ft (3m) and fracture porosity of 0.1% in the following examples. The formation is initially at a constant pressure of 5,000 psi and is subjected to a constant production rate of 6,000 STB/day at the producing well, starting at time of zero. Input data used for simulating pressure-transient responses of Darcy and non-Darcy flow in the numerical models for the naturally fractured reservoir is listed in Table 2. Fig 4 shows log-log plots of wellbore pressure drop,  $\Delta p_{wf}$ , and pressure derivative (logarithmic) versus time predicted by Forchheimer and BCM models assuming Darcy flow for a vertical well in a naturally fractured reservoir. The non-Darcy flow coefficient,  $\beta$ , of Forchheimer model is set zero and the minimum permeability parameter in BCM is set equal to Darcy permeability in both matrix and natural fractures. Results shown in Fig 4 indicate excellent agreement between Forchheimer, BCM models and known Darcy flow transient responses (pressure and its derivative) under the same Darcy flow conditions.

In the following example, we compare results from Forchheimer and BCM models for two non-Darcy flow cases. In the BCM, the minimum permeability in the fracture is equal to 0.001 of the fracture permeability ( $k_{fmin} = 0.001 k_f$ ) and the characteristic length,  $\tau$ , is 20 ft<sup>-1</sup>. Using Equation (13) the estimated equivalent  $\beta$  factor of Forchheimer model under the same flow conditions is 4.624E+09 ft<sup>-1</sup>. Results shown in Fig 5 indicate excellent agreement between pressure-transient responses of non-Darcy flow predicted by the numerical models of BCM and Forchheimer. The pressure derivative (Fig 5) indicates that under severe non-Darcy flow effects the characteristics of early-times pressure-transient responses of non-Darcy flow. Thus non-Darcy flow effects alter characteristics of early-time pressure-transient responses and double-porosity behavior known in Darcy flow and change the slope of the first straight-line.

## **Non-Darcy Flow Effects in Single-Porosity Reservoirs**

The dimensionless wellbore pressure  $(p_{wD})$ , time  $(t_D)$ , wellbore-storage  $(C_D)$  and skin factor (S) are defined by Equation (16) to Equation (19), respectively, for constant production in a single-porosity radial reservoir.

$$p_{wD} = \frac{k_d h}{141.2 q \beta \mu} \left( p_i - p_{wf} \right)$$
(16)

$$t_D = \frac{0.0002637 \, k_d \, t}{\phi \, \mu \, c_t \, r_w^2} \tag{17}$$

$$C_D = \frac{0.8936 C}{\phi c_t h r_w^2}$$
(18)

$$S = \frac{k_d h \Delta p_s}{141.2 q \beta \mu} \tag{19}$$

The dimensionless parameter,  $k_{mr}$ , of BCM is defined as:

$$k_{mr} = \frac{k_{\min}}{k_d} \tag{20}$$

The dimensionless non-Darcy flow parameter,  $\tau_D$ , is defined based on distance in radial-direction as:

$$\tau_D = \tau r \tag{21}$$

In Equation(21), if we analyze pressure-transient data for wellbore pressure, as the case of this example, then  $r = r_w$  and

$$\tau_{wD} = \tau r_w \tag{22}$$

Input data used for generating pressure transient type-curves in this example are listed in Table 3. In numerical simulation using the non-Darcy flow model (BCM), the porous reservoir system is assumed uniform, radially finite of 100 ft thick and is represented by a 1-D radial grid of 2,010 volume elements with a  $\Delta r$  size that increases logarithmically away from the wellbore ( $r_w = 0.3$  ft). Fig 6 and Fig 7 show example type-curves generated from the numerical model study of effect of non-Darcy flow parameters of BCM ( $k_{mr}$  and  $\tau_D$ ) with combined effect of skin (S = 1) and wellbore storage ( $C_D = 500$ ). Fig 6 shows log-log plot of dimensionless wellbore pressure versus dimensionless time for different values of dimensionless non-Darcy flow parameter  $k_{mr}$  (1, 0.5 and 0.2) with a fixed value of  $\tau_D$  of 0.1, skin factor of 1.0 and  $C_D$  of 500. The early time, unit-slope, straight line indicates clearly the effect of wellbore storage. The value of  $k_{mr}$  of 1.0 is simulating Darcy's flow with skin factor of 1.0. Note that the generated type-curves combine both effects of skin (damage in this example) and non-Darcy flow. As the value of  $k_{mr}$  decreases we can see clearly more increase in pressure drop due to non-Darcy flow.

Fig 7 shows type curve (log-log plot of  $p_{wD}$  versus  $t_D$ ) for different values of dimensionless non-Darcy flow parameter  $\tau_D$  (0.01, 10, 1000) with fixed value of  $k_{mr}$  of 0.5, skin factor of 1.0 and  $C_D$  of 500. As the value of  $\tau_D$  decreases we can notice the increase in pressure drop due to non-Darcy flow effect. However this increase in pressure drop is less than that from the effect of decreasing  $k_{mr}$  (see Fig 6). In actual cases we expect low values of the characteristic length,  $\tau$ , thus the major contributing parameter of non-Darcy flow of BCM, in this example, is the minimum permeability plateau,  $k_{min}$ . However, this is not a general conclusion. In different examples, we have seen that if the value of the characteristic length is high, then changing the minimum permeability does not has any impact on the pressure transient responses. In some cases, depending on the production rate of the well, the value of the characteristic length of BCM controls how significant the non-Darcy flow is.

#### Non-Darcy Flow Effects in Hydraulically Fractured Wells

First we compare pressure transient responses generated by the BCM simulating Darcy flow ( $k_{min} = k_d$ ) with known analytical responses of Darcy flow at different flow patterns in hydraulically fractured well. This comparison was performed for three cases of hydraulic fracture conductivity: (1) low finite-conductivity, (2) high finite-conductivity, and (3) infinite-conductivity. In the numerical model we used a 3-D, XYZ, reservoir grid system for a hydraulically fractured well. We used a refined mesh of 141 grid blocks in *x*- and *y*- directions and 1 layer in *z*-direction (141*x*141*x*1), total of 19,881 volume elements. The input data of the reservoir and hydraulically fractured well used in the numerical simulation is shown in Table 4. Fig 8 and 9 show 2-D view of reservoir grid system used for numerical simulation of a hydraulically fractured well of infinite and finite fracture conductivity, respectively. In all examples presented in this paper, we simulate a vertical hydraulic fracture. For a hydraulically fractured well, the dimensionless time is given by

$$t_{Dxf} = \frac{0.0002637 k_d t}{\phi \,\mu \,c_t \, x_f^2} \tag{23}$$

The first comparison case is for a low finite-conductivity fracture of  $C_{fD}$  of 10 (Case 1) and fracture half-length,  $x_{f}$ , of 400 ft. Fig 10 shows a log-log plot of dimensionless wellbore pressure and pressure derivative versus dimensionless time for Case 1 for both numerical model and analytical solutions. In the numerical model of BCM, we set  $k_{min}$  parameter equal  $k_d$  for hydraulic fracture elements (i.e.  $k_{fmin} = k_{fd}$ ). As shown in Fig 10, pressure transient responses of the numerical model of BCM matches the analytical solutions for the three flow periods. The fracture-bilinear flow exists at early times and ends at approximately  $t_{Dxf}$  of 0.01. The formation-linear flow is not clear and lasts only for short time due to low fracture conductivity and it is only obvious in wells with high conductivity fractures ( $C_{fD} \ge 100$ ). The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 2. The second comparison case is for a high finite-conductivity fracture of  $C_{fD}$  of 100 (Case 2) and fracture half-length of 40 ft. As shown in Fig 11, pressure transient responses of BCM matches the analytical solutions for the two flow periods. The fracture-bilinear flow period ends too early because of high fracture conductivity and for actual pressure transient data it does not exist for high finite-conductivity fractures. The formation-linear flow exists and ends at approximately  $t_{Dxf}$  of 0.017. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 0.017. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 0.017. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 0.017. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 0.017. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 0.017. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 1.5. The third comparison case is for an infinite-conductivity fracture (Case 3) and fracture half-length of 1,000 ft. As shown in Fig 12, pressure transient responses

of BCM match analytical solutions of linear and pseudoradial flow periods. The fracture-bilinear flow period does not exist because of infinite-conductivity fracture. The formation-linear flow exists and ends at approximately  $t_{Dxf}$  of 0.1. The pseudoradial-flow period starts at approximately  $t_{Dxf}$  of 2.5.

In the following examples, we study the effect of the minimum permeability parameter and the characteristic length of BCM on pressure transient responses of a hydraulically fractured well for finite- and infinite-conductivity fractures. The input data used in the numerical model to simulate pressure transient responses for all non-Darcy flow cases are shown in Table 4. We present simulation cases for two finite-conductivity fractures ( $C_{fD} = 1$  and 100). The fracture half-length,  $x_f$ , for Case 1 ( $C_{fD} = 1$ ) is 400 ft and for Case 2 ( $C_{fD} = 100$ ) is 40 ft. In these simulation cases, wellbore storage and skin effects are not considered ( $C_D = 0$ ,  $S_f = 0$ ).

Fig 13 shows log-log plots of dimensionless wellbore pressure and derivative versus dimensionless time of Case 1 for three values of  $k_{finr}$  (0.01, 0.1 and 1). The value of the dimensionless characteristic length,  $\tau_D$ , used is 100. We have seen that for values of  $\tau_D$  greater than 1,000 no significant change in pressure transient responses of non-Darcy flow compared to Darcy flow for any value of  $k_{finr}$ . As shown in Fig 13 as the value of  $k_{finr}$  (or  $k_{finin}$ ) decreases, the pressure drop increases due to higher non-Darcy flow effects. The pressure derivative curves change to higher values with higher non-Darcy flow effects. The change in pressure transient responses, because of decreasing in the value of  $k_{finr}$  (increasing non-Darcy flow effects), is similar to the pressure change due to decrease in the fracture conductivity in Darcy flow case.

Fig 14 shows a log-log plot of dimensionless wellbore pressure and derivative versus dimensionless time of Case 2 for three values of  $k_{finr}$  (0.01, 0.1, 0.5 and 1). As the value of  $k_{finr}$  decreases, the pressure drop increases due to high non-Darcy flow effects. The pressure derivative curves change to higher values with higher non-Darcy flow effects. Although we are using a lower value of  $\tau_D$  of 100, the value of  $k_{finr}$  of 0.5 matches pressure transient responses of Darcy flow case ( $k_{finr} = 1$ ) for most of times except at early times. This indicates that  $k_{finr}$ , in this example, needs to be less than 0.5 for significant non-Darcy flow effects. Fig 15 shows log-log plots of dimensionless wellbore pressure and its derivative versus dimensionless time of Case 2 for two values of  $k_{finr}$  (0.1 and 1) with an additional case simulating Darcy flow for  $C_{fD}$  of 11.24. Pressure transient responses (pressure and derivative) of the non-Darcy flow case ( $k_{finr} = 0.1$  and  $\tau_D = 100$  for  $C_{fD} = 100$ ) match responses of Darcy flow case ( $k_{finr} = 1$ ,  $C_{fD} = 0.1$ ) as shown in Fig 15. This indicates that non-Darcy flow effects reduce the actual fracture conductivity (assuming Darcy flow) to lower values, in this example from  $C_{fD}$  of 100 to 11.24. If we examine the pressure transient responses and pressure derivative (Fig 15) we find that responses due to non-Darcy flow of higher fracture conductivity matches exactly responses of Darcy flow of lower fracture conductivity. Therefore, if pressure transient data is analyzed assuming Darcy flow, when non-Darcy flow exist, incorrect estimates of fracture conductivity and fracture properties will be obtained.

In the following example we present simulation cases for finite-conductivity fracture of  $C_{fD}$  of 100 studying the effect of the parameter  $\tau$  of BCM on pressure transient responses. Fig 16 shows log-log plots of dimensionless wellbore pressure and its derivative versus dimensionless time for  $C_{fD}$  of 100 for three values of  $\tau_D$  (10, 100 and 1000). The value of  $k_{fmr}$  used is 0.1. As shown in Fig 16, as the value of  $\tau_D$  decreases, the pressure drop increases due to higher non-Darcy flow effects. The pressure derivative curves change to higher values with higher non-Darcy flow effects. For values of  $\tau_D$  greater than 1,000 we expect lower non-Darcy flow effects and the same responses as Darcy flow. Pressure transient responses (pressure and its derivative) of non-Darcy flow for  $\tau_D = 10$  is close to responses of  $\tau_D = 100$  because of constant value of  $k_{fmr}$  of 0.1 used in both cases. In this example, if  $\tau_D$  is reduced below 10, pressure transient responses will match the case of  $\tau_D$  of 10 for constant value of  $k_{fmr}$ of 0.1. The change in pressure transient responses, because of decreasing in the value of  $\tau_D$  (increasing non-Darcy flow effects), is similar to the pressure change due to decrease in the fracture conductivity for Darcy flow case.

In the following examples, we present simulation cases studying the effect of non-Darcy flow parameters ( $k_{min}$  and  $\tau$ ) of BCM for infinite-conductivity hydraulic fractures on pressure-transient type curves. For all simulation cases the fracture half-length,  $x_{fs}$  is 1,000 ft and the fracture width,  $w_{fs}$  is 0.02 ft. In these simulation cases, wellbore storage and skin effects are not considered ( $C_D = 0$ ,  $S_f = 0$ ). Fig 17 shows log-log plots of dimensionless wellbore pressure and its derivative versus dimensionless time of for infinite-conductivity fracture for four values of  $k_{fmr}$  (0.001, 0.01, 0.1 and 1). The value of the dimensionless characteristic length,  $\tau_D$ , used in all cases is 10 to simulate high non-Darcy flow effects. As shown in Fig 17 as the value of  $k_{fmr}$  decreases, the pressure drop increases due to higher non-Darcy flow effects. The pressure derivative curves change to higher values with higher non-Darcy flow effects) is similar to the pressure transient responses because of decreasing the value of  $k_{fmr}$  (increasing non-Darcy flow effects) is similar to the pressure change due to decreasing the fracture conductivity in Darcy flow case. The pressure transient responses of non-Darcy flow case of  $k_{fmr}$  of 0.01 match responses of Darcy flow of  $C_{fD}$  of 100. Also, the pressure transient responses of non-Darcy flow case of  $k_{fmr}$  of 0.01 is close to responses of Darcy flow of  $C_{fD}$  of 10. This indicates that non-Darcy flow reduces the fracture conductivity. Therefore we conclude that if non-Darcy flow exists because of high fluid velocity near the wellbore, the infinite-conductivity fracture may behave as a fracture with lower finite-conductivity.

Fig 18 shows log-log plots of dimensionless wellbore pressure and its derivative versus dimensionless time of infiniteconductivity fracture for five values of  $\tau_D$  (1, 10, 100, 1000 and 10,000). The value of  $k_{fmr}$  used is 0.01. As shown in Fig 18 as the value of  $\tau_D$  decreases the pressure drop increases due to higher non-Darcy flow effects. The pressure derivative curves change to higher values with higher non-Darcy flow effects. For values of  $\tau_D$  greater than 1,000, we expect low effect of nonDarcy flow behavior. And for values of  $\tau_D$  greater than 10,000, we expect pressure transient responses similar to Darcy flow for any value of  $k_{fnnr}$ . We have noticed that if  $\tau_D$  is less than 1,000 then the parameter  $k_{fnnr}$  become more sensitive. Pressure transient responses (pressure and its derivative) of non-Darcy flow for  $\tau_D$  of 1 is close to responses of  $\tau_D$  of 10 because of constant value of  $k_{fnnr}$  of 0.01 used in both cases. In this example, if  $\tau_D$  is reduced below 1, pressure transient responses will match the case of  $\tau_D$  of 1 for constant value of  $k_{fnnr}$  of 0.01. In general, reducing the value of  $\tau_D$  (the characteristic length), increasing non-Darcy flow effects, reduces the fracture conductivity similar to the effect of reducing the value of  $k_{fnnr}$ parameter ( $k_{fmin}$ ). In general, the characteristic length is a more sensitive parameter for non-Darcy flow, while the minimum permeability is sensitive at lower values of the characteristic length. Non-Darcy flow effects may cause infinite-conductivity fracture to behave as fractures of low finite-conductivity. If pressure transient data is analysed conventionally under these flow conditions, incorrect estimates of fracture conductivity and properties will be obtained.

#### **Non-Darcy Flow Effects in Naturally Fractured Reservoirs**

For pressure-transient type curves in naturally fractured reservoirs, the dimensionless wellbore pressure is given by Equation (16), the dimensionless time is given by Equation (24), and the dimensionless production rate by Equation (25).

$$t_{Dxf} = \frac{0.0002637 k_f t}{\left(\phi_m c_{tm} + \phi_f c_{tf}\right) \mu r_w^2}$$
(24)  
$$a_D = \frac{q}{Q_D}$$
(25)

$$q_D = \frac{1}{q_{\text{max}}} \tag{25}$$

Fig 19 shows a log-log plot of wellbore pressure drop and its derivative versus time for Darcy flow case and three cases of non-Darcy flow. In simulating Darcy flow case, the minimum permeability in natural fractures is set equal to the fracture permeability. The first non-Darcy flow case, Case 1, is for  $k_{fmin}$  of 0.001 of  $k_f$  and  $\tau$  is 100 ft<sup>-1</sup> and the second non-Darcy flow case, Case 2, is for  $k_{fmin}$  of 0.001 of  $k_f$  and  $\tau$  is 20 ft<sup>-1</sup>. The third non-Darcy flow case, Case 3, is for  $k_{fmin}$  of 0.1 of  $k_f$  and  $\tau$  is 20 ft<sup>-1</sup>. We have seen that if  $\tau$  is greater than 1,000 ft<sup>-1</sup>, there is no significant effect of non-Darcy flow on pressure-transient responses. The same conclusion was obtained before when using the BCM for analyzing pressure-transient behavior of non-Darcy flow of vertical wells in single-porosity reservoirs (Al-Otaibi and Wu, 2010) and hydraulically fractured wells. If we compare between Cases 1 and 2, we find that the pressure drop due to non-Darcy flow increases as the value of  $\tau$  decreases under the same value of  $k_{fmin}$ . Also, we find if  $k_{fmin}$  decreases the pressure drop due to non-Darcy flow increases as we compare between Cases 2 and 3 under the same value of parameter  $\tau$ . Non-Darcy flow effects change the early times characteristics of double-porosity behavior compared to Darcy flow. As shown in Fig 19, the pressure derivative curves show clearly the effect of non-Darcy flow on early time pressure-transient responses of a naturally fractured reservoir. Non-Darcy flow behavior affects the early time characteristics of double-porosity behavior and changes the slope and derivative responses depending on how severe non-Darcy flow effect is. Thus, under severe effects of non-Darcy flow, the analysis of pressure-transient data with early time responses assuming Darcy flow may lead to significant errors in estimating naturally fractured reservoir parameters.

The BCM uses two parameters ( $k_{min}$  and  $\tau$ ) to describe non-Darcy flow behavior while the Forchheimer model uses only one parameter ( $\beta$ ). We know that is not possible to combine non-Darcy flow parameters of BCM into one parameter for typecurve analysis of pressure-transient data. To illustrate this point, we developed a dimensionless relationship that combines non-Darcy flow parameters of BCM into one dimensionless parameter ( $F_{nD}$ ). The new dimensionless non-Darcy flow parameter of the BCM is given by

$$F_{nD} = k_{fmr} \tau_D = \frac{k_{f\min} \tau r_w}{k_f}$$
(26)

In Equation (26) the wellbore radius,  $r_w$ , and the permeability of the fracture,  $k_f$ , are constants and the variables are the non-Darcy flow parameters,  $k_{fmin}$  and  $\tau$ . In the following example we show that it is not possible to use such relation, Equation (26) and we must use the two parameters,  $k_{fmr}$  and  $\tau_D$  of the BCM to model non-Darcy flow in porous fractured media. The input data used in the following type-curve examples for numerical simulation is shown in Table 5.

We use input data shown in Table 5 with interporosity flow coefficient,  $\lambda$ , of  $1 \times 10^{-5}$  and storativity ratio,  $\omega$ , of 0.01 in this example. Fig 20 shows a log-log plot of dimensionless wellbore pressure and its derivative versus dimensionless time (type curve) for two non-Darcy flow cases. For the first non-Darcy flow case, Case 1,  $k_{fnrr}$  is 0.001 and  $\tau_D$  is 100 and for the second case, Case 2,  $k_{fnrr}$  is 0.1 and  $\tau_D$  is 10. Using Equation (26), to find value of  $F_{nD}$  parameter for the two non-Darcy flow cases, we obtain the same value of  $F_{nD}$  of 1. Although Cases 1 and 2 have the same value of  $F_{nD}$  parameter, results shown in Fig 20 indicate that pressure-transient responses of Cases 1 and 2 are not the same. Thus, it is not possible to combine non-Darcy flow parameters of the BCM into one parameter for type-curve analysis of pressure-transient data. Case 2 has more significant non-Darcy flow effects compared to Case 1 because of lower value of parameter  $\tau_D$ , which adds more pressure drop and changes

early-time pressure-transient behavior. As we concluded earlier in this study, the characteristic length,  $\tau$ , is the more sensitive parameter of the BCM where at high values of  $\tau$ , the minimum permeability,  $k_{min}$  has no significant impact on pressure-transient responses. For low values of  $\tau$ , however, the minimum permeability has significant impact on pressure-transient responses.

## **Effect of Production Rate**

Type curves shown in Fig 21 show the effect of production rate on pressure-transient responses of non-Darcy flow in a naturally fractured reservoir. Fig 21 shows a log-log plot of dimensionless wellbore pressure and its derivative versus dimensionless time (type curve) for two non-Darcy flow cases under two production rates. The wellbore storage and skin damage effects are not considered in these type curves. In the first non-Darcy flow case,  $k_{fmr}$  is 0.1 and  $\tau_D$  is 1,000 which simulates low non-Darcy flow effects and for two production rates,  $q_D$  of 1 and 0.5. In the first non-Darcy flow case,  $k_{fmr}$  is 0.1 and  $\tau_D$  is 1,000 which simulates low non-Darcy flow effects and for two production rates,  $q_D$  of 1 and 0.5. Note that for  $q_D$  of 1 represents maximum production rate of 10,000 STB/day and  $q_D$  of 0.5 represents a production rate of 5,000 STB/day for this particular example. The increase in production rate under the same conditions of non-Darcy flow effects (low or high) causes significant increase in pressure drop at the wellbore and changes the early-time characteristics of pressure-transient responses of a dual-porosity reservoir, as shown in Fig 21. The changes in early-time pressure-transient behavior of the double-porosity reservoir can be more significant for high non-Darcy flow effects under high production rates. This indicates that non-Darcy flow effects on pressure-transient responses, especially, on early-times responses, can be more severe at high production rates.

#### **Effect of Double-Porosity Parameters**

Type curves shown in Fig 22 show the effect of interporosity flow coefficient ( $\lambda$ ), the double-porosity parameter, on pressuretransient responses of non-Darcy flow in a naturally fractured reservoir. Fig 22 shows a log-log plot of dimensionless wellbore pressure and its derivative versus dimensionless time for two cases of non-Darcy flow and two cases of Darcy flow. For non-Darcy flow cases,  $k_{fmr}$  is 0.1 and  $\tau_D$  is 10 which simulates high non-Darcy flow effects and for two cases of interporosity flow coefficient,  $\lambda$  of 1x10<sup>-5</sup> and 1x10<sup>-6</sup>. For Darcy flow cases,  $k_{fmr}$  is 1, and for two cases of interporosity flow coefficient,  $\lambda$  of 1x10<sup>-5</sup> and 1x10<sup>-6</sup>. In all cases, the storativity ratio,  $\omega$ , is 0.01. As expected, reducing the value of  $\lambda$  shift the concave up curve, characteristics of a double-porosity reservoir, of the pressure derivative to the right for both Darcy and non-Darcy flow cases. Non-Darcy flow effects changes the early-time pressure-transient behavior of the double-porosity reservoir compared to Darcy flow for all cases of  $\lambda$  as shown in the pressure derivative curves in Fig 22. However, non-Darcy flow does not change the intermediate- and late-time characteristics of the pressure-transient behavior of the double-porosity reservoir for all cases of interporosity flow coefficient which matches the Darcy flow behavior.

Type curves shown in Fig 23 show the effect of storativity ratio ( $\omega$ ), the double-porosity parameter, on pressure-transient responses of non-Darcy flow in a naturally fractured reservoir. For non-Darcy flow cases,  $k_{fmr}$  is 0.1 and  $\tau_D$  is 10 which simulates high non-Darcy flow effects and for two cases of storativity ratio,  $\omega$  of 0.01 and 0.1. For Darcy flow cases,  $k_{fmr}$  is 1, and two cases of storativity ratio,  $\omega$  of 0.01 and 0.1 is considered. In all cases, the interporosity flow coefficient,  $\lambda$ , is  $1 \times 10^{-5}$ . Increasing the value of  $\omega$  moves the concave up curve, characteristics of double-porosity reservoir, of the pressure derivative to the top for both Darcy and non-Darcy flow cases. Non-Darcy flow effects impact the early-time pressure-transient behavior of the double-porosity reservoir compared to Darcy flow for all cases of  $\omega$  as shown in the pressure derivative curves in Fig 23. However, non-Darcy flow does not change the intermediate- and late-time characteristics of the pressure-transient behavior of the double-porosity reservoir for all cases of storativity ratio which matches the Darcy flow behavior.

### Effect of Wellbore Storage and Skin Damage

Type curves shown in Fig 24 show the combined effect of wellbore storage and non-Darcy flow on pressure-transient responses of a naturally fractured reservoir. Fig 24 shows four cases of wellbore storage ( $C_D$  of 0, 10, 100 and 1,000). For non-Darcy flow,  $k_{finr}$  is 0.1 and  $\tau_D$  is 10 which simulates high non-Darcy flow effects, the storativity ratio,  $\omega$ , is 0.01 and the interporosity flow coefficient,  $\lambda$ , is  $1\times10^{-5}$ . The wellbore storage can be identified clearly by the unit-slope line on a log-log plot (Fig 24). As concluded earlier in this chapter, non-Darcy flow effects impact mainly the early-time behavior of pressure-transient responses of the double-porosity reservoir. The wellbore storage effects usually complicate the analysis of the pressure-transient data in naturally fractured reservoirs. When the value of the wellbore storage coefficient increases, it is more difficult to identify the double-porosity reservoir responses. The wellbore storage masks the early-time responses of pressure-transient data and therefore may be difficult to identify non-Darcy flow effects on early-time responses. If the wellbore storage is significant, case of  $C_D$  of 1,000 in this example, the double-porosity reservoir behavior may be masked as well as indicated in pressure derivative curves in Fig 24. Thus for high wellbore storage effects pressure-transient responses of the double-porosity reservoir storage effects pressure-transient responses of the double-porosity reservoir storage effects pressure-transient responses of the double-porosity reservoir behavior may be masked as well as indicated in pressure derivative curves in Fig 24. Thus for high wellbore storage effects with non-Darcy flow may complicate the analysis of pressure-transient data for pressure buildup and drawdown tests, wellbore storage effects with non-Darcy flow may complicate the analysis of pressure-transient data and increases the problem of solution uniqueness of type-curve analysis.

Type curves shown in Fig 25 show the combined effect of skin damage and non-Darcy flow on pressure-transient responses of a naturally fractured reservoir. Fig 25 shows four cases of skin damage (S of 0, 2, 5 and 10). For non-Darcy flow,

 $k_{fmr}$  is 0.1 and  $\tau_D$  is 10 which simulates high non-Darcy flow effects, the storativity ratio,  $\omega$ , is 0.01, the interporosity flow coefficient,  $\lambda$ , is  $1 \times 10^{-5}$  and the dimensionless wellbore storage coefficient,  $C_D$ , is 10. The skin damage adds an additional pressure drop and does not change the characteristics of early-time responses of the pressure-transient behavior of the double-porosity reservoir. As the value of the skin factor increases the pressure drop at the wellbore increases because of physical formation damage, however, the shape and characteristics of the pressure derivative curves does not impact significantly for early- and late-time pressure-transient responses. Note that in this example because of wellbore storage effects, the early-time responses cannot be identified in Fig 25. However, we have studied the effect of skin damage with non-Darcy flow without wellbore storage effects earlier in this chapter and concluded that skin damage does not change the early-time characteristics of the double-porosity reservoir. The combined effect or co-existence of non-Darcy flow, skin damage, and wellbore storage may further complicate the analysis of a single-rate well test in a naturally fractured reservoir using type-curves. This is because many related matching parameters and solution uniqueness problem usually exist in well-testing type curves analysis.

#### **Analysis of Multi-Rate Test**

The combined effect or co-existence of physical skin damage and non-Darcy flow usually complicates the analysis of actual pressure transient data using a single-rate test. This is because it is very difficult to differentiate between the skin effect and non-Darcy flow behavior using a single flow rate test (pressure drawdown or buildup test) in a naturally fractured reservoir. In previous sections, we found that the late-time behavior of pressure-transient responses of non-Darcy flow is similar to physical skin behavior and the only difference is in early-time responses if non-Darcy flow behavior is significant. Furthermore, the early-time behavior of non-Darcy flow may be masked by wellbore storage effects. If physical skin does not exist, which is typically not the case; it is possible to estimate the skin factor due to non-Darcy flow, similar to estimating the physical skin, and further use the type-curve regression to analyze the data using BCM or Forchheimer models. In presence of combined effects of skin and non-Darcy flow, a multi-rate test (minimum of two variable flow rates) is required for an accurate estimate of the physical skin damage and non-Darcy flow skin. The total skin factor,  $S_{t}$ , can be estimated using Equation (27). The total skin factor, Equation (28), is the sum of physical skin, S, and the skin due to non-Darcy flow,  $S_{ND}$ .

$$S = 1.15I\left[\frac{\Delta p_{1hr}}{m} - \log\left(\frac{\bar{k}}{\left(\phi_f c_f + \phi_m c_m\right)\mu r_w^2}\right) + 3.23\right]$$
(27)

$$S_t = S + S_{ND} \tag{28}$$

In this example, we use the input data of the double-porosity reservoir shown in Table 2 to simulate a multi-rate well test under combined effect of non-Darcy flow and skin damage using the BCM. Synthetic pressure transient responses are generated for a high non-Darcy flow case of  $k_{finr}$  is 0.1 and  $\tau$  is 10 ft<sup>-1</sup>. Four different flow tests, simulating four pressure drawdown tests at variable production rates (2000, 3000, 4000 and 6000 STB/day) are considered in which each flow test lasts for 10 hours. In each flow test, three cases of physical skin damage are considered (S = 0, 2 and 4). Also, in each flow test we simulate Darcy flow responses without skin damage to show the significance of non-Darcy flow effects.

Fig 26 shows the wellbore pressure versus time for the multi-rate well test with non-Darcy flow effects. A quick look at Fig 26 shows that when the production rate increases the pressure drop due to non-Darcy flow and formation damage increases. The increase in pressure drop due non-Darcy flow is more noticeable compared to the increase in pressure drop due to formation damage ends to increase the pressure drop due to formation damage when the production rate increases. Each flow test is analyzed using the standard straight-line analysis technique to estimate the effective permeability and the total skin factor for each non-Darcy flow effects but the estimated effective permeability is less than the input fracture permeability due to non-Darcy flow effects but the estimated error is insignificant which depends on how severe is non-Darcy flow. The estimated total skin factor does change in each case because of the combined effect of non-Darcy flow and formation damage. The total skin factor,  $S_t$ , is estimated using Equation (27). The estimated total skin is the sum of the skin due to formation damage (S) and non-Darcy flow skin ( $S_{ND}$ ). Thus if total skin and physical skin are known, then the non-Darcy flow skin can be estimated using Equation (28).

Results of the analysis of pressure-transient responses for all cases of physical skin and non-Darcy flow considered in the multi-rate test are shown in Fig 27. Fig 27 show plots of the estimated total skin factor versus production rate for all cases considered in the multi-rate test. The non-Darcy flow skin is a function of the production rate, as the production rate increases the skin factor due to non-Darcy flow increases with or without physical skin damage. However, the physical skin of formation damage tends to decrease the estimated value of the non-Darcy flow skin, thus as the value of physical skin increases the estimated value of non-Darcy flow skin decreases for all cases. The slope of the straight line of the estimated total skin factor versus production rate is approximately constant for all cases of physical skin damage can be estimated from the intercept of the straight-line.

## **Field Examples**

In the first field example, we show analysis results of a pressure buildup test for high-rate oil producer well in a single-porosity reservoir in Kuwait using BCM. Analysis of pressure buildup test data shows that  $S_t = -0.244$  and C = 0.1108 bbl/psi using standard well testing techniques. Although the estimated skin factor is negative which indicates a stimulation case, non-Darcy flow effects may exist and affects the well performance. Type curves generated by the numerical model of BCM were used to match the actual field data assuming Darcy and non-Darcy flow. An equivalent shut-in time suggested by Agarwal (1980) is used to account for the effects of producing time on pressure-buildup. Fig 28 shows a log-log plot of wellbore pressure drop and its derivative versus equivalent shut-in time for actual pressure buildup data and the numerical model for two matched cases of Darcy and non-Darcy flow. Note that the matching process requires the knowledge of all reservoir properties except the physical skin factor and non-Darcy flow parameters.

The physical skin factor is not known because no multi-rate test was available for this well, thus the estimated skin factor from the buildup test represents a total skin factor. In this example, we should emphasize that the estimated total skin factor  $(S_t)$  from a pressure buildup test was assumed to be mainly due to non-Darcy flow effects  $(S_t = S_{ND})$ , because no multi-rate test was available for this well and it was stimulated before the pressure transient tests. Therefore, in this particular example, the estimated non-Darcy flow parameters may represent low values of  $k_{min}$  and  $\tau$  (high non-Darcy flow). It was difficult to match some of the data at early times for Darcy and non-Darcy flow cases. Furthermore, using the numerical model type curves, it is possible to obtain many cases of non-Darcy flow and physical skin factor of -0.244. The second matching case, Case 2, is for non-Darcy flow ( $k_{min} = 1.2$  md and  $\tau = 49.98$  ft<sup>-1</sup>) and a skin factor of -2.63. The results indicate that if non-Darcy flow effects exist then the actual physical skin factor of the well is -2.63 which represents a highly stimulated well. Thus, the productivity decline may be due to non-Darcy flow effects and not the physical skin damage. These results should be confirmed by the analysis of a multi-rate test to estimate actual skin factors of physical skin and non-Darcy flow.

In the second field example, we show analysis results of actual pressure buildup test for a hydraulically fractured well in Kuwait using the numerical model BCM. Type curves generated by the BCM were used to match the actual field data assuming Darcy and non-Darcy flow. Fig 29 shows a log-log plot of wellbore pressure drop and its derivative versus equivalent shut-in time for actual pressure buildup data and the numerical model for two cases of Darcy and non-Darcy flow. It was difficult to match some of the data at early times for Darcy and non-Darcy flow cases. Analysis of pressure buildup test data shows that the average permeability of the formation is 2.47 md and the wellbore storage coefficient is 0.1235 bbl/psi. It was possible to obtain many cases of Darcy and non-Darcy flow parameters that match the actual field data by changing many parameters during type-curve matching using the numerical model. In this example, we show only two cases that match the actual field data for a finite-conductivity fracture with Darcy flow and an infinite-conductivity fracture with non-Darcy flow. As shown in Fig 29, the field data was matched with a type curve of a finite-conductivity fracture ( $x_{f} = 100$  ft) assuming Darcy flow. Also, the field data was matched with a type curve of an infinite-conductivity fracture ( $x_{f} = 100$  ft) assuming non-Darcy flow ( $k_{fmr} = 0.01$  and  $\tau = 30.1$  ft<sup>-1</sup>). In both cases, the fracture-skin factor is not considered (i.e.  $S_{f} = 0$ ). The results indicate that if non-Darcy flow effects exist then an infinite-conductivity hydraulic fracture may behave as a fracture of lower finite-conductivity. The analysis of pressure-transient data under non-Darcy flow conditions assuming Darcy flow and using conventional methods will result in incorrect estimates of fracture conductivity.

In the third field example, we use type curves generated by BCM to analyze non-Darcy flow behavior on a pressuretransient data for a high flow rate well in a naturally fractured reservoir in Kuwait. Analysis of pressure buildup test data shows that the estimated average permeability is 1,812 md, the interporosity flow coefficient ( $\lambda$ ) is 2.149x10<sup>-7</sup>, the storativity ratio ( $\omega$ ) is 0.0025, and the total estimated skin factor ( $S_t$ ) is 5.29. Type curves generated by the numerical model of BCM were used to match the actual field data assuming Darcy and non-Darcy flow. An equivalent shut-in time is used to account for the effects of producing time on pressure-buildup tests. Fig 30 shows a log-log plot of wellbore pressure drop and its derivative versus equivalent shut-in time for actual pressure buildup data and the numerical model for two matched cases of Darcy and non-Darcy flow. Note that the matching process requires the knowledge of all reservoir properties except the physical skin factor and non-Darcy flow parameters. Again, it was difficult to match some of the data at early times for Darcy and non-Darcy flow cases. In this example, the physical skin factor is not known because no multi-rate test was available for this well, thus the estimated skin factor from the buildup test represents a total skin factor. Furthermore, using the numerical model type curves, it is possible to obtain many cases of non-Darcy flow and physical skin that all match the actual field data. The first matching case, Case 1, is for Darcy flow and a physical skin factor of 5.29. The second matching case, Case 2, is for non-Darcy flow ( $k_{min} = 17.9$  md and  $\tau = 127$  ft<sup>-1</sup>) and a skin factor of 1.48. The results indicate that if non-Darcy flow effects exist then the actual physical skin factor is less than the skin factor of non-Darcy flow. Thus, the non-Darcy flow, not the physical skin damage, is the main reason of the well productivity decline. These results should be confirmed by the analysis of a multi-rate test to estimate actual skin factors of physical skin and non-Darcy flow.

#### Conclusions

Non-Darcy flow is very important in flow calculations in porous and fractured reservoirs, especially in hydraulic fractures and stimulation design. The motivation for the work presented in this paper to investigate, validate and extend the Barree-Conway flow model (BCM) for pressure-transient analysis of wells involving hydraulic fractures and in naturally fractured reservoirs.

We developed and used 3D numerical models according to Forchheimer and BCM specifically for pressure-transient analysis of single-phase fluid flow in porous and fractured reservoirs. The developed numerical models are capable of simulating all near wellbore effects coupled with the non-Darcy flow behavior. An equivalent non-Darcy flow coefficient was derived based on the apparent permeability of non-Darcy flow for both Forchheimer and BCM equations. The numerical model of non-Darcy flow according to BCM have been used to model and interpret actual field pressure-transient data in both porous-medium and fractured reservoirs.

The specific conclusions obtained from the work presented in this paper are as follow:

- 1. The Barree-Conway model (BCM) can be applied, similar to the Forchheimer model, to analyze and interpret pressure-transient responses of non-Darcy flow in porous and fractured reservoirs. The BCM predicts similar pressure-transient responses of non-Darcy flow to those as predicted by the Forchheimer model under the same fluid flow conditions for vertical wells in porous or fractured reservoirs.
- 2. The non-Darcy flow parameters of the BCM may not be all estimated from single-rate pressure-transient tests in single-porosity reservoirs using conventional analyses. However, they can be estimated by a matching process based on non-linear optimization algorithm incorporated into the developed numerical model, provided that the mechanical skin is known in prior from analysis of multi-rate pressure-transient test (minimum of two flow rate tests).
- 3. Type curves generated by the BCM are provided to demonstrate a methodology for modeling single-phase pressuretransient behavior of non-Darcy flow in single-porosity reservoirs, hydraulically fractured wells, and naturally fractured reservoirs.
- 4. The BCM parameter of characteristic length,  $\tau$ , is more sensitive than the minimum permeability,  $k_{min}$ , parameter for all flow rates considered in this study. The minimum permeability parameter is only sensitive at low values of the characteristic length or at extremely high flow rates.
- 5. Barree and Conway related their model parameter  $\tau$  to the particle size of the porous or fractured system which should corresponds to high values. However, in numerical simulation of pressure-transient tests, we found that the meaning of this parameter is related to small effective radius (in terms of 10<sup>-4</sup> to 10<sup>-1</sup> ft depending on flow conditions of non-Darcy flow and the type of the reservoir and the well) and it must has a small value for significant non-Darcy flow to occur.
- 6. In simulating multi-rate tests in single- and double-porosity reservoirs, increase in the value of the mechanical skin due to formation damage does not significantly affect the estimated value of the skin factor due to non-Darcy flow at a given constant production rate. In most cases, increase in the skin damage slightly decreases the skin factor due to non-Darcy flow at a given constant production rate.
- 7. In numerical simulation of finite-conductivity hydraulic fractures using the BCM, the change in pressure transient responses because of decrease in the value of the minimum permeability and/or the characteristic length parameters in the fracture for high non-Darcy flow effects is similar to the pressure change due to decrease in the fracture conductivity in Darcy flow case. The non-Darcy flow behavior affects the characteristics and duration of flow patterns in hydraulically fractured wells and hence the analysis of pressure transient data. The analysis of pressure-transient responses of non-Darcy flow using conventional methods results in lower estimate of the fracture conductivity and incorrect fracture properties.
- 8. The combined effect of non-Darcy flow and skin damage changes the characteristics of early time pressure-transient responses of naturally fractured reservoirs. The skin damage effect alone does not change the characteristics of early time pressure-transient responses with Darcy or non-Darcy flow.
- 9. In naturally fractured reservoirs, it is possible to match using type curves the late-time pressure-transient responses of high non-Darcy flow effects. However, under the same conditions, it is difficult to match the early-time pressure-transient responses of high non-Darcy flow effects.

Parameter	Value	Unit
Darcy Permeability	$k_d = 100$	md
Minimum Permeability	$k_{min} = 1$	md
Characteristic Length	$\tau = 100$	1/ft
Equivalent $\beta$ (Forchheimer)	$\beta = 9.064 \text{E} + 09$	1/ft
Viscosity	$\mu = 1.2$	ср
Porosity	$\phi = 0.2$	
Density	$\rho = 60$	lb/ft <sup>3</sup>
Total Compressibility	$C_t = 10 \times 10^{-6}$	psi <sup>-1</sup>
Production Rate	q = 10,000	STB/d
Formation Thickness	h = 100	ft
Initial Pressure	$p_i = 5,000$	psi
Reservoir Drainage Radius	$r_e = 10,000$	ft
Wellbore Radius	$r_w = 0.33$	ft

#### Table 1 - Input data used for comparison of BCM and Forchheimer numerical models

Table 2 - Inpu	it used in nur	nerical mode	ls for pressure-
transient res	ponses in a na	aturally fract	ured reservoir

Parameter	Value	Unit
Matrix Permeability	$k_m = 1$	md
Fracture Permeability	$k_f = 1,000$	md
Matrix Porosity	$\phi_m = 0.15$	
Fracture Porosity	$\phi_{f} = 0.001$	
Viscosity	$\mu = 1.2$	cp
Density	$\rho = 60$	lb/ft <sup>3</sup>
Total Compressibility in	$C_{tf} = 1 \times 10^{-6}$	psi <sup>-1</sup>
Fracture		
Total Compressibility in	$C_{tm} = 1 \times 10^{-6}$	psi <sup>-1</sup>
Matrix		
Production Rate	q = 6,000	STB/d
Formation Thickness	h = 50	Ft
Initial Pressure	$p_i = 5,000$	psi

Reservoir drainage radius	$r_e = 10,000$	Ft
Wellbore Radius	$r_w = 0.33$	ft

 Table 3 - Input data used for generating pressure-transient

 type-curves of non-Darcy flow in single-porosity reservoir

 using BCM

Parameter	Value	Unit
$k_{min}$ relative to $k_d$	$k_{mr} = 1, 0.5, 0.2$	
Dimensionless	$\tau_{wD} = 0.1, 10,$	
Characteristic Length	100	
Dimensionless Wellbore	$C_D = 500$	
Storage		
Skin Factor	S = 1	
Viscosity	μ = 1.2	ср
Porosity	$\phi = 0.2$	
Density	$\rho = 60$	lb/ft <sup>3</sup>
Total Compressibility	$C_t = 10 \times 10^{-6}$	psi <sup>-1</sup>
Production Rate	<i>q</i> = 5,000	STB/d
Formation Thickness	<i>h</i> = 100	ft
Initial Pressure	$p_i = 5,000$	psi
Reservoir drainage radius	$r_e = 10,000$	ft
Wellbore Radius	$r_{w} = 0.3$	ft

 Table 4 - Input data used for simulating pressure-transient

 responses of a hydraulically fractured well using BCM for

 the verification examples

Parameter	Value	Unit
Reservoir and Fluid Data		
Darcy Permeability	$k_d = 9.02$	md
Minimum to Darcy Permeability	$k_{mr} = 1$ (Darcy Flow)	
Ratio		
Porosity	$\phi = 0.2$	
Formation Thickness	<i>h</i> = 100	ft
Total Compressibility	$C_t = 10 \times 10^{-6}$	psi <sup>-1</sup>
Wellbore Radius	$r_{w} = 0.35$	ft
Initial Pressure	$p_i = 5,000$	psi
Production Rate	<i>q</i> = 5,032	STB/d
Fluid Viscosity	$\mu = 1.2$	ср
Fluid Density	$\rho = 60$	lb/ft <sup>3</sup>
Hydraulic Fracture Data		
Fracture Half-Length	$x_f = 400, 40, 1000$	ft
Fracture Width	$W_f = 0.02$	ft
Fracture Conductivity	$C_{fD} = 10, 100, \text{ infinite}$	

 Table 5 - Input data used in numerical model for generating pressure-transient type curve of non-Darcy flow in a naturally fractured reservoir using BCM

Parameter	Value	Unit
Matrix Permeability	<i>k</i> <sub>m</sub> = 1	md
Fracture Permeability	$k_f = 6,600$	md
Matrix Porosity	$\phi_m = 0.10$	
Fracture Porosity	$\phi_{\rm f} = 0.001$	
Viscosity	μ = 1.2	ср
Total Compressibility in Fracture	$C_{tf} = 1 \times 10^{-6}$	psi <sup>-1</sup>
Total Compressibility in Matrix	$C_{tm} = 1 \times 10^{-6}$	psi <sup>-1</sup>
Production Rate	<i>q</i> = 10,000	STB/d
Formation Thickness	<i>h</i> = 50	ft
Initial Pressure	$p_i = 5,000$	psi
Reservoir drainage radius	$r_e = 10,000$	ft





Wellbore Radius

Fig. 1 Experimental results of Forchheimer and BCM models for single-phase non-Darcy experiments by Lai et al., Colorado School of Mines (2009)



Fig. 2 Space discretization and flow-term evaluation in the IFD method (Pruess et al., 1999) used in the numerical simulator



Fig. 3 Comparison of pressure-transient responses of non-Darcy flow of BCM and Forchheimer models for a fully penetrating well in a single-porosity reservoir



Fig. 4 Log-Log plot of pressure-transient responses of Forchheimer and BCM models assuming Darcy flow in a naturally fractured reservoir



Fig. 5 Log-Log plot of pressure-transient responses of non-Darcy flow in a naturally fractured reservoir using Forchheimer and BCM models



Fig. 6 Effect of non-Darcy flow parameter  $k_{\rm mr}$  of BCM on pressure transient responses (type curves) in a single-porosity reservoir



Fig. 7 Effect of non-Darcy flow parameter  $\tau_D$  of BCM on pressure transient responses (type curves) in a single-porosity reservoir



Fig. 8 Reservoir grid system for numerical simulation of an infiniteconductivity hydraulically fractured well (2-D View)



Fig. 9 Reservoir grid system for numerical simulation of a finiteconductivity hydraulically fractured well (2-D View)



Fig. 10 Pressure-transient responses of a hydraulically fractured well for comparison case 1 (C  $_{\rm ID}$  = 10)



Fig. 11 Pressure-transient responses of a hydraulically fractured well for comparison case 2 ( $C_{\rm ID}$  = 100)



Fig. 12 Pressure-transient responses of a hydraulically fractured well for comparison case 3 (Infinite-Conductivity)



Fig. 13 Effect of  $k_{\rm fmr}$  on pressure-transient responses of a hydraulically fractured well for Case 1,  $C_{\rm ID}$  = 1



Fig. 14 Effect of  $k_{fmr}$  on pressure-transient responses of a hydraulically fractured well for Case 2,  $C_{ID}$  = 100



Fig. 15 Effect of non-Darcy flow and reduced fracture conductivity on pressure-transient responses of a hydraulically fractured well



Fig. 16 Effect of  $\tau_D$  , on pressure-transient responses of a hydraulically fractured well for  $C_{\rm ID}$  = 100



Fig. 17 Effect of non-Darcy flow parameter ( $k_{tmr}$ ) and reduced fracture conductivity on pressure-transient responses of a hydraulically fractured well (infinite-conductivity)



Fig. 18 Effect of non-Darcy flow parameter  $(\tau_D)$  and reduced fracture conductivity on pressure-transient responses of a hydraulically fractured well (infinite-conductivity)



Fig. 19 Log-Log plot of pressure-transient responses showing effect of non-Darcy flow in a naturally fractured reservoir using BCM



Fig. 20 Type curves studying effect of combining non-Darcy flow parameters of the BCM in a naturally fractured reservoir



Fig. 21 Type curves studying effect of non-Darcy flow and production rate on pressure-transient behavior of a naturally fractured reservoir using BCM



Fig. 22 Type curves showing effect of non-Darcy flow and interporosity flow coefficient on pressure-transient behavior of a naturally fractured reservoir using BCM



Fig. 23 Type curves showing effect of non-Darcy flow and storativity ratio on pressure-transient behavior of a naturally fractured reservoir using BCM



Fig. 24 Type curves studying effect of non-Darcy flow and wellborestorage on pressure-transient behavior of a naturally fractured reservoir using BCM



Fig. 25 Type curves studying effect of non-Darcy flow and skin damage on pressure-transient behavior of a naturally fractured reservoir using BCM



Fig. 26 A multi-rate test in a naturally fractured reservoir



Fig. 27 Estimated total skin versus production rate for the multi-rate well test in a naturally fractured reservoir



Fig. 28 Type curve matching of a pressure buildup test data from single-porosity reservoir using the BCM



Fig. 29 Type curve matching of a pressure buildup test for a hydraulically fractured well using the BCM

## Nomenclature

v

- p = pressure, psi
- k = formation permeability, Darcy or md
- = superficial velocity, ft/s
- t = time, hour
- q =volumetric flow rate, bbl/day
- V =volume, bbl
- F = mass flow term, lbm/day
- Q = mass sink/source term
- A = area of interface, ft<sup>2</sup>
- D = depth, ft
- R = residual, dimensionless
- h = formation thickness, ft
- r = radius or distance in radial direction, ft
- L = characteristic dimension of a matrix block
- n = number of normal sets of planes limiting the less-permeable medium (n = 1, 2 or 3)
- $c = \text{compressibility, psi^{-1}}$
- C = wellbore storage coefficient, bbl/psi
- S = skin factor, dimensionless
- $S_t$  = total skin factor, dimensionless



Fig. 30 Type curve matching of a pressure buildup test for a vertical well in naturally fractured reservoir using BCM

 $x_f$  = fracture-half length, ft

#### Greek Symbols

- $\mu$  = fluid viscosity, cp
- $\rho$  = fluid density, lbm/ft<sup>3</sup>
- $\beta$  = non-Darcy flow coefficient, ft<sup>-1</sup>
- $\tau$  = characteristic length, ft<sup>-1</sup>
- $\phi$  = formation porosity, fraction
- $\lambda$  = mobility of flowing fluid phase, md/cp; or interporosity flow coefficient, dimensionless
- $\gamma$  = transmissivity of flow
- $\Phi$  = flow potential, psi
- $\omega$  = storativity ratio, dimensionless

#### Superscript

n = current time level

n+1 = next time level to be solved

#### Subscript

- D = Darcy
- mr = minimum relative
- $\beta$  = fluid phase (could be oil or gas)
- i, j = arbitrary element or node
- ij+1/2 = proper averaging of properties at the interface between the two elements i, j
- N = total number of nodes, elements or grid blocks
- $\eta_i$  = set of neighboring elements (j) (non-fractured or fractured) to which element i is directly connected
- app = apparent
- *min* = minimum
- p = perforated
- w = wellbore
- m = matrix
- f =fracture
- i = initial conditions, i.e. at time equal zero
- wf = wellbore flowing
- D = dimensionless
- s = skin zone
- lhr = at time equal one hour
- max = maximum
- e = outer boundary

### References

- Agarwal, R.G. (1980). A New Method to Account for Producing Time Effects when Drawdown Type Curves are Used to Analyze Pressure-Buildup and Other Test Data. SPE 9289, SPE Annual Technical Conference and Exhibition, Dallas, 21-24 September.
- Al-Otaibi, A.M. and Wu, Y.S. (2010). Transient Behavior and Analysis of Non-Darcy Flow in Porous and Fractured Reservoirs According
- to the Barree and Conway Model. SPE 133533, SPE Western North America Regional Meeting, Anaheim, California, 26-30 May. Barenblatt, G.I., Zheltov, I.P. and Kochina, I.N. (1960). Basic Concepts in the Theory of Seepage of Homogeneous Liquids in Fissured
- Rocks. PMM, Sov. Appl. Math. Mech., 24(5), 852-864.
- Barree, R.D. and Conway, M.W. (2004). Beyond Beta Factors: A Complete Model for Darcy, Forchheimer and Trans-Forchheimer Flow in Porous Media. SPE 89325, SPE Annual Technical Conference and Exhibition, Houston, Texas 26-29 September.
- Blacker, L.K. (1982). An Analysis of Rate-Sensitive Skin in Oil Wells. SPE 11187, SPE Annual Technical Conference and Exhibition, New Orleans, 26–29 September.
- Carman, P.C. (1937). Fluid Flow through Granular Beds. Transactions Institution of Chemical Engineers 15 (150).
- Fand, R.M., Kim, B.Y.K, Lam, A.C.C. and Phan, R.T. (1987). Resistance to the Flow of Fluids through Simple and Complex Porous Media whose Matrices are Composed of Randomly Packed Spheres. Transactions of the ASME 109 (268).
- Forchheimer, P. (1901). Wasserbewegung durch Bode. Zeits Deutsch, Vol. 45, 1781.
- Gidley, J.L. (1991). A Method for Correcting Dimensionless Fracture Conductivity for Non-Darcy Flow Effects. SPEPE, Nov. 1991, 391.
- Guppy, K.H., Cinco-Ley, H., Ramey, Jr., H.J. and Samaniego-V., F. (1982a). Non-Darcy Flow in Wells with Finite-Conductivity Vertical Fractures. SPEJ, Oct. 1982, 681.
- Guppy, K.H., Cinco-Ley, H., and Ramey, Jr., H.J. (1982b). Pressure Buildup Analysis of Fractured Wells Producing at High Flow Rates. *JPT*, Nov. 1982, 2656.
- Holditch, S.A. and Morse, R.A. (1976). The Effects of Non-Darcy Flow on the Behavior of Hydraulically Fractured Gas Wells. JPT, Oct.

1976, 1169.

- Kazemi, H. (1969). Pressure Transient Analysis of Naturally Fractured Reservoirs with Uniform Fracture Distribution. SPEJ, 451-62. Trans., AIME, 246.
- Lai, B., Miskimins, J.L. and Wu, Y.S. (2009). Non-Darcy Porous Media Flow According to the Barree and Conway Model: Laboratory and Numerical Modeling Studies. SPE 122611, SPE Rocky Mountain Petroleum Technology Conference, Denver, Colorado 14-16 April.
- Lee, W.J. and Holditch, S.A. (1981). Fracture Evaluation with Pressure Transient Testing in Low-Permeability Gas Reservoirs. *JPT*, Sep. 1981, 1776.
- Lopez-H., H.D. (2007). Experimental Analysis and Macroscopic and Pore-level Flow Simulations to Compare Non-Darcy Flow Models in Porous Media. *PhD dissertation*, Colorado School of Mines, Golden, Colorado.
- McGuire, W.J. and Sikora, V.J. (1960). The Effect of Vertical Fractures on Well Productivity. Trans., AIME 218, 401.
- Montillet, A. (2004). Flow through a Finite Packed Bed of Spheres: A Note on the Limit of Applicability of the Forchheimer Type Equation. Journal of Fluids Engineering 126 (139).
- Pruess, K. (1983). GMINC A Mesh Generator for Flow Simulations in Fractured Reservoirs. Research Report No. LBL-15227, Lawrence Berkeley Laboratory, Berkeley, CA, March 1983.
- Pruess, K. (1991). TOUGH2 A General Purpose Numerical Simulator for Multiphase Fluid and Heat Flow. Research Report No. LBL-29400, Lawrence Berkeley Laboratory, Berkeley, CA, May 1991.
- Pruess, K. and Narasimhan, T.N. (1985). A practical method for modeling fluid and heat flow in fractured porous media. Soc. Pet. Eng. J., 25, pp.14-26.
- Tavares, C.A.P., Kazemi, H. and Ozkan, E. (2004). Combined Effect of Non-Darcy Flow and Formation Damage on Gas Well Performance of Dual-Porosity and Dual-Permeability Reservoirs. SPE 90623, SPE Annual Technical Conference and Exhibition, Houston, TX, 26-29 September.
- Umnuayponwiwat, S., Ozkan, E., Pearson, C.M. and Vincent, M. (2000). Effect of Non-Darcy Flow on the Interpretation of Transient Pressure Responses of Hydraulically Fractured Wells. SPE 63176, SPE Annual Technical Conference and Exhibition, Dallas, 1-4 October.
- Villalobos-L., H., Rodriguez, F. and Samaniego-V., F. (1989). Pressure Transient Behavior of Naturally Fractured Reservoirs Considering the Effect of High-Velocity Flow. SPE 19787, SPE Annual Technical Conference and Exhibition, San Antonio, TX, 8-11 October.
- Warren, J.E. and Root, P.J. (1963). The Behavior of Naturally Fractured Reservoirs. Soc. Pet. Eng. J., Trans., AIME, Sep. 1963, 228, pp. 245-255.
- Wattenbarger, R.A. and Ramey, H.J. Jr (1969). Well Test Interpretations in Vertically Fractured Gas Wells. JPT, May 1969, 625.
- Wu, Y.S. (1998). MSFLOW: Multiphase Subsurface Flow Model of Oil, Gas and Water in Porous and Fractured Media with Water Shut-off Capability. *Documentation and User's Guide*, Walnut Creek, California.
- Wu, Y.S. (2002). Numerical Simulation of Single-Phase and Multiphase Non-Darcy Flow in Porous Media and Fractured Reservoirs. Transport in Porous Media, Vol. 49, No. 2, 209-240, Nov. 2002.
- Wu, Y.S. and K. Pruess (1988). A multiple-porosity method for simulation of naturally fractured petroleum reservoirs. SPE Reservoir Engineering, 3, 327-336.