SPE 122612

Simulation of Multiphase Non-Darcy Flow in Porous and Fractured Media

Yu-Shu Wu, Bitao Lai, and Jennifer L. Miskimins, Colorado School of Mines; Yuan Di, Peking University

This paper was prepared for presentation at the 2009 SPE Annual Technical Conference and Exhibition held in New Orleans, Louisiana, USA, 4–7 October 2009.

Abstract

A Buckley and Leverett type analytical solution is derived for non-Darcy displacement of immiscible fluids in porous media, in which non-Darcy flow is described using the general model proposed by Barree and Conway. Recent laboratory studies and analyses have shown that the Barree and Conway model is able to describe the entire range of relationships between rate and potential gradient from low- to high-flow rates through porous media, including those in transitional zones. We also present a general mathematical and numerical model for incorporating the Barree and Conway model to simulate multiphase non-Darcy flow in porous and fractured media, while flow in fractured rock is handled using a general multi-continuum approach. The numerical solution of the proposed multiphase, non-Darcy flow model is based on a discretization scheme using an unstructured grid with regular or irregular meshes for multi-dimensional simulation. The final discretized nonlinear equations are handled fully implicitly with the Newton iteration. As an application example, we use the analytical solution to verify the numerical solution for and to obtain some insight into one-dimensional non-Darcy displacement of two immiscible fluids according to the Barree and Conway model. Overall, this work provides an improved platform for modeling multiphase non-Darcy flow in oil and gas reservoirs, including complex fractured systems such as shale gas reservoirs.

1. Introduction

Darcy's Law has been used exclusively in studies of porous-medium flow in reservoirs, however, there is considerable evidence that high-velocity non-Darcy flow occurs in many subsurface systems, such as in the flow near wells of oil or gas production, groundwater pumping, and liquid waste injection. Darcy's law, describing a linear relationship between volumetric flow rate (Darcy velocity) and pressure (or potential) gradient, has been the fundamental principle in analyzing flow processes in porous media. Any deviation from this linear relation may be defined as non-Darcy flow. In this paper, our concern is only with the non-Darcy flow behavior caused by high flow velocities. Effects of non-Darcy or high-velocity flow regimes in porous media have been observed and investigated for decades (e.g., Tek et al., 1962; Scheidegger, 1972; Katz and Lee, 1990; Wu, 2002). Most studies performed on non-Darcy flow in porous media in the early time have focused mostly on single-phase-flow conditions in peteroleum engineering (Tek et al., 1962; Swift and Kiel, 1962; Lee et al. 1987). Some investigations have been conducted for non-Darcy flow in fractured reservoirs (Skjetne et al., 1999) and for non-Darcy flow into highly permeable fractured wells (e.g., Guppy et al., 1981, 1982). Other studies have concentrated on finding and validating correlations of non-Darcy flow coefficients (e.g., Liu et al., 1995).

In analyzing non-Darcy flow through porous median, the *Forchheimer* equation (1901) has been excpusively used to describe non-Darcy porous meda flow, and has been extended to multiphase flow conditions (*Evans et al*., 1987; *Evans and Evans*, 1988; *Liu et al*., 1995; Wu, 2001 and 2002). Recent laboratory studies and analyses have shown that the Barree and Conway model is able to describe the entire range of relationships between flow rate and potential gradient from low- to high-flow rates through porous media, including those in transitional zones (Barree and Conway, 2004 and 2007; Lopez, 2007). In this paper, we derive a Buckley and Leverett type analytical solution for one-dimensional non-Darcy displacement of immiscible fluids in porous media using the Barree and Conwaywe model. We also present a general numerical model for incorporating the Barree and Conway model to simulate multiphase non-Darcy flow in porous and fractured media.

This paper represents a continual study of our previous investigation of single-phase non-Darcy flow in reservoirs according to the Barree abd Conway model (Lai et al. 2009). The objective of this study is to develop a mathematical method for quantitative analysis of multiphase non-Darcy flow through heterogeneous porous and fractured rocks using the Barree and Conway's model. The numerical solution of the proposed mathematical model is based on a discretization scheme using an unstructured grid with regular or irregular meshes for multi-dimensional simulation. The final discretized nonlinear equations are handled fully implicitly with the Newton iteration. The flow in fractured rock is handled using a general multi-continuum approach. As an application example, we use the analytical solution to verify the numerical solution for and to obtain some insight into one-dimensional non-Darcy displacement of two immiscible fluids according to the Barree and Conway model. Overall, this work provides an improved platform for modeling multiphase non-Darcy flow in oil and gas reservoirs,

including complex fractured systems such as shale gas reservoirs.

2. Mathematical Model

A multiphase system in a porous or fractured reservoir is assumed to be similar to the black oil model, composed of three phases: oil, gas, and water. For simplicity, the three fluid components, water, oil, and gas are assumed to be present only in their associated phases. Each phase flows in response to pressure, gravitational, and capillary forces according to the multiphase extension of the Barree and Conway model (Barree and Conway, 2007) for non-Darcy flow. In an isothermal system containing three mass components, three mass-balance equations are needed to fully describe the system, as described in an arbitrary flow region of a porous or fractured domain for flow of phase β (β = w for water, β = o oil, and β = g for gas),

$$\frac{\partial}{\partial t}(\phi S_\beta \rho_\beta) = -\nabla \bullet (\rho_\beta \mathbf{v}_\beta) + q_\beta \tag{2.1}$$

where ρ_β is the density of fluid β; $\mathbf{v}_\beta$ is the volumetric velocity vector of fluid β; S_β is the saturation of fluid β; ϕ is the effective porosity of formation; t is time; and q_β is the sink/source term of phase (component) β per unit volume of formation, representing mass exchange through injection/production wells or due to fracture and matrix interactions.

Volumetric flow rate (namely Darcy velocity with Darcy flow) for non-Darcy flow of each fluid may be described using the multiphase extension of the Barree and Conway's model, extended to a vector form for multidimensional flow (see Apendix A):

$$-\nabla\Phi_\beta = \frac{\mu_\beta \mathbf{v}_\beta}{k_d k_{r\beta}\left(k_{mr} + \frac{(1-k_{mr})\mu_\beta \tau}{\mu_\beta \tau + \rho_\beta |\mathbf{v}_\beta|}\right)} \tag{2.2}$$

where $\nabla\Phi_\beta$ is the flow potential gradient, defined as:

$$\nabla\Phi_\beta = (\nabla P_\beta - \rho_\beta g \nabla D) \tag{2.3}$$

where P_β is the pressure of the fluid; g is gravitational acceleration; and D is the depth from a datum. In Equation (2.2), k_d is constant Darcy or absolute permeability; k_{mr} is the minimum permeability ratio at high rate, relative to Daryc permeability (fraction); $k_{r\beta}$ is the relative permeability to fluid β; μ_β is viscosity of fluid β; and τ is characteristic length.

Equation (2.1), the governing of mass balance for three phases, needs to be supplemented with constitutive equations, which express all the secondary variables and parameters as functions of a set of primary thermodynamic variables of interest. The following relationships will be used to complete the description of multiphase flow through porous media:

$$S_w + S_o + S_g = 1 \tag{2.4}$$

The capillary pressures relate pressures between the phases. The aqueous- and gas-phase pressures are related by

$$P_w = P_g - P_{cgw}(S_w), \tag{2.5}$$

where P_{cgw} is the gas-water capillary pressure in a three-phase system and assumed to be a function of water saturation only. The NAPL pressure is related to the gas phase pressure by

$$P_o = P_g - P_{cgo}(S_w, S_o) \tag{2.6}$$

where P_{cgo} is the gas-oil capillary pressure in a three-phase system, which is a function of both water and oil saturations. For formations, the wettability order is (1) aqueous phase, (2) oil phase, and (3) gas phase. The gas-water capillary pressure is usually stronger than the gas-oil capillary pressure. In a three-phase system, the oil-water capillary pressure, P_{cow}, may be defined as

$$P_{cow} = P_{cgw} - P_{cgo} = P_o - P_w \tag{2.7}$$

The relative permeabilities are assumed to be functions of fluid saturations only, i.e., not affatced by non-Darcy flow behavior. The relative permeability to the water phase is taken to be described by

$$k_{rw} = k_{rw}(S_w) \tag{2.8}$$

to the oil phase by

$$k_{ro} = k_{ro}(S_w, S_g) \tag{2.9}$$

and to the gas phase by

$$k_{rg} = k_{rg}(S_g) \tag{2.10}$$

The densities of water, oil, and gas, as well as their viscosities can in general be treated as functions of pressure.

3. Numerical Formulation and Solution

Equations (2.1) and (2.2), as described by the Barree and Conway's model, for multiphase non-Darcy flow of gas, oil and water in porous media, are highly nonlinear and in general needs to be solved numerically. In this work, the methodology for using a numerical approach to simulate the non-Darcy flow consists of the following three steps: (1) spatial discretization of the mass conservation equation; (2) time discretization; and (3) iterative approaches to solve the resulting nonlinear, discrete algebraic equations. A mass-conserving discretization scheme, based on finite or integral finite-difference or finite-element methods (Pruess et al. 1999) is used and discussed here. Specifically, non-Darcy flow equations, as discussed in Section 2, have been implemented into a general-purpose, three-phase reservoir simulator, the MSFLOW code (Wu, 1998) As implemented in the code, Equation (2.1) can be discretized in space using an integral finite-difference or control-volume finite-element scheme for a porous and/or fractured medium. The time discretization is carried out with a backward, first-order, finite-difference scheme. The discrete nonlinear equations for water, oil, and gas flow at node i are written as follows:

$$\left\{\left(\phi\, S_\beta \rho_\beta\right)_i^{n+1} - \left(\phi\, S_\beta \rho_\beta\right)_i^{n}\right\}\frac{V_i}{\Delta t} = \sum_{j\in\eta_i}\left(\text{flow}_\beta\right)_{ij}^{n+1} + Q_{\beta i}^{n+1} \tag{3.1}$$

where n denotes the previous time level; n+1 is the current time level; V_i is the volume of element i (i=1, 2, 3, …, N, N being the total number of elements of the grid); Δt is the time step size; η_i contains the set of neighboring elements (j), porous or fractured block, to which element i is directly connected; and "flow_β" is a mass flow term between elements i and j for fluid β, defined by Equation (2.2) implicitly. For flow between two grid blocks, the mass flow term, "flow_β", can be evaluated directly (See Appendix A) as,

$$\text{flow}_{\beta,ij} = A_{ij}\rho_\beta v_\beta = \frac{A_{ij}}{2\mu_\beta}\left\{-[\mu_\beta^2 S_\beta \tau - \Delta\Phi_{\beta,ij} k_d k_{r\beta} k_{m} \rho_\beta] + \sqrt{[\mu_\beta^2 S_\beta \tau - \Delta\Phi_{\beta,ij} k_d k_{r\beta} k_{m} \rho_\beta]^2 + 4\mu_\beta \rho_\beta \Delta\Phi_{\beta,ij} k_d k_{r\beta} \mu_\beta S_\beta \tau}\right\} \tag{3.2}$$

where A_{ij} is the common interface area between connected elements i and j; all the parameters, such permeability, relative permeability, density, and viscosity needs a proper averaging or weighting of properties at the interface between the two elements i and j; A_{ij} is the common interface area between the connected blocks or nodes i and j; and the disctete flow potential gradient is defined in an integral findinite difference as,

$$\Delta\Phi_{\beta,ij} = \frac{\left(P_{\beta,i} - \rho_{\beta,ij+1/2}\, g\, D_i\right) - \left(P_{\beta,j} - \rho_{\beta,ij+1/2}\, g\, D_j\right)}{D_i + D_j} \tag{3.3}$$

In (3.2), the mass sink/source term at element i, $Q_{\beta i}$ for phase β, is defined as

$$Q_{\beta i} = q_{\beta i} V_i \tag{3.4}$$

In the model formulation, Darcy permeability, relative permeability, and other non-Darcy flow parameters, such as minimum permeability ratio, k_{mr}, and characteristic length, τ, are all considered as flow properties of the porous media and need to be averaged between connected elements in calculating the mass flow terms. In general, weighting approaches used are that absolute permeability is harmonically weighted along the connection between elements i and j, relative permeability is upstream weighted, and non-Darcy flow coefficients are arithmetically averaged.

Newton/Raphson iterations are used to solve Equation (3.1). For a three-phase flow system, 3 × N coupled nonlinear equations must be solved, including three equations at each element for the three mass-balance equations of water, oil, and gas, respectively. The three primary variables (x_1, x_2, x_3) selected for each element are oil pressure, oil saturation, and gas saturation, respectively. In terms of the three primary variables, the Newton/Raphson scheme gives rise to

$$\sum_m \frac{\partial R_i^{\beta,n+1}\left(x_{m,p}\right)}{\partial x_m}\left(\delta x_{m,p+1}\right) = -R_i^{\beta,n+1}\left(x_{m,p}\right) \qquad \text{for } m = 1, 2, \text{ and } 3 \tag{3.5}$$

where index m = 1, 2, and 3 indicates the primary variable 1, 2, or 3, respectively; p is the iteration level; and i = 1, 2, 3, …, N, the nodal index. The primary variables are updated after each iteration:

$$x_{m,p+1} = x_{m,p} + \delta x_{m,p+1} \tag{3.7}$$

A numerical method is used to construct the Jacobian matrix for Equation (3.5), as outlined by Forsyth et al. (1995).

Boundary Condition: Similarly to Darcy flow handling, first-type or Dirichlet boundary conditions denote constant or time-dependent phase pressure, and saturation conditions. These types of boundary conditions can be treated using the large-volume or inactive-node method (Pruess, 1991), in which a constant pressure/saturation node may be specified with a huge volume while keeping all the other geometric properties of the mesh unchanged. However, caution should be taken in (1) identifying phase conditions when specifying the "initial condition" for the large-volume boundary node and (2) distinguishing upstream/injection from downstream/production nodes. Once specified, primary variables will be fixed at the big-volume boundary nodes, and the code handles these boundary nodes exactly like any other computational nodes.

Flux-type or Neuman boundary conditions are treated as sink/source terms, depending on the pumping (production) or injection condition, which can be directly added to Equation (3.1). This treatment of flux-type boundary conditions is especially useful for a situation where flux distribution along the boundary is known, such as dealing with a single-node well. More general treatment of multilayered well-boundary conditions is discussed in Wu et al. (1996) and Wu (2000b).

4. Handling Non-Darcy Flow in Fractured Media

The technique used in the current model for handling non-Darcy flow through fractured rock follows the dual-continuum methodology (Warren and Root, 1963; Kazemi,1969; Pruess, 1991; Pruess and Narasimhan, 1985; Wu, 2002). The method treats fracture and matrix flow and interactions using a multi-continuum numerical approach, including the double- or multiporosity method (Wu and Pruess, 1988), the dual-permeability method, and the more general "multiple interacting continua" (MINC) method (Pruess and Narasimhan, 1985). As shown in Wu and Qin (2009), the generalized dual-continuum, MINC method, can handle any flow processes of fractured media with matrix size varying from as large as the model domain of interest to as small as a representative elementary volume (REV) of zero volume. In general, the fracture network can be continuous in a pattern, randomly distributed or discrete.

The non-Darcy flow formulation, Equations (2.1) and (2.2), and (3.1) and (3.2), as discussed above, is applicable to both single-continuum and multi-continua media. Using the dual-continuum concept, Equations (2.1) and (3.1) can be used to describe multiphase flow, respectively, both in fractures and inside matrix blocks when dealing with fractured reservoirs. A special attention needs to be paid to treating fracture/matrix flow terms with Equations (3.1) and (3.2) for estimation of mass exchange at fracture/matrix interfaces using a double-porosity approach. In particular, special attention should be paid to selecting characteristic length of non-Darcy flow distance between fractures and matrix crossing the interface for the double-porosity or the nested discretizations may be approximated using the results for Darcy flow (Warren and Root, 1965; Pruess, 1983; Wu, 2002). The flow between fractures and matrix may be still evaluated using Equation (3.2) and the characteristic distance for flow crossing fracture/matrix interfaces for 1-D, 2-D and 3-D dimensions of rectangular matrix blocks, characteristic distances, based on quasi-steady flow assumption(Wu, 2002).

When handling flow through a fractured rock using the numerical formulation using the generalized multicontonuum approach, the problem essentially becomes how to generate a mesh that represents both the fracture and matrix systems. Several fracture-matrix subgridding schemes exist for designing different meshes for different fracture-matrix conceptual models (Pruess, 1983). Once a proper mesh of a fracture-matrix system is generated, fracture and matrix blocks are specified to represent fracture or matrix domains, separately. Formally, they are treated in exactly the same way in the solution of the discretized model. However, physically consistent fracture and matrix properties and modeling conditions must be appropriately specified for fracture and matrix systems, respectively.

4. Buckley-Leverett Analytical Solution for Two-Phase Non-Darcy Displacement

Buckley and Leverett (1942) established the fundamental principle for Darcy flow and displacement of immiscible fluids through porous media in their classical study of fractional flow theory. Their solution involves the noncapillary displacement process of two incompressible, immiscible fluids in a one-dimensional, homogeneous system. The Buckley-Leverett fractional flow theory has been applied and generalized to study enhanced oil recovery (EOR) problems (e.g., Patton et al. 1971; Hirasaki and Pope, 1974; Pope, 1980; Larson and Hirasaki, 1978; Hirasaki, 1981). An extension to more than two immiscible phases dubbed "coherence theory" was described by Helfferich (1981). The more recent example in the development of the Buckley-Leverett theory is the extension to non-Newtonian fluid flow and displacement (Wu et al., 1991 and 1992), and non-Darcy displacement according to the *Forchheimer* model (Wu, 2001).

This paper presents a Buchley-Leverett type analytical solution describing the displacement mechanism of non-Darcy multiphase flow in porous media according to the Barree and Conway model. The analysis approach follows upon the work for multiphase non-Newtonian fluid flow and displacement in porous media (Wu et al., 1991 and 1992; Wu, 2001) and results in an analytical solution that includes effects of non-Darcy multiphase displacement. The details on deriving the two-phase displacement solution is given in Appendix B, and also discussed in Appendix B is a practical procedure for evaluating the behavior of the analytical solution, which is similar to the graphic method by Welge (1952) for solving the Buckley-Leverett problem. The analytical solution and the resulting procedure can be regarded as an extension of the Buckley-Leverett theory to analyzing the Barree and Conway non-Darcy flow problem of two-phase immiscible fluids in porous media.

The analytical solution results of Appendix B reveal that the saturation profile and displacement efficiency are controlled not only by relative permeabilities, as in the Buckley-Leverett solution, but also by the and parameters of non-Darcy flow equations as well as injection rates.

4. Application and Verification

In this section, the Buckley-Leverett analytical solution is used to give us some insight to non-Darcy flow and displacement phenomena. The physical flow model is a one-dimensional linear porous medium, which is at first saturated uniformly with a nonwetting fluid ($S_o = 0.8$) and a wetting fluid ($S_w = S_{wr} = 0.2$). A constant volumetric injection rate of the wetting fluid is imposed at the inlet ($x = 0$), starting from $t = 0$. The relative permeability curves used for all the calculations in this paper are shown in Figure 1. The properties of the rock and fluids used are listed in Table 1.

For a given displacement system with constant injection rate, the solution (B.13) shows that non-Darcy fluid displacement in

a porous medium is characterized not only by relative permeability data, as in Buckley-Leverett displacement, but also by non-Darcy flow parameters of the two fluids, as introduced in (2.2), (B.9), and (B.10). Using the results from the analytical solution, some fundamental aspects of non-Darcy fluid displacement will be established. Figure 2, determined using Equation (B.10) for the flow system, shows that pressure gradients change significantly as a function of saturation for the non-Darcy flow system. At both high and low values for the wetting phase saturation, the pressure gradients become relatively smaller because the total flow resistance decreases as the flow is close to single-phase flow condition.

The resulting fractional flow and its derivative curves are shown in Figure 3. Note that fractional flow curves change also with the non-Darcy model parameters due to the change in pressure gradient and flow rate for different non-Darcy flow parameters under the same saturation. Saturation profiles of non-Darcy displacement after a 10-hour injection period are plotted in Figure 4, showing a typical behavior of non-Darcy displacement according to the Barree and Conway model.

As an application example, the analytical solution is used to examine the validity of the numerical method, as discussed in Section 3, which is implemented in a general-purpose, three-phase reservoir simulator, the MSFLOW code (Wu, 1998) for modeling multiphase non-Darcy flow and displacement processes according to the Barree and Conway model. To reduce the effects of discretization on numerical simulation results, very fine, uniform mesh spacing (Δx = 0.01 m) is chosen. A one-dimensional 5 m linear domain is discretized into 500 one-dimensional uniform gridblocks. The flow description and the parameters for this problem are identical to those, in Table 1, for the case of evaluation of analytical solution. The comparison between the analytical and numerical solutions is shown in Figure 5. The figure indicates that the numerical results are in excellent agreement with the analytical prediction of the non-Darcy displacement for the entire wetting-phase sweeping zone. Except at the shock, advancing saturation front, the numerical solution deviates only slightly from the analytical solution, resulting from a typical "smearing front" phenomenon of numerical dispersion effects when matching the Buckley-Leverett solution using numerical results (Aziz and Settari, 1979). The comparison between the analytical and numerical solutions is shown in Figure 5.

6. Summary and Conclusions

This paper presents a general mathematical model and numerical approach for incorporating the Barree and Conway model to simulate multiphase, multidimensional non-Darcy flow in porous and fractured media. Both analytical and numerical approaches are discussed in this study. In particular, we derive a Buckley and Leverett type analytical solution for one-dimensional non-Darcy displacement of immiscible fluids in porous media with the Barree and Conway non-Darcy flow model. In numerical solution, the multiphase non-Darcy flow formulation is implemented into a general purpose reservoir simulator using an unstructured grid with regular or irregular meshes for multi-dimensional simulation, while flow in fractured rock is handled using a general multi-continuum approach.

The analytical solution for non-Darcy displacement is based on the assumptions, similar to those used for the classical Buckley-Leverett solution. The analytical solution provides some insight into the physics of displacement involving non-Darcy flow, a more complicated process than the Darcy displacement, as described by the Buckley-Leverett solution. Multiphase non-Darcy flow and displacement are controlled not only by relative permeability curves, such as in Darcy displacement, but also by non-Darcy flow relations and model parameters as well as injection or flow rates. As an example of application, the analytical solution is applied to verify the numerical formulation of a numerical simulator for modeling multiphase non-Darcy flow.

Acknowledgements

The authors wish to thank the members of the Fracturing, Acidizing, Stimulation Technology (FAST) Consortium located at the Colorado School of Mines and the Stimlab Proppant Consortium for their support. The authors would also like to thank the support from Sinopec Inc. of China through the National Basic Research Program of China (973 Program).

Nomenclature

A	=	Cross section area of flow, m^2
A_{ij}	=	Common interface area between the connected blocks or nodes i and j
D	=	Depth from a datum
d_i	=	Distance from the center of block i to the common interface of blocks i and j
$flow_\beta$	=	mass flux of fluid f (kg/s)
g	=	Gravitational acceleration constant
k_{min}	=	Minimum permeability at high rate, darcies
k_{mr}	=	Minimum permeability relative to Darcy permeability, fraction
$k_{r\beta}$	=	Relative permeability to fluid β
N	=	Total number of nodes/elements/gridblocks of the grid
P_β	=	Pressure of fluid β, Pa
P_{cgo}	=	Gas-oil capillary pressure (Pa)
P_{cgw}	=	Gas-water capillary pressure (Pa)
P_{cow}	=	Oil-water capillary pressure (Pa)
$\partial P/\partial L$	=	Potential gradient, Pa/m
Q	=	Fluid volumetric flow rate, m^3/sec
Q_i	=	Mass sink/source term at element i, for the fluid

S_β = Saturation of fluid β
Δt = Time step size
v = Volumatric velocity, m/sec
β = phase index (β = w for wetting and β = n for non-wetting phase)
ρ = Fluid density, kg/m^3
μ_β = Viscosity of fluid β, Pa.s
τ = Characteristic length, m/10000
ϕ = Effective porosity of the medium
$\nabla\Phi_\beta$ = Flow potential gradient for fluid β, Pa/m

References

Aziz, K. and A. Settari, *Petroleum Reservoir Simulation,* Applied Science, London, 1979.

Barree, R.D. and, M.W. Conway, Multuiphase Non-Darcy Flow in Proppant Packs. Paper SPE 109561, presented at the 2007 Annual Technical Conference and Exhibition, Anaheim, CA, 11-14 November, 2007.

Barree, R. D. and, M. W. Conway, Beyond Beta Factors: A Complete Model for Darcy, Forchheimer and Trans-Forchheimer Flow in Porous Media. Paper SPE 89325 presented at the 2004 Annual Technical Conference and Exhibition, Houston, Texas 26-29 September, 2004.

Buckley, S. E. and M. C. Leverett, Mechanism of fluid displacement in sands, *Trans.*, AIME, Vol. 146, 107-116, 1942.

Cardwell, W.T. Jr., The Meaning of the triple value in noncapillary Buckley-Leverett theory, *Trans.*, AIME, Vol. 216, 271-276, 1959.

Evans, E. V. and R. D. Evans, Influence of an immobile or mobile saturation on non-Darcy compressible flow of real gases in propped fractures, *J. Petroleum Technology,* Vol. 40, No. 10, pp.1343-1351, 1988.

Evans, R. D., C. S. Hudson and J. E. Greenlee, The effect of an immobile liquid saturation on the non-Darcy flow coefficient in porous media, *J. SPE Production Engineering, Trans*. AIME, Vol. 283, pp.331-338, 1987.

Forchheimer, P., *Wasserbewegung durch Bode*, ZVDI (1901) 45, 1901.

Forsyth, P. A., Y. S. Wu and K. Pruess, Robust Numerical Methods for Saturated-unsaturated Flow with Dry Initial Conditions in Heterogeneous Media. *Advance in Water Resources* **18**, p. 25-38, 1995.

Guppy, K. H., H. Cinco-Ley, H. J. Ramey, Jr. and F. Samaniego, Non-Darcy flow in wells with finite-conductivity vertical fractures, *Soc. Pet. Eng. J.*, pp.681-698, 1982.

Guppy, K. H., H. Cinco-Ley, and H. J. Ramey, Jr., Effects of non-Darcy flow on the constant-pressure production of fractured wells, *Soc. Pet. Eng. J.*, pp.390-400, 1981.

Helfferich, F. G., Theory of multicomponent, multiphase displacement in porous media, *Soc. Pet. Eng. J.*, 51-62, 1981.

Hirasaki, G. J., Application of the theory of multicomponent, multiphase displacement to three-component, two-phase surfactant flooding, *Soc. Pet. Eng. J.*, 191-204, 1981.

Hirasaki, G. J. and G. A. Pope, Analysis of factors influencing mobility and adsorption in the flow of polymer solution through porous media, *Soc. Pet. Eng. J.*, 337-346, 1974.

Katz, D. L. and R. L. Lee, *Natural Gas Engineering, Production and Storage*, Chemical Engineering Series, McGraw-Hill Book Co. Inc., New York, 1990.

Kazemi, H., Pressure Transient Analysis of Naturally Fractured Reservoirs with Uniform Fracture Distribution. *SPEJ*, 451-62. *Trans., AIME*, 246, 1969.

Lai, B, J. L. Miskimins, and Y. S. Wu, Non-Darcy Porous Media Flow According to the Barree and Conway Model: Laboratory and Numerical Modeling Studies, SPE-122611, Prsented at the 2009 Rocky Mountain Petroleum Technology Conference, 14-16 April, 2009, Denver, CO, 2009.

Larson, R.G. and G. J. Hirasaki, Analysis of the physical mechanisms in surfactant flooding, *Soc. Pet. Eng. J.*, 42-58, 1978.

Lee, R. L., R. W. Logan and M. R. Tek: Effetcs of turbulence on transient flow of real gas through porous media, *SPE Formation Evaluation*, pp.108-120, 1987.

Liu, X., F. Civan and R. D. Evans, Correlations of the non-Darcy flow coefficient, *J. Canadian Petroleum Technology,* Vol. 34, No. 10, pp.50-54, 1995.

Lopez. H. D., Experimental Analysis and Macroscopic and Pore-level Flow Simulations to Compare Non-Darcy Flow Models in Porous Media. PhD dissertation, Colorado School of Mines, Golden, Colorado, 2007.

Patton, J. T., K. H. Coats and G.T Colegrove, Prediction of polymer flood performance, *Soc. Pet. Eng. J., Trans.*, AIME, Vol. 251, 72-84, 1971.

Pope, G. A., The application of fractional flow theory to enhanced oil recovery, *Soc. Pet. Eng. J.*, 191-205, 1980.

Pruess, K., C. Oldenburg, and G. Moridis, *TOUGH2 User's Guide, Version 2.0,* Report LBNL-43134, Berkeley, California: Lawrence Berkeley National Laboratory, 1999.

Pruess, K., TOUGH2 - A General-Purpose Numerical simulator for multiphase fluid and heat flow, Report LBL-29400, Lawrence Berkeley National Laboratory, Berkeley, California, 1991.

Pruess, K. and Narasimhan, T. N., A practical method for modeling fluid and heat flow in fractured porous media, *Soc. Pet. Eng. J., 25*, pp.14-26, 1985.

Pruess, K., GMINC - A mesh generator for flow simulations in fractured reservoirs, Report LBL-15227, Berkeley, California: Lawrence Berkeley National Laboratory, 1983.

Scheidegger, A. E. *The Physics of Flow through Porous Media*, University of Toronto Press, 1972.

Skjetne, E., T. K. Statoil and J. S. Gudmundsson, Experiments and modeling of high-velocity pressure loss in sandstone fractures, SPE 56414, Presented at the 1999 SPE Annual Technical Conference and Exhibition, Houston, Texas, 3-6, October, 1999.

Swift, G. W. and O. G. Kiel, The prediction of gas-well performance including the effects of non-Darcy flow, *J. Petroleum Technology, Trans*. AIME, Vol. 222, 791-798, 1962.

Tek, M. R., K. H. Coats and D. L. Katz, The effects of turbulence on flow of natural gas through porous reservoirs, *J. Petroleum Technology, Tran.s* AIME, Vol. 222, pp.799-806, 1962.

Warren, J.E., and P. J. Root, The behavior of naturally fractured reservoirs, *Soc. Pet. Eng. J., Trans., AIME*, pp.245-255, 228, 1963.

Welge, H. J., A simplified method for computing oil recovery by gas or water drive, *Trans.* AIME, Vol. 195, 91-98, 1952.

Willhite, G.P., *Waterflooding*, SPE textbook Series, Society of Petroleum Engineers, Richardson, TX, 1986.

Wu, Y. S. and G. Qin, A General Numerical Approach for Modeling Multiphase Flow and Transport in Fractured Porous Media, *Communications in Computational Physics*, (accepted), 2009.

Wu, Y. S., Numerical Simulation of Single-Phase and Multiphase Non-Darcy Flow in Porous and Fractured Reservoirs. *Transport in Porous Media* **49** (2): 209-240, 2002.

Wu, Y. S., Non-Darcy Displacement of Immiscibel Fluids in Porous Media, *Water Resources Research*, Vol. 37, No. 12, pp.2943-2950, 2001

Wu, Y. S. A virtual node method for handling wellbore boundary conditions in modeling multiphase flow in porous and fractured media, LBNL-42882, *Water Resources Research*, Vol. 36, N0. 3, pp.807-814, 2000.

Wu, Y. S., P. A. Forsyth and H. Jiang, A consistent approach for applying numerical boundary conditions for subsurface flow, *Journal of Contaminant Hydrology* **23**: 157-185. 1996.

Wu, Y. S., MSFLOW: *Multiphase Subsurface Flow Model of Oil, Gas and Water in Porous and Fractured Media with Water Shut-off Capability, DOCUMENTATION and User's Guide*, Walnut Creek, California, 1998.

Wu, Y. S. and K. Pruess, and P. A.Witherspoon, Flow and displacement of Bingham non-Newtonian fluids in porous media, *SPE Reservoir Engineering*, 369-376, 1992.

Wu, Y. S., K. Pruess, and P. A. Witherspoon, Displacement of a Newtonian fluid by a non-Newtonian fluid in a porous medium, *Transport in Porous Media*, 6, 115-142, 1991.

Wu, Y. S. and K. Pruess, A multiple-porosity method for simulation of naturally fractured petroleum reservoirs, *SPE Reservoir Engineering*, 3, pp. 327-336, 1988.

Table 1 Parameters for the non-Darcy Displacement Example.

Parameter	Value	Unit
Effective Porosity	$\phi = 0.30$	
Darcy permeability	$k_d = 10$	Darcy
Minimum permeability	$k_{min} = 0.1, 1.0$	Darcy
Characteristic length	τ=10,000	1/m
Wetting Phase Density	$\rho_w = 1{,}000$	kg/m^3
Wetting Phase Viscosity	$\mu_w = 1.0 \times 10^{-3}$	Pa•s
Nonwetting Phase Density	$\rho_n = 800$	kg/m^3
Nonwetting Phase Viscosity	$\mu_n = 5.0 \times 10^{-3}$	Pa•s
non-Darcy Flow Constant	$C_\beta = 3.2 \times 10^{--6}$	$m^{3/2}$
Injection Rate	$q = 1.0 \times 10^{-5}$	m^3/s

Appendix A. Relationship of One-Dimensional flow rate versus pressure gradient

Th Barree and Conway equation (Barree and Conway, 2007) presents a one-dimensional model for pressure gradient versus multiphase flow rate. In the two-phase non-Darcy flow model, the pressure drop of each phase (e.g., gas phase) is written as:

$$\left(\frac{\partial P}{\partial L}\right)_g = \left(\frac{\mu v}{k_{g\_eff}}\right)_g \qquad \text{(A.1)}$$

where the effective permeability of gas, k_{g_eff} , can be written as:

$$k_{g\_eff} = k_d k_{rg} k_{rm} + \frac{(1-k_{rm})k_d k_{rg}}{\left(1+\frac{\rho_g v_g}{\mu_g S_g \tau}\right)} \tag{A.2}$$

Substituting (A.2) into (A.1), we have the one dimensional form of the Barree and Conway non-Dracy floiw equation, pressure gradient as a function of flow velocity as,

$$\left(\frac{\partial P}{\partial L}\right)_g = \frac{\mu_g v_g}{k_d k_{rg} k_{rm} + \frac{(1-k_{rm})k_d k_{rg}}{\left(1+\frac{\rho_g v_g}{\mu_g S_g \tau}\right)}} \tag{A.3}$$

If we replace pressure gradient by potential gradient and extend one-dimensional velocity in (A.3) to a multidimensional vector, we have a general form of Equation (2.2) for correlating potential gradient and flow rate with the Barree and Conway model.

For incorporation of the Barree and Conway model into the continuity or mass conservation equation (2.1), it is more convenient to use a relationship of expressing flow rate in terms of pressure or potential gradient. Under one-dimensional flow condition along x-directon, solving the flow velocity from (2.2) in terms of potentional gradient leads to,

$$v_\beta = \frac{-[\mu_\beta^2 S_\beta \tau - \left(-\frac{\partial \Phi_\beta}{\partial x}\right) k_d k_{r\beta} k_{rm} \rho_\beta] + \sqrt{[\mu_\beta^2 S_\beta \tau - \left(-\frac{\partial \Phi_\beta}{\partial x}\right) k_d k_{r\beta} k_{rm} \rho_\beta]^2 + 4\mu_\beta \rho_\beta \left(-\frac{\partial \Phi_\beta}{\partial x}\right) k_d k_{r\beta} \mu_\beta S_\beta \tau}}{2\mu_\beta \rho_\beta} \tag{A.4}$$

Note that Equation (A.4) is used in this paper to replace Darcy's law for correlating flow rate and potential gradient according to the Barree and Conway non-Darcy flow model.

Appendix B. Derivation of Buckley-Levereet Analytical Solution

For the derivation of the analytical solution, we assume the following Buckley-Leverett flow conditions for one-dimensional non-Darcy flow of two immiscible fluids:

- Both fluids and the porous medium are incompressible.
- Capillary pressure gradient is negligible.
- Gravity segregation effect is negligible (i.e., stable displacement exists near the displacement front).
- One-dimensional flow and displacement is along the x-coordinate of a semi-infinite linear flow system with a constant cross-sectional area (A).

Among these assumptions, incompressibility of fluids and formation is critical to deriving the Buckley-Leverett solution. This assumption provides a good approximation to displacement processes of two liquids (e.g., oil and water) in porous media, because of the small compressibilities of the two fluids. For gas and liquid displacement, however, this assumption may pose certain limitation to the resulting solution, when large pressure gradients buildup in a flow system

Under the Buckley-Leverett flow condition, Equation (2.1) can then be changed for two-phase displacement of the wetting (β=w) and nonwetting phase (β=n) as follows:

$$-\frac{\partial v_\beta}{\partial x} = \phi \frac{\partial S_\beta}{\partial t} \tag{B.1}$$

For the one-dimensional flow, v_β can be determined from Equation (2.2) or (A.4) as,

$$v_\beta = \frac{1}{2\mu_\beta \rho_\beta}\left(-[\mu_\beta^2 S_\beta \tau - \left(\frac{\partial P}{\partial x} + \rho_\beta g \sin(\alpha)\right) k_d k_{r\beta} k_{rm} \rho_\beta] + \right)$$
$$+ \frac{1}{2\mu_\beta \rho_\beta}\left(\sqrt{[\mu_\beta^2 S_\beta \tau - \left(\frac{\partial P}{\partial x} + \rho_\beta g \sin(\alpha)\right) k_d k_{r\beta} k_{rm} \rho_\beta]^2 + 4\mu_\beta \rho_\beta \left(\frac{\partial P}{\partial x} + \rho_\beta g \sin(\alpha)\right) k_d k_{r\beta} \mu_\beta S_\beta \tau}\right) \tag{B.2}$$

where $\left(\frac{\partial P}{\partial x}\right)$ is a component of the pressure gradient along the x-coordinate - the same for the wetting or nonwetting phase ; g

is the gravitational acceleration constant, and α is the angle between the horizontal plane and the flow direction (the x-coordinate).

To complete the mathematical description of the physical problem, the initial and boundary conditions must be specified. For simplicity in derivation, the system is initially assumed to be uniformly saturated with both wetting and nonwetting fluids. The wetting phase is at its residual saturation, and a nonwetting fluid, such as oil or gas, is at its maximum saturation in the system, as follows:

$$S_n(x, t=0) = 1 - S_{wr} \tag{B.3}$$

where S_{wr} is the initial, residual wetting-phase saturation. Wetting fluid, such as water, is continuously being injected at a known rate q. Therefore, the boundary conditions at the inlet (x = 0) are:

$$v_w(x=0, t) = \frac{q}{A} \tag{B.4}$$

where v_w is flow rate or flux of water across a unit area of the one-dimentional system; and A is cross-sectional area of the one-dimensional flow system and

$$v_n(x=0, t) = 0 \tag{B.5}$$

The derivation of the analytical solution follows the work by Wu et al. (1991), in which the fractional flow concept is used to simplify the governing Equations (3.1) in terms of saturation only. The fractional flow of a fluid phase is defined as a volume fraction of the phase flowing at a location x and time t to the total volume of the flowing phases (Willhite, 1986). The fractional flow can be written as

$$f_\beta = \frac{v_\beta}{v_w + v_n} = \frac{v_\beta}{v} \tag{B.6}$$

where the total flow flux is

$$v = v_w + v_n \tag{B.7}$$

From volume balance due to incompressibility of the system, we have

$$f_w + f_n = 1 \tag{B.8}$$

The fractional flow function for the wetting phase may be written in the following form:

$$f_w = \frac{1}{1 + \frac{v_n}{v_w}} \tag{B.9}$$

when the flux v_w and v_n for wetting and non-wetting phases are defined in Equation (B.2).

Equation (B.9) as well as (B.2) indicates that the fractional flow f_w of the wetting phase is a function of both saturation and pressure gradient. However, for a given injection rate at a time, and for given fluid and rock properties of a porous material, the pressure gradient at a given time can be shown by the following to be a function of saturation only under the Buckley-Leverett flow condition:

$$\begin{aligned} & q - \frac{A}{2\mu_w\rho_w}\left(-[\mu_w^2 S_w \tau - \left(\frac{\partial P}{\partial x} + \rho_w g \sin(\alpha)\right) k_d k_{rw} k_{rm} \rho_w] + \right) \\ & \quad - \frac{1}{2\mu_w\rho_w}\left(\sqrt{[\mu_w^2 S_w \tau - \left(\frac{\partial P}{\partial x} + \rho_w g \sin(\alpha)\right) k_d k_{rw} k_{rm} \rho_w]^2 + 4\mu_w\rho_w\left(\frac{\partial P}{\partial x} + \rho_w g \sin(\alpha)\right) k_d k_{rw} \mu_w S_w \tau}\right) \\ & \quad - \frac{A}{2\mu_n\rho_n}\left(-[\mu_n^2 S_n \tau - \left(\frac{\partial P}{\partial x} + \rho_n g \sin(\alpha)\right) k_d k_m k_{rm} \rho_n] + \right) \\ & - \frac{1}{2\mu_n\rho_n}\left(\sqrt{[\mu_n^2 S_n \tau - \left(\frac{\partial P}{\partial x} + \rho_n g \sin(\alpha)\right) k_d k_m k_{rm} \rho_n]^2 + 4\mu_n\rho_n\left(\frac{\partial P}{\partial x} + \rho_n g \sin(\alpha)\right) k_d k_m \mu_n S_n \tau}\right) = 0 \end{aligned} \tag{B.10}$$

Equation (B.10) shows that the pressure gradient and the saturation are inter-dependent on each other for this particular displacement system of Buckley-Leverett flow. Therefore, Equation (3.10) implicitly defines the pressure gradient in the system as a function of saturation.

The governing equation, Equation (B.1), subject to the boundary and initial conditions described in Equations (B.3) - (B.5) can be solved for the frontal advance equation (e,g., Wu et al., 1991; Wu 2001):

$$\left(\frac{dx}{dt}\right)_{S_w} = \frac{q(t)}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_t \tag{B.11}$$

Note that (B.11) has the same form as the Buckley-Leverett equation. However, the dependence of the fractional flow f_w for the non-Darcy displacement on saturation is different. The fractional flow, f_w, is related to saturation not only through the relative permeability functions, as in the case of Buckley and Leverett solution, but also through the pressure gradient, as described by Equation (B.10).

Equation (B.11) shows that, for a given time and a given injection rate, a particular wetting fluid saturation profile propagates through the porous medium at a constant velocity. As in the Buckley-Leverett theory, the saturation for a vanishing capillary pressure gradient will in general become a triple-valued function of distance near the displacement front (Cardwell, 1959). Equation (B.11) will then fail to describe the velocity of the shock saturation front, since $(\partial f_w/\partial S_w)$ does not exist on the front because of the discontinuity in S_w at that point. The location x_{S_w} of any saturation S_w traveling from the inlet at time t can be determined by integrating Equation (B.11) with respect to time, yielding

$$x_{S_w} = \frac{q \times t}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_{S_w} \tag{B.13}$$

Direct use of Equation (B.13), given x and t, will result in a multiple-valued saturation distribution, which can be handled by a mass balance calculation, as in the Buckley-Leverett solution. An alternative graphic method of Welge (1952) can be shown (Wu et al., 1991) to apply to calculating the above solution in this case. The only additional step in applying this method is to take into account the contribution of the pressure-gradient dependence to the non-Darcy displacement, using a fractional flow curve. Therefore, the wetting-phase saturation at the displacement saturation front may be determined by

$$\left(\frac{\partial f_w}{\partial S_w}\right)_{S_F} = \frac{(f_w)_{S_F} - (f_w)_{S_{wr}}}{S_F - S_{wr}} \tag{B.14}$$

The average saturation in the displaced zone is given by

$$\left(\frac{\partial f_w}{\partial S_w}\right)_{S_F} = \frac{1}{\bar{S}_w - S_{wr}} \tag{B.15}$$

where $\bar{S}_w$ is the average saturation of the wetting phase in the swept zone behind the sharp displacement front. Then, the complete saturation profile can be determined using Equation (B.13) for a given non-Darcy displacement problem with constant injection rate according to the Barree and Conway model.

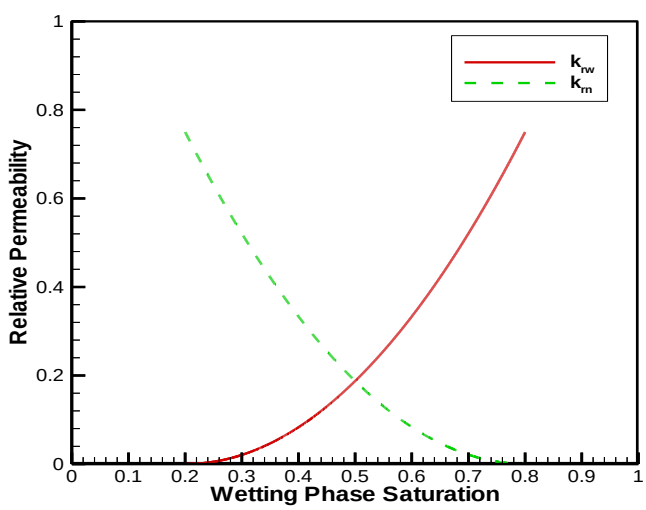

Figure 1. Relative-permeability curves used in analytical and numerical solutions for Barree and Conway non-Darcy displacement.

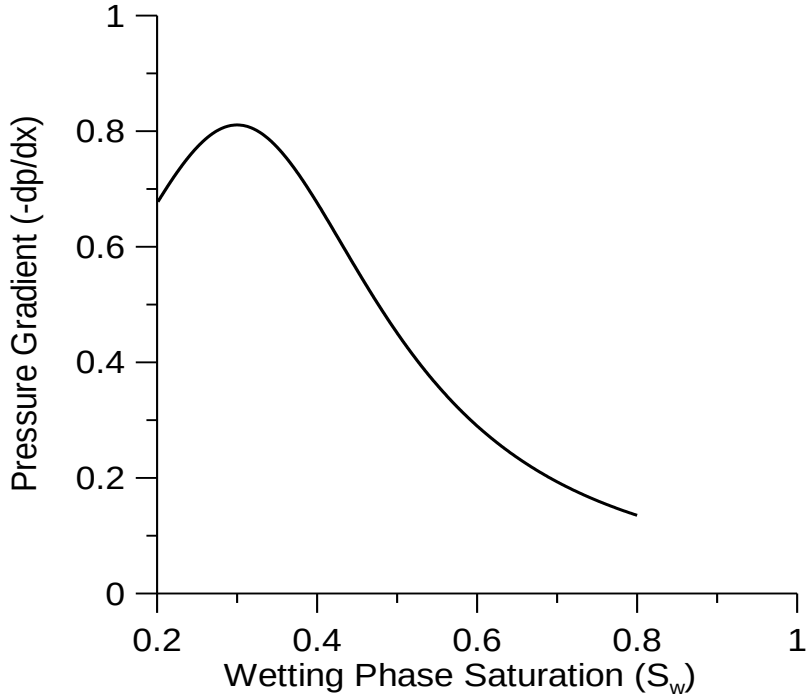

Figure 2. Pressure gradients versus displacing wetting phase saturation in the non-Darcy displacement system

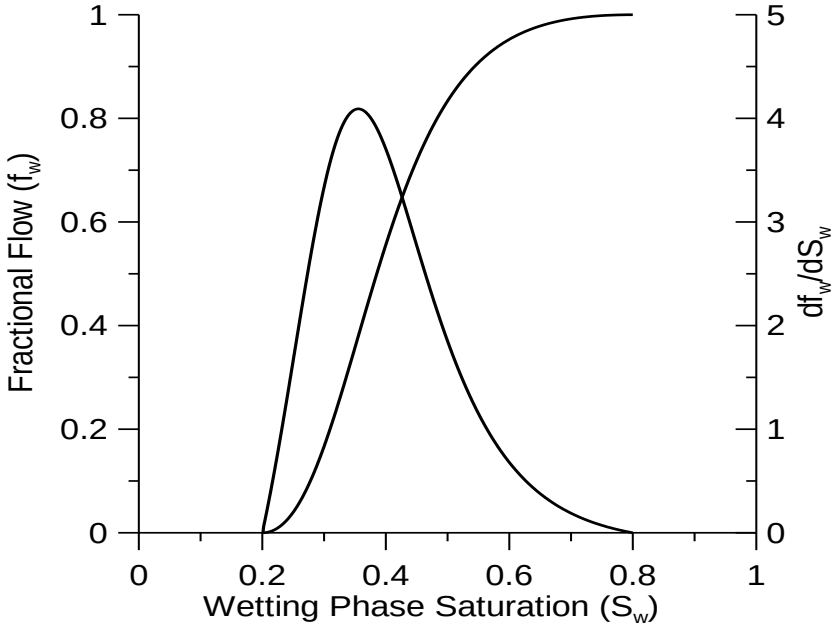

Figure 3. Fractional flow and its derivative curves with respect to wetting phase saturation in the non-Darcy displacement system

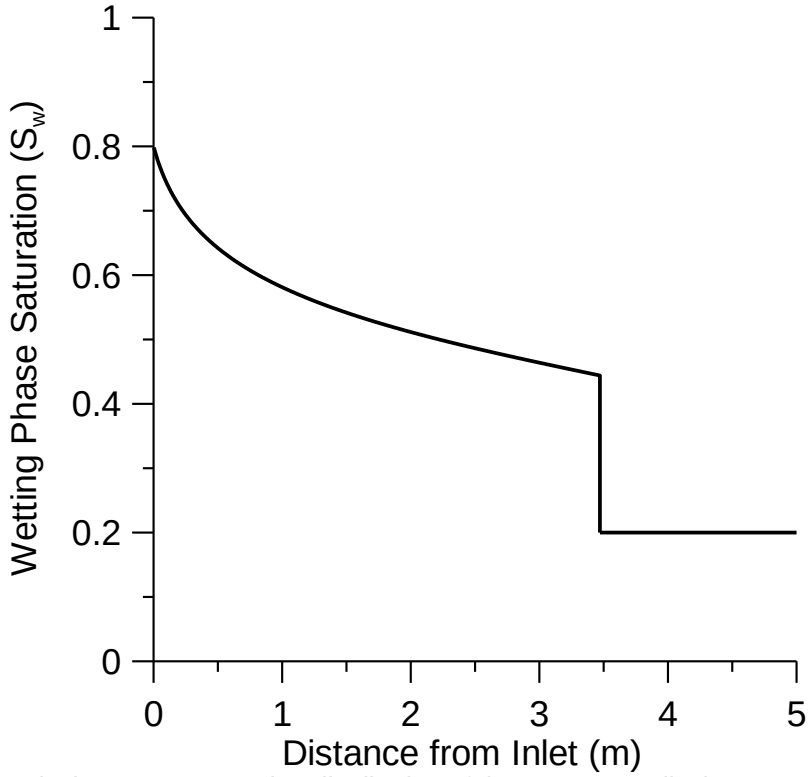

Figure 4. Displacement saturation distribution of the non-Darcy displacement system after 10 hours of injection

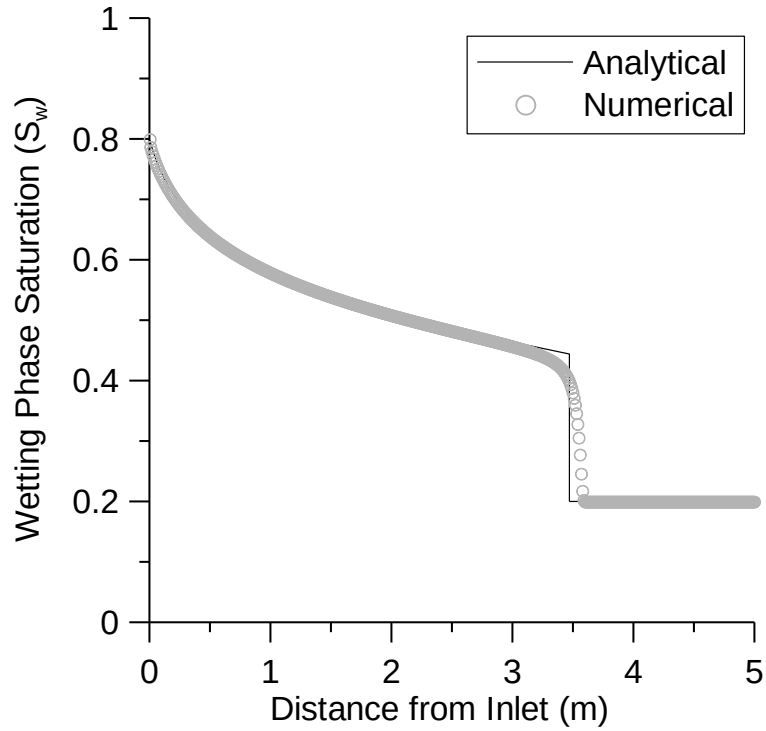

Figure 5. Comparison between displacement saturation profiles calculated from analytical and numerical solutions after 10 hours of injection