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Accurate and Efficient Simulation of Fracture–Matrix Interaction in Shale Gas Reservoirs

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Abstract Gas production from low-permeability shale formations relies on natural or man-made fractures for gas flow pathways to production wells. Shale gas reservoir simulation includes fracture–matrix flow and fracture–matrix interactions as they are key. Attention has focused on modeling fractures in shale gas production, yet there have been few studies carried out to address how to accurately simulate fracture–matrix interaction in unconventional low-permeability gas reservoirs. The classic double porosity model and the MINC method represent studies designed to accurately simulate fracture–matrix interaction; however, methods continue to encounter issues causing them to fall short of the accuracy sought. Applicability of the classic double porosity model with a constant shape factor to low-permeability reservoir simulation is questionable and has not been validated as the required pseudo-steady-state flow condition timing may not sufficiently satisfy shale gas reservoirs. The MINC method treats inter-porosity flow in a fully transient mode by further subdividing individual blocks with a number of 1-D nested meshes. The MINC concept, however, assumes that fracture–matrix flow is controlled only by the distance to the nearest fracture surrounding the matrix block and is shown to be no longer applicable after the early rapid transient period of fracture–matrix flow. A comparative investigation of commonly used fractured reservoir simulation approaches for applicability to fracture–matrix interaction in unconventional reservoirs is presented in this paper. A new nested subdividing concept, referred to as the Schwarz–Christoffel conformal mapping approach, will be introduced. The new method is able to accurately and efficiently simulate the matrix–fracture interaction for the entire tran-

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sient flow by combining the MINC and Schwarz–Christoffel conformal mapping concepts of gridding inside the matrix. The theoretical development, benchmarking, and application of the new modeling approach explanations follow.

Keywords Double porosity model · MINC · Schwarz–Christoffel conformal mapping · Analytical solution · Shale gas reservoir simulation

Lists of Symbols

A	Interface area of fracture–matrix (m^2)
c_t	Total gas compressibility coupled with gas sorption term (Pa^{-1})
D	Distance between flow potentials (m)
L	Dimension of the block (m)
m_g	Gas sorption term (kg/m^3)
k, k_m, k_g	Permeability, permeability of matrix, and gas permeability (m^2)
P_L	Langmuir isotherm pressure (Pa)
P_m, P_f	Gas pressure at matrix and fracture, respectively (Pa)
$P_{m,b}$	Gas pressure at matrix coupled with the Klinkenberg effect (Pa)
$\bar{P}_{m,b}$	Averaged gas pressure at matrix coupled with the Klinkenberg effect (Pa)
P_i	Gas pressure at initial condition (Pa)
q	Sink/source term ($kg/(m^3\ s)$)
r	Distance from the block center at radial direction (m)
t	Time (s)
T	Transmissibility term between two blocks
v	Velocity of fluid (m/s)
V_L	Langmuir volume (m^3/kg)
V_m	Volume of block (m^3)
$\emptyset$, $\emptyset_m$	Porosity, matrix porosity
ρ	Density of the fluid (kg/m^3)
ρ_g	Gas density at standard condition (kg/m^3)
ρ_R	Rock bulk density at standard condition (kg/m^3)
β	Gas density-related coefficient ($kg/(m^3\ Pa)$)
μ	Viscosity of gas (Pa s)
σ	Shape factor ($1/m^2$)

Subscripts

f	Fracture
g	Gas
i	Initial condition
i, j	Blocks i and j
m	Matrix
x, y, z	x, y, z-coordinate direction

1 Introduction

The matrix provides most of the pore volume for storage in fractured reservoirs and contributes little to the global flow, while the fracture supplies the flow or transmissivity with negligible contributions to reservoir porosity. Barenblatt et al. (1960) and by Warren and Root (1963) introduced the dual-porosity model which treats a naturally fractured reservoir with irregular matrix, vugs and fractures as an equivalent, homogeneous model. Compared to conventional reservoirs, gas production in ultra-low-permeability unconventional reservoirs is driven by highly non-linear flow equations (Wu et al. 2014), such as the gas Klinkenberg effect (e.g., Javadpour 2009), high gas compressibility, and adsorption and desorption on organic surfaces (e.g., Sigal et al. 2013; Javadpour et al. 2007). Gas production driven by non-linear flow equations involves several processes due to the presence of multi-scale fracture network, and the heterogeneous nature of a porous/fractured and stress-sensitive rock.

Several conceptual models have been developed to model fracture–matrix interaction for flow in fractured porous media. Models share the same scheme, but possess different assumptions or concepts dealing with flow inside the matrix. Explicitly dividing the matrix and fractures utilizing the discrete fracture model is an accurate option, but is limited in field scale simulation. The discrete fracture model generally requires an exceptional number of grid blocks to represent matrix and fracture systems, increasing the computational cost (Gong and Qin 2012). Lack of detailed knowledge for fracture and matrix geometric properties and their spatial distributions at given subsurface sites necessitates multiple realizations, rendering explicit fracture simulations nearly impossible for field application.

The classic double porosity model may not be applicable in a shale gas reservoir simulation as transient flow may be long lasting. Efforts to improve transient flow have been established as Pruess and Narasimhan (1982, 1985) defined a Multiple Interacting Continuum (MINC) method to improve the transient flow simulation; however, when the flow stabilizes, an error occurs. Pruess and Wu (1989) introduced a semi-analytical solution utilizing a trial function developed by Vinsome and Westerveld (1980) to estimate the pressure distribution in a matrix block when simulating slightly compressible fluid flow. Zimmerman et al. (1993) introduced another type of semi-analytical treatment of fracture/matrix flow using a non-linear ordinary differential equation to calculate interaction term. Sarma and Aziz (2006) derived a time-dependent shape factor for simulation of naturally fractured reservoirs. Lu et al. (2008) introduced transfer functions for multiphase flow in fractured reservoirs that captures fluid expansion, diffusion, and displacement. Improvements to the classical double model have been accomplished as a result of these efforts. Challenges in shale gas reservoir simulation persist, however, in determining integral solutions dealing with the gas compressibility term, the gas Klinkenberg effect, and sorption term. Non-linear physics of the shale gas reservoir design do not allow for the derivation of an accurate form of analytical solutions. Karimi-Fard et al. (2006) proposed a subgridding technique which is constructed numerically by using iso-pressure surfaces of fine grid pressure solution. Ding et al. (2014) verified the iso-pressure method combined with the MINC method being accurate in shale gas reservoir simulation, but advance iso-pressure surface generation was inefficient.

The flow behavior in a single square/cubical node is analyzed to determine methods for accurately handling flow from the matrix to the surrounding fracture and to understand critical issues affecting gas recovery from the matrix. Four model types are discussed, the classic double porosity model, the MINC model, the Schwarz–Christoffel conformal mapping approach, and the method combining the MINC and the Schwarz–Christoffel conformal mapping approach. The combined MINC and the Schwarz–Christoffel mapping approach

method demonstrates a significant improvement to the classic double porosity model and the MINC method.

2 Mathematical Model

Flow behavior in shale gas reservoirs is characterized by single gas phase or multiphase (gas, gas condensate, and/or brine). Wu et al. (2014) presented the mass balance equation of shale gas flow in an arbitrary flow region of a porous or fractured unconventional low-permeability reservoir. Models are tested only in single gas phase flow condition for purposes of this paper. The mass balance equation can be simplified as follows:

$$\frac{\partial}{\partial t}\left(\emptyset\rho + m_{\mathrm{g}}\right) = -\nabla \cdot (\rho v) + q, \tag{2.1}$$

where m_{g} is the gas sorption term utilizing the Langmuir isotherm equation to account for the gas adsorbed/desorbed on the solid surface (Langmuir 1916); the Klinkenberg effect (Klinkenberg 1941) is considered as a contribution to the permeability by modifying absolute permeability for the gas phase as a function of gas pressure in the velocity term v (Wu et al. 1998).

2.1 Classic Double Porosity Model

The classic double porosity model with a shape factor is widely employed in conventional naturally fractured reservoir simulation, but is valid only when flow turns stable or reaches pseudo-steady state. Warren and Root (1963), Kazemi et al. (1976), and many others (e.g., Coats 1989; Thomas et al. 1983) provided shape factors for the model. Lim and Aziz (1995) derived a shape factor from an analytical solution of liquid flow in a porous medium, and Mathias and Zimmerman (2003) verified the factor in the Laplace domain for liquid flow. An analytical solution for gas flow in a square (cubic) system is derived with two major assumptions: flow is approximated in a square/cubical system to radial/spherical flow, and the sorption term is linearized on the right-hand-side (RHS) of the governing equation. The same shape factors can be derived for shale gas flow when the same treatment derived by Lim and Aziz (1995) is applied for the analytical solution. Shape factors verified in this paper are summarized in Table 1.

2.2 MINC Model

Pruess and Narasimhan (1982, 1985) presented the MINC theory to improve the simulation of transient flow. The theory assumes that the fracture–matrix flow is controlled by the distance to nearest fractures only. Under this assumption, the matrix is divided into a set of nested sub-blocks and, for a square/cubical system, the sub-blocks are square/cubic. The MINC model is accurate if the flow potential at each sub-block can be represented by the pressure at

Table 1 Summary of the shape factors for two/three-dimensional system derived from various authors

Dimension	Warren and Root (1963)	Kazemi et al. (1976)	Lim and Aziz (1995)
2	$32/L^2$	$8/L^2$	$18.17/L^2$
3	$60/L^2$	$12/L^2$	$25.67/L^2$

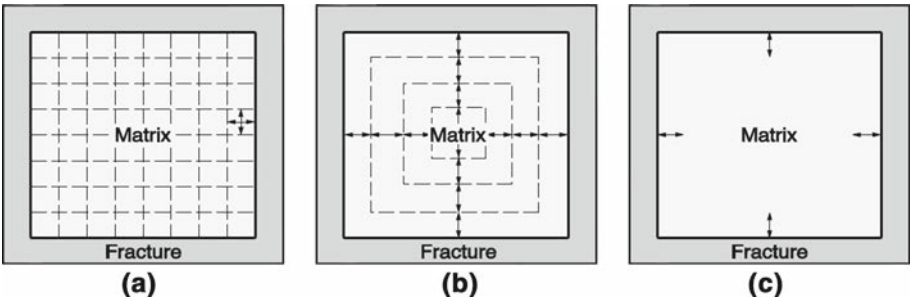

Fig. 1 Schematic of different inter-porosity flow concepts **a** explicit discretization **b** nested discretization **c** double porosity discretization (Wu and Pruess 1988)

the center of each block. The schematics of the MINC and the double porosity discretization are illustrated in Fig. 1.

2.3 Schwarz–Christoffel Conformal Mapping Approach

Fluid flow in matrices is driven by potential gradient and moves in the downward direction of the potential field. As in a disk-shaped matrix, flow streamlines are rays from the origin, and the orthogonal equipotential lines are concentric and homothetic to the disk boundary. The orthogonal field consists of fluid flow streamlines and equipotential curves. Schwarz–Christoffel conformal mapping can transfer the orthogonal field in the disk shape into any other polygons. The Schwarz–Christoffel transformation is widely utilized in the potential theory and its applications, including minimal surfaces and fluid dynamics, most notably those utilizing the 2D Laplace equation. The Schwarz–Christoffel mapping is a conformal transformation of the upper half-plane into a polygon (e.g., Driscoll 1996, 2005; Delillo et al. 2006). The Riemann mapping theorem, with consideration of a polygon in a complex plane, implies there exists a one-to-one conformal mapping that is capable of mapping the upper half-plane to the polygon.

2.3.1 Schwarz–Christoffel Mapping Mesh Generation

The transformation is non-linearly constrained along the side of the target polygon. Axes of symmetry are initially mapped to determine the vertexes, as symmetry is advantageous for reducing computational work. Consequently, the outer boundary can be determined, and integration can occur along different rays from the disk to the square. Mapped curves now represent flow streamlines in the square system, and concentric circles are able to be later mapped to the square. Perpendicularity in the disk system is kept on the square system, as displayed in Fig. 2.

Sub-blocks near the boundary are controlled more by the fracture, and the shape resembles a square (but is not square). The location away from the fracture creates sub-block more like a circle than a square. Schwarz–Christoffel mapping is an applicable theory on the 2-D plane and can be extended to 3-D according to the symmetry, as illustrated in Fig. 3.

2.3.2 Transmissibility Calculation between Neighboring Sub-Blocks

Generation of the sub-block by using the Schwarz–Christoffel mapping essentially follows the MINC theory (Pruess 1983). Block number and the volume fractions of each sub-block are

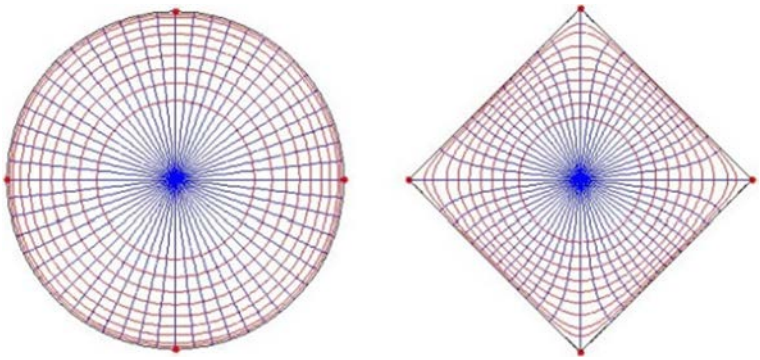

Fig. 2 Schematic of the Schwarz–Christoffel mapping from disk shape onto the square system: rays from the origin represent the flow streamlines, and concentric curves represent the iso-pressure curves

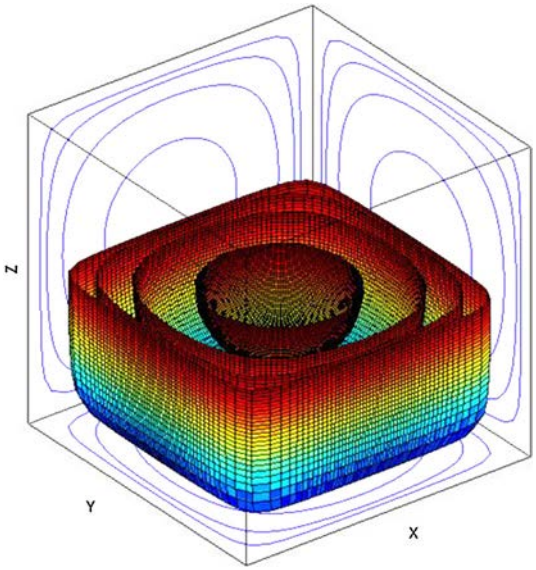

Fig. 3 Schematic of the Schwarz–Christoffel mapping nested blocks of a three-dimensional system extended from the 2-D system (*blue curves* on each plane)

predefined, and utilize an iterative method to identify the sub-block boundaries (Pruess et al. 1999). The Schwarz–Christoffel mapping provides coordinate values of all the points in the interface allowing the interface area to be determined by a summation method. The distance between two sub-blocks is determined by the interface area and the sub-block volumes. Flux between two-block is governed by the transmissibility term, and the transmissibility between two gird blocks is calculated with a weighted harmonic average as

$$T_{ij} = A_{ij} \frac{k_i k_j}{D_j k_i + D_i k_j}, \tag{2.2}$$

where A_{ij} is the area of the interface between the blocks i and j; D_i and D_j are, respectively, the distances from the block centers i and j to their interface; k_i and k_j are, respectively, the absolute permeability along the direction toward the interface in the blocks i and j.

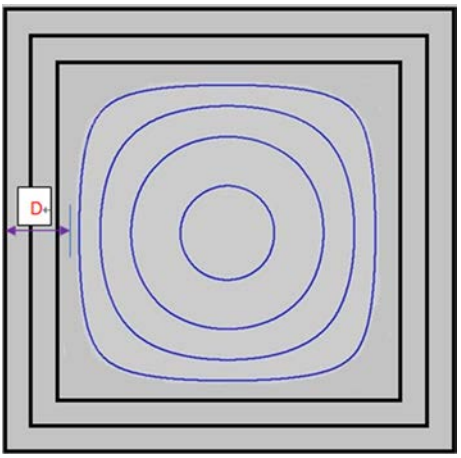

Fig. 4 Schematic of the MINC combines the Schwarz–Christoffel Conformal Mapping Concepts: MINC nested elements from the exterior to transition location "*D*" and the Schwarz–Christoffel mapping elements from "*D*" to the center of the block

2.4 MINC Combines the Schwarz–Christoffel Conformal Mapping Concepts (MINC-SCCM)

The MINC concept assumes that fracture–matrix flow is controlled only by the distance to the nearest fracture surrounding the matrix block, indicated as integral to the early rapid transient period of fracture–matrix flow for the MINC solution to remain combined with the true solution (Fig. 7). Early in the flow, the matrix has not been fully penetrated, and the inner portion of the matrix acts as an infinite source feeding the outer portion. During early production, pressure behavior resembles it in an infinite reservoir (Matthews 1986). The Schwarz–Christoffel conformal mapping concept, however, divides the matrix based on the theory of flow potential and is indicated to apply following the early rapid transient period (Fig. 7).

Proceeding with the concept of the MINC concept in early stages and switching to the Schwarz–Christoffel conformal mapping concepts after the early transient flow period seem to be a clear step. However, this method is complicated to implement. A semi-analytical solution for matrix–fracture flow has utilized a concept referred to as penetration depth (where penetration depth and time are equivalent) for a pressure disturbance at the block surface thus, a function of time at certain reservoir conditions (e.g., Pruess and Wu 1989; Moridis 2002). Considering this solution, a method is introduced dividing the outer portion of the matrix utilizing the MINC concept, while the inner portion is divided on the theory of the Schwarz–Christoffel mapping concept (Fig. 4).

2.4.1 Transition Location from the MINC to the Schwarz–Christoffel Mapping Concept Determination

The transition "*D*", which is a distance from the block surface, from the MINC to the Schwarz–Christoffel mapping concept is determined by the Darcy velocity law and the shape factor, as determined by an analytical solution. The analytical solution describes the flow inside the matrix block in a fully transient way (Appendix). Though the classic double porosity model is not accurate in shale gas simulation, the analytical solution derived Lim and Aziz shape factors (Eqs. 5.20, 5.23), which nearly match the reference solutions when flow turns stabilize (Fig. 6). The MINC concept may be applied to divide the matrix from the block

surface to the location "*D*", and the Schwarz–Christoffel mapping concept may be applied to divide the remaining matrix. In the double porosity model, mass out of the matrix equals mass flowing into the fracture, or vice versa. Consider gas flow rate for a linear system with P_m at the inlet and P_f at the outlet. The linear flow equation can be written as follows for compressible gas flow according to the Darcy's law:

$$q = \beta \frac{k_g A}{2\mu} \frac{(P_m^2 - P_f^2)}{D}. \tag{2.3}$$

In the two-dimensional system, interface area between the matrix and the fracture is $A = 4L^2$; D indicates the potential distance from the matrix to the fracture; β is a gas density-related coefficient, defined as $\beta = M/ZRT$. The classic double porosity model calculates the inter-porosity transfer rate to be proportional to the pressure difference between the matrix and the fracture, based on the pseudo-steady-state flow assumption (Warren and Root 1963). The resulting transfer rate between the matrix and the fracture in a control volume produces the equation below:

$$q = \sigma \cdot \beta \frac{k_g}{2\mu} \left(P_m^2 - P_f^2\right) V_m. \tag{2.4}$$

Substitute Lim and Aziz shape factor (Eq. 5.18) to Eq. 2.4, and equal to Eq. 2.3, leading to

$$D = \frac{4}{18.17} L. \tag{2.5}$$

Note that, D (Eqs. 2.5, 2.6) provides only an approximate location of the potential center and is not a factor in direct calculation. Slight errors originating from assumptions to "D" are acceptable. The transition distance from the fracture obtained from the analytical solution (use Eqs. 5.23, 2.3 and 2.4) for the cubical system with three sets of fracture calculated by

$$D = \frac{6}{25.67} L. \tag{2.6}$$

3 Results and Analysis

Concept evaluation utilizes a single node system for its simplified structure and limited variables. Pure methane single-phase gas flow in a square/cubical system is calculated taking into account Darcy flow, gas sorption on the solid surface, and the Klinkenberg effect. The low-permeable square-shaped matrix is connected to two/three sets of conductive fractures (Table 2), while fracture pressure remains constant at 3.45E06 Pa and system dimension is 10 m. Prevention and/or reduction of inconsistencies were minimized by keeping the number of sub-blocks in a matrix and the volumes of each sub-block the same in all nested cases. Cases with 5, 10, and 35 sub-blocks were tested, and in some cases, sub-blocks were even increased to 100. Although there is a lack of rigorous proofs, the numerical results move toward the true solution as the number of sub-blocks increases. An important result revealed that errors in the MINC model or the Schwarz–Christoffel mapping model cannot be completely eliminated by raising the sub-block number. Results of cases with 10 sub-blocks are presented in this paper.

The reference solution in Fig. 6 is obtained by fine grid simulation utilizing the MSFLOW-UG simulator (Wu 2012). In the two-dimensional case, the matrix is divided into 48 segments in both X and Y directions with only one layer in Z direction. In the three-dimensional case,

Table 2 Case study data

Matrix permeability	9.87E−20	m^2	Porosity	5	%
Fracture permeability	9.87E−12	m^2	Langmuir pressure	950	Pa
Initial reservoir pressure	2.76E+07	Pa	Langmuir volume	1.41E−03	m^3/kg
Fracture pressure	3.45E+06	Pa	Klinkenberg coefficient	1.60E+05	Pa

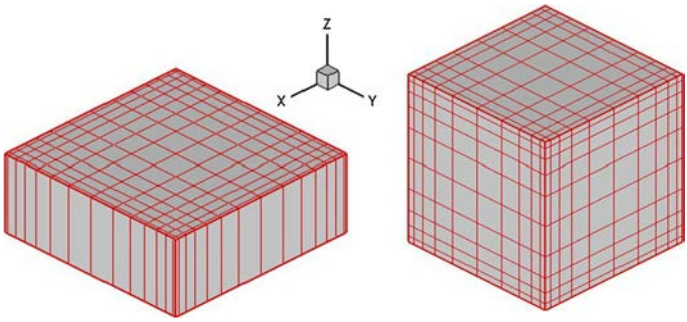

Fig. 5 Schematic of the fine grid discretization of the two- (*left*)/three- (*right*) dimensional system

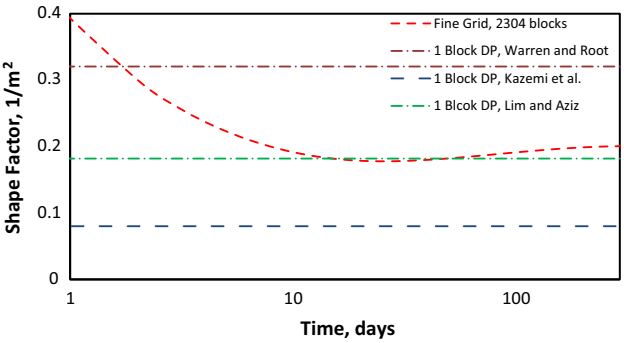

Fig. 6 Shape factor comparison of the double porosity model for the system with two-sets of fracture

the matrix is divided into 32 segments in every direction (X, Y, and Z). Figure 5 presents simple schemes of the fine grid discretization for the two- and three-dimensional systems.

3.1 Flow Behavior in the Matrix

Production rate and matrix average pressure are utilized to inversely calculate the shape factor which is applied to weight the geometrical effect. Calculations of shape factor resulting from the fine grid simulation (i.e., reference solution) accurately describe the flow behavior in the fractured system as it transits from a high value to a relatively stable and low value (Fig. 6). Pressure drawdown initiates at the matrix and fracture juncture, advancing into the matrix until porosity transfer reaches equilibrium. Upon reaching equilibrium, the center of flow in the matrix ceases to advance, while pressure in the matrix decreases linearly with time causing the flow to attain a pseudo-steady state (Serra 1983).

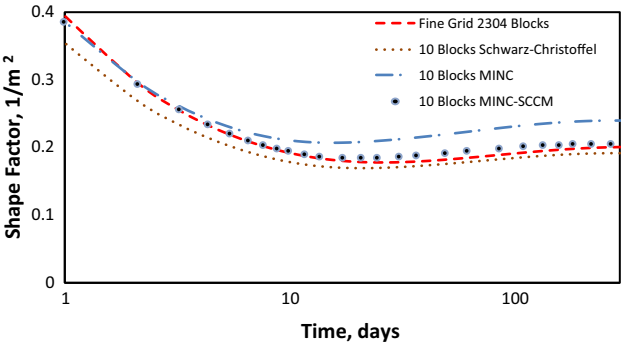

Fig. 7 Shape factor comparison of the nested sub-blocks model for the system with two-sets of fractures (shape factor is calculated by production rate and average matrix pressure)

Based on the pseudo-steady-state flow assumption with application of a constant shape factor, the double porosity model neglects transient flow behavior and does not adequately manage the matrix–fracture interaction. The double porosity model with Warren and Root or Kazemi et al.'s shape factors, even under the stable flow conditions, do not adequately compare to the reference solution (Fig. 6). In contrast, the Lim and Aziz shape factor nearly matches the reference solution upon flow stabilization (Fig. 6), which indicates the analytical solutions are accurately acceptable.

3.2 Results Analysis of Nested Sub-Blocks Cases

The MINC model's solution closely aligns with the reference solution at early times (Fig. 7), subsequently diverging following early transient flow and, finally not converging to the "true" solution with refinement of block subdivision. The MINC subdivides only in the X, Y, and Z directions toward the block boundary thus, the circular direction around the ring is not preserved. Comparatively, the Schwarz–Christoffel mapping approach is superior to both the double porosity and MINC models at later times. Errors in early stages of the Schwarz–Christoffel mapping approach, however, limit the overall advantages. The ability to eliminate early stage errors with a refinement of the block is prevented due to the square shape of the iso-pressure curves other than the Schwarz–Christoffel conformal mapping shapes (e.g., Cai 2014; Ding et al. 2014).

The MINC model and the Schwarz–Christoffel mapping model are accurate at different times. The MINC-SCCM combination model takes advantage of both concepts and can accurately handle the gas flow in the matrix for the entire production period (Fig. 7). Cumulative production curves (Fig. 8) also indicate the precise work of the MINC-SCCM combination. Shape factors and cumulative gas production from the three-dimensional case are not plotted in this paper, because they do not deviate from patterns of the two-dimensional case (more plots can be found in Cai 2014).

4 Discussion

Transient flow regimes may be prolonged in shale gas reservoirs, and the widely used classical double porosity model may not be applicable in the shale gas reservoir simulation. Fracture–matrix flow simulated by 1D MINC flow with the nested sub-blocks design was introduced

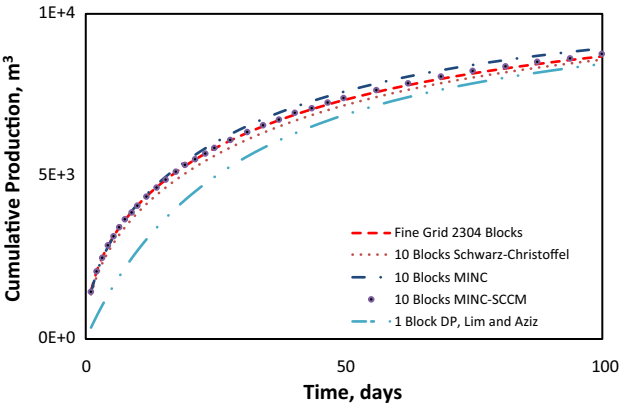

Fig. 8 Cumulative production of the nested sub-block model and the double porosity model of system with two-sets of fractures

by Pruess and Narasimhan. The theory of the MINC approach is simple, and computationally acceptable with pressure behavior essentially emulating that in infinite reservoirs. Flow is controlled only by the nearest fracture with flow streamlines along axes, which follows the MINC assumptions, making the MINC concept more suitable for the discretization of the matrix outer portion.

Flow streamlines begin to radiate out from the block center rather than along the axes as gas is produced from the reservoir following the initial transient period. This alteration in flow streamlines derails perpendicularity with the iso-pressure curves and effective application of the MINC concept as flow does not converge with the correct solution. Preservation of perpendicularity following the initial transient stage is pursued via the Schwarz–Christoffel mapping concept. The Schwarz–Christoffel conforming model is applied to stabilize flow, thus retaining perpendicularity. The transition from the MINC concept to the Schwarz–Christoffel conforming mapping model occurs in a transition location as determined by an analytical solution.

Transition location "*D*" is determined by an analytical solution with flow in the matrix described in a fully transient way. Two major assumptions exist in the derivation process: approximate the flow in a square/cubical system to a cylindrical/spherical flow and linearize the pressure term on the right side of the governing equation. The simplified analytical solution utilizes only the first term of the infinite summation series to produce an approximate solution for identifying transition location "*D*." Approximating transition location "*D*" is acceptable as "*D*" is not imperative in calculations thus, does not affect node discretization. Combining the MINC and Schwarz–Christoffel mapping concept methods allows characterization of majority flow at different times while defining precision flow processes during the entire flow period. The combination operates to maximize advantages of both methods for attainment of accurate results (Fig. 8).

5 Summary and Conclusions

This paper summarizes current major matrix–fracture interaction modeling concepts, introduces a new nested subdividing concept, evaluates several models at shale gas reservoir conditions, and recommends a combination method. Limitations of the classic double porosity model, the MINC model, and the Schwarz–Christoffel conformal mapping concept are

discussed. In shale gas reservoirs, flow may become stable or reach a pseudo-steady state after a long period of transient flow due to the low matrix permeability inherent to these reservoirs, the Klinkenberg effect, and the sorption of as molecules in the matrix. The classic double porosity model does not take into account simulation of the transient flow, resulting in a significant error in the early transient flow stage. Transient flow is negligible short in conventional reservoirs and the classic double porosity model functions accurately. Shale gas reservoirs, characterized by low matrix permeability, experience significantly prolonged and distinct flow regimes. Transient and stable flow periods must both be fully addressed to adequately manage the flow in such a reservoir. The MINC model and the Schwarz–Christoffel conformal mapping model do not fully consider the flow characteristics at different times and are not sufficient to manage the shale gas reservoir simulation. Combining the MINC and Schwarz–Christoffel mapping concepts of gridding inside the matrix allows majority flow characterization at different times and is able to accurately and efficiently simulate the matrix–fracture interaction for the entire transient flow. The block is discretized with MINC nested elements from the exterior to an analytically derived transition location ("*D*"), from this location to the center of the block, the Schwarz–Christoffel conformal mapping approach is applied to complete the task.

Extensively used models include idealized blocks, i.e., square and rectangular shape blocks, but the MINC concept, as well as the Schwarz–Christoffel mapping concept, can be extended to any arbitrary polygonal shape in theory. The feasibility of methods applied in other shapes should be tested and verified in our future work. Additionally, for a wider application, the MINC-SCCM model should be extended and evaluated under more complex conditions, such as multi-component/phase, and multi-scale conditions.

Appendix

Analytical Solutions of the Gas Flow in a Two-Dimensional System

Flow in a square system can be approximated to a cylindrical flow. The governing equation of a single-phase gas radial flow through a fracture–matrix system can be derived by combining a mass balance on the control volume with the dual-continuum concept (Lai et al. 1983; Wu and Pan 2003). The interface area for any two adjacent control volumes at a distance of r from the center:

$$A_r = 2\pi r h \tag{5.1}$$

where, subscript r is the distance from the block center at radial direction; and h is the formation thickness. Then control volume at radial distance r over distance dr:

$$V_r = A_r \cdot \mathrm{d}r = 2\pi r h \cdot \mathrm{d}r \tag{5.2}$$

Consider an ideal gas flow in a homogenous, isothermal, and incompressible porous media. The gas mass balance equation for a control volume inside the matrix is written as,

$$\rho q_r A_r - \left[\rho q_r A_r + \frac{\partial}{\partial r}\left(\rho q_r A_r\right)\mathrm{d}r\right] = \frac{\partial}{\partial t}\left[V_r\left(\rho \emptyset_{\mathrm{m}} + \frac{\alpha P_{\mathrm{m}}}{P_{\mathrm{m}} + P_{\mathrm{L}}}\right)\right] \tag{5.3}$$

where, α is the gas adsorption related coefficient,

$$\alpha = \rho_{\mathrm{k}} \rho_{\mathrm{g}} V_{\mathrm{L}} \tag{5.4}$$

where, ρ_{k} and ρ_{g} respectively are kerogen density and gas density at standard condition; V_{L} and P_{L} are the Langmuir volume and Langmuir pressure. Flow rate (Darcy 1856) at

surface of any control volume is proportional to the pressure gradient at the surface and the permeability of the matrix, inversely proportional to the fluid viscosity, and with considering the Klinkenberg effect contribution to the gas permeability,

$$q_r = -\frac{k_{\mathrm{m}}(1+\frac{b_{\mathrm{m}}}{P_{\mathrm{m}}})}{\mu_{\mathrm{g}}}\frac{\partial P_{\mathrm{m}}}{\partial r} \tag{5.5}$$

The ideal gas law defines the gas density as a function of pressure with a constant coefficient,

$$\rho = \beta P_{\mathrm{m}} \tag{5.6}$$

The unit of the Klinkenberg coefficient is consistent with pressure. Define an equivalent pressure coupling gas pressure and Klinkenberg coefficient as follows,

$$P_{\mathrm{m,b}} = P_{\mathrm{m}} + b_{\mathrm{m}} \tag{5.7}$$

Substitute (5.4) (5.5) and (5.6) into (5.3),

$$\frac{\partial}{\partial r}\left(\beta P_{\mathrm{m}} \cdot 2\pi r h \frac{k_{\mathrm{m}}(1+\frac{b_{\mathrm{m}}}{P_{\mathrm{m}}})}{\mu_{\mathrm{g}}}\frac{\partial P_{\mathrm{m}}}{\partial r}\right)\mathrm{d}r = \frac{\partial}{\partial t}\left[2\pi r h \cdot \mathrm{d}r\left(\beta P_{\mathrm{m}}\emptyset_{\mathrm{m}} + \frac{\alpha P_{\mathrm{m}}}{P_{\mathrm{m}}+P_{\mathrm{L}}}\right)\right] \tag{5.8}$$

Use (5.7), leading to,

$$\frac{k_{\mathrm{m}}}{\mu_{\mathrm{g}}}\frac{1}{r}\frac{\partial}{\partial r}\left[r P_{\mathrm{m,b}}\frac{\partial P_{\mathrm{m}}}{\partial r}\right] = \frac{\partial}{\partial t}\left[\emptyset_{\mathrm{m}} P_{\mathrm{m}} + \frac{\alpha P_{\mathrm{m}}}{\beta\,(P_{\mathrm{m}}+P_{\mathrm{L}})}\right] \tag{5.9}$$

In order to linearization of the above equation, define an averaged value of the gas pressure in the coefficient of RHS,

$$\bar{P}_{\mathrm{m,b}} = \bar{P}_{\mathrm{m}} + b_{\mathrm{m}} \tag{5.10}$$

In terms of $P^2_{\mathrm{m,b}}$ and the averaged matrix gas pressure, the governing equation can be linearized as follows

$$\frac{k_{\mathrm{m}}}{\mu_{\mathrm{g}}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P^2_{\mathrm{m,b}}}{\partial r}\right) = \emptyset_{\mathrm{m}}\left[\frac{1}{\bar{P}_{\mathrm{m,b}}} + \frac{\alpha P_{\mathrm{L}}}{\beta\emptyset_{\mathrm{m}}\bar{P}_{\mathrm{m,b}}(\bar{P}_{\mathrm{m,b}} - b_{\mathrm{m}} + P_{\mathrm{L}})^2}\right]\frac{\partial P^2_{\mathrm{m,b}}}{\partial t} \tag{5.11}$$

or,

$$\frac{\partial P^2_{\mathrm{m,b}}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_{\mathrm{m}}}{\emptyset_{\mathrm{m}}\mu_{\mathrm{g}}C_{\mathrm{t}}}\frac{\partial P^2_{\mathrm{m,b}}}{\partial r}\right) \tag{5.12}$$

C_{t} is a total gas compressibility, which includes the gas sorption term and the gas compressibility.

$$C_{\mathrm{t}} = \frac{1}{\bar{P}_{\mathrm{m,b}}} + \frac{\alpha P_{\mathrm{L}}}{\beta\emptyset_{\mathrm{m}}\bar{P}_{\mathrm{m,b}}(\bar{P}_{\mathrm{m,b}} - b + P_{\mathrm{L}})^2} \tag{5.13}$$

The initial matrix pressure is P_i; the boundary pressure is constant at P_{f}. The analytical solution can be expressed as follows (Crank 1975; Lim and Aziz 1995) in term of square pressure,

$$\frac{\bar{P}^2_{\mathrm{m,b}} - P^2_i}{P^2_{\mathrm{f,b}} - P^2_i} = 1 - \sum_{n=1}^{\infty}\frac{4}{R^2\,\propto^2_n}\mathrm{Exp}\left[\frac{-\propto^2_n k_{\mathrm{m}} t}{\emptyset_{\mathrm{m}}\mu C_{\mathrm{t}}}\right] \tag{5.14}$$

where, $\propto_n$ is the root of

$$J_0(R\cdot\propto_n) = 0 \tag{5.15}$$

J_0 is the Bessel J function of the first kind of order zero.

Shape Factor Derivation for Gas Flow in a Two-Dimensional System with Lim and Aziz Assumptions (1995)

Lim and Aziz (1995) suggested only using the first term of the infinite summation series to make an approximate solution. In the double porosity model, mass out of the matrix equals mass flowing into the fracture, or vice versa. As the square system for example, the matrix–fracture transfer rate can be expressed relating to the accumulation in the matrix as follows,

$$q = \rho \emptyset c_{\mathrm{t}} \frac{\partial \bar{P}_{\mathrm{m,b}}}{\partial t} V_{\mathrm{m}} \tag{5.16}$$

Use (5.14) and (5.16), leading to,

$$q = V_{\mathrm{m}} \frac{5.78}{R^2} \frac{k_{\mathrm{m}}}{2\mu} \beta (P_{\mathrm{m,b}}^2 - P_{\mathrm{f}}^2) \tag{5.17}$$

Darcy's law suggests the flow rate from the matrix to the fracture is proportional to the pressure gradient at the matrix–fracture interface,

$$q = \rho \frac{k_{\mathrm{m}}}{\mu} A \frac{\partial \bar{P}_{\mathrm{m,b}}}{\partial x}|_{x=0} \tag{5.18}$$

Leads to the matrix–fracture transfer rate as an equation with a shape factor,

$$q = \sigma \frac{k_{\mathrm{m}}}{2\mu} \beta (P_{\mathrm{m,b}}^2 - P_{\mathrm{f}}^2) V_{\mathrm{m}} \tag{5.19}$$

The volume is the basis of equating the square and the cylindrical geometries, leading to $R = 0.56L$. L is the fracture dimension of the square system; R is the radius of the equivalent cylindrical system. Therefore, the shape factor can be easily determined

$$\sigma = \frac{18.17}{L^2} \tag{5.20}$$

Analytical Solutions and Shape Factor for Gas Flow in a Three-Dimensional System

Pressure diffusion in a cubical system surrounded by three sets of fractures can be approximated by that of a spherical system. Follow the same steps above, leading to the pressure diffusion equation in a spherical geometry,

$$\frac{\partial^2 \partial^2_{P_{\mathrm{m,b}}}}{\partial r^2} + \frac{2}{r} \frac{\partial P_{\mathrm{m,b}}^2}{\partial r} = C_{\mathrm{t}} \frac{\partial P_{\mathrm{m,b}}^2}{\partial t} \tag{5.21}$$

The analytical solution gives as follows (Crank 1975; Zimmerman et al. 1992; Lim and Aziz 1995),

$$\frac{\bar{P}_{\mathrm{m,b}}^2 - P_i^2}{P_{\mathrm{f,b}}^2 - P_i^2} = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} Exp \left[\frac{-n^2 \pi^2 k_{\mathrm{m}} t}{\emptyset_{\mathrm{m}} \mu C_{\mathrm{t}} R^2} \right] \tag{5.22}$$

The equivalent radius equals to $0.62L$.With the same assumption and treatment on the analytical solution, the shape factor for a cubical matrix with three sets of fractures can be written as follows,

$$\sigma = \frac{25.67}{L^2} \tag{5.23}$$

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