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Efficient simulation of hydraulic fractured wells in unconventional reservoirs



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ABSTRACT

Reservoir simulators remain essential tools for improved reservoir management in order to optimize hydraulic fracturing design in unconventional low permeability reservoirs. However, the commonlyused simulator requires too much computational efforts for the description of local phenomena in the vicinity of the hydraulic fractures, and is practically infeasible for field applications, due to large number of gridblocks involved in a full-field simulation with many multi-fractured complex wells. Therefore, coarse grid simulations have been widely used, but the techniques for the coarse grid simulation of fractured wells need to be improved.

In this paper, we present efficient numerical methods to handle both long-term well performance and transient behavior simulations for hydraulic fractured wells in unconventional reservoirs with a coarse grid. To simulate correctly the long-term behavior, transmissibilities around the fracture and the connection factor between a fractured block and the fracture are computed, based on a (pseudo)-steadystate near-fracture solution. This approach provides not only an accurate long-term well production calculation, but also a correct pressure distribution in the near-fracture region. In unconventional reservoirs, the transient effects cannot be ignored, due to the very low reservoir permeability and nearwell/near-fracture physical processes such as fracturing fluid induced formation damage. In order to handle the transient effect, the coupled modeling technique is used, and its efficiency is demonstrated with a tight-gas reservoir.

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1. Introduction

With the increasing demand of hydrocarbon facing current energy shortage, the unconventional resources, such as tight gas and shale gas plays, are becoming more and more important. Significant progress has been made in the past decade for economic development of unconventional petroleum resources. Economic production from unconventional tight and shale gas reservoirs depends upon artificial well stimulation like hydraulic fracturing. Now, reservoir modeling and simulation become increasingly important in order to optimize and improve reservoir management of a tight or shale field by designing fracture length and spacing, and well spacing or infill well location or trajectory, etc.

From a numerical modeling point of view, a single hydraulic fracture can be handled for research purpose using a very refined grid for flow simulation. However, unconventional reservoir

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http://dx.doi.org/10.1016/j.petrol.2014.09.005 0920-4105/© 2014 Elsevier B.V. All rights reserved. developments include many wells with multi-hydraulic fractures. These fractures are long with a length of several hundred meters and narrow with a width of only several millimeters. Besides, fracturing in unconventional reservoirs induces usually a complex fracture network by reactivating reservoir natural fractures (see, for example, Delorme et al., 2013; Norbeck et al., 2014). A fine grid numerical model to simulate these field cases requires too much CPU time and it is generally impractical or even impossible to perform fine grid simulations in field applications. So, there is a need to model hydraulically fractured reservoirs with coarse gridblocks.

To simulate hydraulic fractures with a reservoir simulator, some authors suggested using an equivalent wellbore radius or a negative skin factor with a coarse grid (see, for example, Lefevre et al., 1993). But this approach does not produce "elliptical shape" pressure distribution around the fracture and can generate a negative well connection factor for the reservoir simulator. Elahmady and Wattenberger (2006) proposed to use pseudopermeabilities to simulate flow perpendicular and along the fracture directions. Zhou and King (2011) used an upscaling method to handle fractured wells in heterogeneous media. Many

Nomenclature		y_1 y_2	coordinates coordinates
C C _f F FCI K P P ₀ O	a constant dimensionless fracture conductivity flow across a block edge fracture connection index permeability pressure pressure at a wellblock or a fracture block well flow rate	$egin{array}{c} W \ W_0 \ arphi \ arphi \ arphi \ arphi \ arphi \ arphi \ arphi^+ \ arphi^- \ \Delta x \end{array}$	fracture width wellblock potential fracture boundary fracture boundary outside the fracture fracture boundary inside the fracture gridblock size
r ₀ r	equivalent wellblock radius wellbore radius	Subscri	pts
PI S T x x_1 x_2 X_f y	(numerical) productivity index skin transmissibility a 2D point with $x=(x_1, x_2)$ coordinates coordinates fracture half length a 2D point with $y=(y_1, y_2)$	f i res s w 0	fracture gridblock <i>i</i> gridblock <i>j</i> reservoir steady-state or pseudo-steady-state solution well wellblock or a fracture block

papers can be found in the literature to discuss the simulation of fractured wells with coarse grids and to propose some practical solutions (see, for example, Sadrpanah et al., 2006; Gataullin, 2008; Abacioglu et al., 2009; Burgoyne and Little, 2012).

The problem of well modeling has been discussed for a long time. Peaceman's (1978, 1983, 1993) model has been widely used for reservoir simulations. But that model has its limitations and cannot be used for the fractured well simulation. To improve the well modeling, Ding et al. (1995), Ding (1996) and Ding and Jeannin (2001) proposed to modify the near-well numerical scheme, based on a steady-state or a pseudo-steady-state flow regime, to adapt the singular flow behavior in the near-well region with coarse grid simulations. This concept was also extended for the modeling of fractured wells by Ding and Chaput (1999).

For unconventional reservoir simulations, it is essential to model correctly long conductive fractures such as hydraulic fractures. Li and Lee (2008) proposed to use a transport index (fracture connection index) between a matrix block and the fracture to handle embedded discrete fractures. They assumed that the pressure varies linearly in the normal direction to each fracture and they computed an average distance between a fractured block and the fracture for the transport index calculation. The same formula was used by Hajibeygi et al. (2011) in hierarchical fracture model for conductive fracture modeling and by Moinfar et al. (2013) for the simulation of hydraulically fractured unconventional reservoirs. In unconventional reservoir simulations, even with the presence of complex fracture networks, it is common to simulate large hydraulic fractures explicitly using a discrete fracture approach and homogenize short/diffused fractures for a single or dual continuum (see, for example, Moinfar et al., 2013; Wu et al., 2013). The flow transfer between a grid block and a hydraulic fracture as well as between grid blocks near a large fracture needs to be simulated with precisions. However, assuming linear pressure variation is not very accurate near the fracture extremities and in the zone where several fractures are intersected, especially on large gridblocks. In this paper, we propose to compute the fracture connection index (FCI) based on steady-state or pseudo steady-state pressure solution with integral representation. Our approach can handle complex cases like flow modeling near fracture extremities, and it can also be extended to compute flow transports between grid blocks near a fracture. Examples are presented to handle hydraulically fractured wells in unconventional reservoirs. Note that some formation damage issues can also be handled through the modification of connection factors (or skin) between a fractured block and the fracture.

Another issue related to the fractured well simulation in unconventional reservoirs is the modeling of transient flow behavior. In a conventional reservoir, the transient period is generally very short, and a pseudo-steady-state flow regime is quickly reached in the near-well/near-fracture region. The transient behavior can generally be neglected in conventional reservoir simulations, except for transient well testing. However, the transient behavior cannot be ignored in low permeability unconventional tight and shale gas reservoirs. Artus and Fructus (2012) used very fine meshes to simulate transient behavior in shale-gas reservoirs and they showed the necessity to improve transmissibility calculation even for very fine mesh connections around the fractures, especially near the fracture extremities.

Coarse gridblock is generally not adapted to the modeling of transient behavior (the size of the gridblock is too large compared to variation length of physical variables such as pressure, saturation) and the description of near-well/near-fracture physical processes. To simulate the transient behavior, Blanc et al. (1999) proposed a transient well index, based on an analytical solution, to handle well testing problem for vertical wells. Al-Mohannadi et al. (2007) and Aguilar et al. (2007) used time-dependent well index to simulate horizontal and multilateral wells. Archer (2010) also shows the necessity of transient well index for the coarse grid simulation. The transient effect becomes particularly important for fracture wells in the unconventional reservoir due to long fracture length and low reservoir permeability or long transient flow period (Medeiros et al. 2007; Ibrahim, 2013).

Transient effects associated with the near-well/near-fracture physical process are generally more complex. For a fractured well in an unconventional reservoir, the typical physical process is the fracturing fluid induced formation damage and cleanup (Bennion et al., 2000; Friedel, 2004; Ding and Renard, 2005; Cheng, 2012; Agrawal and Sharma, 2013; Bertoncello et al., 2014) or possible non-Darcy flow inside the fracture towards the well (Wu, 2002). All these studies require the use of very fine gridblocks around the fractures, and it is difficult to investigate well production behavior in a full-field context. Besides, a full-field reservoir simulator might not have all the options to simulate detailed near-well/ near-fracture physical processes. In order to take into account the

transient flow and near-well/near-fracture physics on one side and long-term production behavior on the other side, we propose to use the coupled modeling technique (Ding, 2010; Ding et al., 2012). A coarse grid model is used for the long-term production simulation, and a detailed near-well/near-fracture model is used to simulate near-well/near-fracture physics. These two models are linked through a time-dependent numerical productivity index and/or fracture connection index.

In this paper, we will present first the well model, based on a steady-state (or pseudo-steady-state) flow regime, for the simulation of fractured wells, which is suitable for long-term well productivity calculation. Then the transient behavior related to coarse gridblock size and near-well/near-fracture physical processes is modeled through the technique of coupled modeling. Examples are presented and show that these two combined approaches are well suited for the simulation of fractured wells in unconventional reservoirs.

2. Coarse grid simulation of fractured wells

2.1. Problems of previous fractured well modeling

Peaceman's (1978, 1983, 1993) well model has been widely used in flow simulators. To compute the wellbore pressure (with a constrained well flow rate) or the well flow rate (with an imposed bottom-hole pressure), Peaceman (1978) introduced a numerical productivity index (*PI*) on a wellblock W_0 to relate the wellbore pressure P_{w0} , the well flow rate Q_0 to the simulated wellblock pressure P_0 , by

$$Q_0 = PI(P_0 - P_{w0}), \tag{1}$$

with the numerical PI given by

$$PI = \frac{C}{\ln(r_0/r_w) + S'},\tag{2}$$

where *C* is a positive constant, depending on the well length and the permeability on the wellblock, *S* is the skin factor, r_w is the wellbore radius, and r_0 is the equivalent wellblock radius, which depends on the wellblock size and the well trajectory on the wellblock. For example, for a fully penetrating vertical well in a 2D square gridblock, the equivalent wellblock radius is given by

$$r_0 = 0.2\Delta x \tag{3}$$

with Δx the gridblock size.

It is not difficult to find that Peaceman's formula is not suitable for the fractured well modeling. One solution for the simulation of fractured wells is using the concept of an equivalent wellbore radius, which equals 1/2 of the fracture half length, or a negative skin factor. However, several problems arise with the equivalent wellbore radius or a negative skin factor: (1) the wellblock cannot be too small. If the wellblock size Δx is smaller than 2.5 times of the fracture half length, the numerical PI becomes a negative value. For example, for a fracture with a half length of 100 m, the wellblock size cannot be smaller than 250 m. In terms of skin, it cannot be too small so that $\ln(r_0/r_w) + S$ is negative. This constraint limits many applications. (2) The elliptic pressure distribution in the vicinity of a fractured well is modeled by a radial one. (3) The simulated result is not accurate before a pseudo steady-state regime is reached around the whole fracture. The transient period is as long as the fracture length increasing. (4) The above equivalent wellbore radius is based on the infinite conductivity assumption. The flow inside the fracture is not considered. (5) This model is not valid in a heterogeneous media.



Fig. 1. Flux approximation in the control-volume method.

2.2. Improved well modeling

To improve the well modeling, Ding et al. (1995), Ding (1996) and Ding and Jeannin (2001) proposed to ameliorate the numerical scheme for near-well flow calculations. An accurate near-well flow approximation improves automatically the numerical *PI* formulation. In general, if a near-well steady-state or pseudo-steady-state solution is known, we can determine a suitable numerical scheme by identifying correct flow calculations. Considering a simplified two-point flux approximation scheme:

$$F_{ij} = T_{ij}(P_j - P_i), \tag{4}$$

where P_i and P_j are the pressures on the blocks *i* and *j* respectively, F_{ij} is the flux term across the edge Γ_{ij} (Fig. 1), the transmissibility term T_{ij} can be determined with a (pseudo) steady-state flow solution by

$$\Gamma_{ij} = \frac{F_{ij}^s}{(P_j^s - P_i^s)},$$
5)

where P_i^s and P_j^s are the (pseudo) steady-state pressure on the blocks *i* and *j* respectively, F_{ij}^s is the (pseudo) steady-state flux across the edge Γ_{ij} . The pressure variation in the vicinity of the well is very big due to the singular near-well flow behavior. On the wellblock, we can define the wellblock pressure as a volumetric average pressure or a pressure at a distance r_0 from the well. Once the wellblock pressure is well defined, we can naturally determine the numerical *PI* for Eq. (1) by

$$PI = \frac{Q_0^5}{(P_0^5 - P_w^5)},$$
(6)

where Q_0^s is the well flow rate on the wellblock W_0 under a (pseudo) steady-state regime, P_0^s and P_w^s are respectively the wellblock pressure and the wellbore pressure under the (pseudo) steady-state regime. So, with this approach, the near-well flow can be correctly handled and the numerical *PI* can also be easily determined.

2.3. Improved fractured well modeling

For a fractured well, two problems should be considered in the flow modeling: flow from the reservoir to the fracture, and flow from the fracture to the well. The modeling of flow from the reservoir to the fracture is the most important one. Inspired by the well modeling with a numerical *PI*, we introduce a fracture connection index (*FCI*), which relates the pressure on a fracture block P_0 , the fracture pressure P_{f0} on the same block and the flow rate entering to the fracture Q_{f0} by

$$Q_{f0} = FCI(P_0 - P_{f0}).$$
(7)

Considering a fully penetrated fracture (a 2D problem), a steady-state flow equation in the vicinity of the fracture in a



Fig. 2. Modeling of steady-state flow towards the fracture.

homogeneous medium is given by

$$\frac{\partial^2 P^s}{\partial x_1^2} + \frac{\partial^2 P^s}{\partial x_2^2} = 0 \tag{8}$$

with some boundary conditions at the fracture boundary Γ_f (Fig. 2), for example,

$$P^{s}(x) = P_{f}, \quad x = (x_{1}, x_{2}) \in \Gamma_{f}$$
 (9)

for the Dirichlet condition, or

$$\frac{\partial P^s}{\partial n}(x) = Q_f, \quad x = (x_1, x_2) \in \Gamma_f \tag{10}$$

for the Neumann condition.

Appendix A presents a boundary integral method (sometimes also called semi-analytical method) to solve this steady-state problem. In particular, an analytical solution can be obtained with a constant potential condition (equivalent to the well-known uniform flux condition for the fractures). Once the steady-state solution is obtained, we can determine the transmissibility around the fracture by Eq. (5) and the fracture connection index (*FCI*) by

$$FCI = \frac{Q_{f0}^{s}}{(P_{0}^{s} - P_{f0}^{s})}.$$
(11)

The pressure distribution, under a (pseudo) steady-state flow, depends mainly on the position of the fracture, and much less on the fracture conductivity. The transmissibilities and FCIs, obtained with a constant potential steady-state solution, can adequately be applied to the simulation of fractured wells with a wide range of conductivity values as shown in the examples hereafter.

It is generally recognized the importance of accurate modeling of flow transport between matrix and conductive fractures. Li and Lee (2008) assumed that the pressure varies linearly around the fracture and computed an average distance of a fractured block to the fracture for the matrix-fracture transfer calculation. The same approach is recently applied to flow simulation in unconventional reservoirs (Moinfar et al., 2013; Norbeck et al., 2014) by considering the presence of main hydraulic fractures and a complex fracture network. However, using linear pressure assumption is not suitable for flow modeling near fracture extremities and in the zone of fracture intersections, especially for large gridblocks, Artus and Fructus (2012) illustrated the necessity of considering particular matrix-fracture flow near hydraulic fracture extremities. The formula proposed in this paper improves the matrix-fracture interaction for long conductive fractures, and examples below show the accuracy of our approach.

In heterogeneous media, no analytical or semi-analytical (pseudo) steady-state solution is available. However, we can use a technique similar to the near-well upscaling procedure (Ding, 1995; Mascarenhas and Durlofsky, 2000) by solving a steady-state problem with a fine grid numerical simulation. The obtained fine grid solution is then used to compute the transmissibility around the fracture and fracture connection index (*FCI*) from Eqs. (5) and (11).

With this improved model for the fractured well modeling, the problems mentioned above are solved, in particular, (1) the concept of an equivalent wellbore radius is not used, so there is no limitation on the gridblock size. (2) The near-fracture pressure and flow distribution can be correctly handled, as the numerical scheme in the near-fracture region is modified to adapt the near-fracture flow behavior. (3) The transient behavior due to long fracture length can be partly reduced, because a pseudo-steadystate regime can be quickly attained in a scale between two neighboring gridblocks, much faster than in the whole fracture region. (4) This approach is not limited to an infinite conductivity fracture. Besides, this approach can be applied to a wide range of fracture conductivities, even if the flow regime is determined with a particular boundary condition (constant potential or other boundary conditions). (5) This concept can be extended to heterogeneous media.

Moreover, this approach can also be used for flow simulation with general discrete fracture network.

Flow inside the fracture has also to be considered, especially, for low conductivity fractures. In general, the simulation uses Darcy's law with a given fracture permeability to simulate flows inside the fracture. In the fracture plane, the well is considered as a sink/source at the intersection point between a horizontal well and the created fracture. So, the technique of well model (Peaceman, 1983; Ding et al., 1995) can be directly applied to handle the radial flow inside the fracture towards a horizontal well. However, if a 1D grid is used inside the fracture (as shown in the examples hereafter), precaution should be taken, because a 1D grid cannot simulate correctly the radial flow behavior towards a horizontal well. Nevertheless, this has almost no impact on high conductivity fractured wells, as the pressure drop is very low along the fracture due to the high fracture conductivity. But errors can be significant in low conductivity fractures. One solution, as illustrated in the example below, is to introduce a skin factor or modify numerical PI for the connection between the wellblock (block containing the intersection of the well and the fracture inside the fracture plane) and the well to mimic a 2D radial flow in the fracture plane.

2.3.1. Remark

For the fractured well modeling, we use numerical PI to represent the connection between the well and the fracture, and *FCI* to represent the connection between the fracture and the reservoir.

2.4. Incorporation of near-well/near-fracture physics and coupled modeling

Many physical processes may change near-well/near-fracture flow and formation properties. Concerning the fractured well, formation damage issue is greatly discussed (Bennion et al., 2000; Friedel, 2004; Ding and Renard, 2005; Agrawal and Sharma, 2013; Bertoncello et al., 2014). In general, formation damage in fractured wells falls into two categories: damage inside the fracture and damage inside the formation. The damage inside the fracture can be caused by proppant crushing, proppant embedment, fracture plugging with chemicals and polymer residues. The damage inside the reservoir is classified by mechanical damage and hydraulic damage. The mechanical damage, which is generally characterized by a reduction of absolute permeability, includes polymer solids deposition near the fracture face, clay swelling, broken gel/fine migration, etc. The hydraulic damage occurs from the increase in water saturation caused by the fracturing fluid invasion into the porous media. It includes the hysteresis of relative permeability and high capillary pressure, water blocking, etc. The hydraulic damage reduces the gas flow in the zone, where the fractured fluids are invaded.

The formation damage can be characterized with skin factors. For a fractured well, if the formation damage is inside the reservoir, a skin is added to *FCI* term for the connection between the reservoir and the fracture. If the formation damage is inside the fracture, a skin is added to numerical *PI* term for the connection between the fracture and the intersected well. In Ding and Renard (2005), they presented a procedure for the determination of long-term numerical *PIs* for formation damage around a well. The similar procedure can be applied to determine *FCIs* for the formation damage near fracture faces. According to the physical problem, we will choose suitable indices (numerical *PI* and/or *FCI*) to integrate the damage effect.

Some physical processes alter progressively the near-fracture formation, and we cannot use constant skin factors (associated with numerical *PI* and/or *FCI*) to mimic this process. One solution is to use variable numerical *PI* and/or *FCI* through a coupled modeling (Ding, 2010). The coupled modeling is initially developed to update numerical *PIs* for a full-field coarse grid simulation. This technique is also suitable for updating *FCIs* for coarse grid fractured well simulations. The coupled modeling is particularly useful in the following two situations: (1) large gridblock size and low permeability; (2) near-well/near-fracture damage process acts in a long period.

In coupled modeling, data exchanges between the reservoir model and the near-fracture model are performed through updating numerical *PIs* and/or *FCIs* for the reservoir model and boundary conditions for the near-fracture model. The updated numerical *PIs* and/or *FCIs* correct inaccuracies in the coarse grid simulation. This technique can improve the modeling of transient effect considerably for fractured wells in full-field reservoir simulations.

3. Numerical examples

3.1. Example 1 – coarse grid simulation of a single fractured well

In this example, we study how to handle a single fractured horizontal well with a coarse grid, and investigate the impact of different reservoir permeabilities, corresponding to a low permeability conventional reservoir, a tight gas reservoir and a shale gas reservoir, on the well performance simulation.

Considering a half-geometry with respect to the fractured well as shown in Fig. 3. A fracture with half length of 125 m and width 0.5 cm is first simulated with a very fine grid, starting with gridblock size of 0.25 cm around the fracture (Fig. 3a), and the reservoir thickness is discretized by 5 layers with gridblock sizes of 10 m. The horizontal well is drilled in the middle layer. This reservoir zone is also discretized with a uniform 2D coarse grid with block size of 50 m in the *x* and *y* directions and only 1 layer in

the *z*-direction (Fig. 3b). For the coarse grid, the fractured well model presented above with a constant potential boundary condition is applied to handle the near-fracture flow by modifying the transmissibilities. It is assumed that only the fracture contributes the production. The reservoir water is immobile with irreducible water saturation of 0.2. Only gas flow is considered.

First, a low permeability "conventional" reservoir with permeability of 2 mD is considered. Fig. 4 presents a comparison between fine and coarse grid simulations on the gas production for a fracture permeability of 5 D, 50 D and 5000 D (or a dimensionless fracture conductivity C_f =0.1, 1 and 100). The dimensionless fracture conductivity is defined by

$$C_f = \frac{K_f}{K_{res}} \frac{W}{X_f}.$$
 (12)

with K_f the fracture permeability, K_{res} the reservoir permeability, w the fracture width, and X_f the fracture half length. The maximum gas production is limited to $10^6 \text{ m}^3/\text{day}$. For the low fracture conductivity case, a significant difference between the coarse and fine grid simulations is observed, while for high fractures conductivities, the presented model is perfectly suitable for the coarse grid simulation. The transient period is less than 1 day due to high reservoir permeability.

In the coarse grid simulation, the fracture is discretized with a 1D grid. For high fracture conductivity, all gas in the fracture is quickly produced by the well, and the pressure drop near the well (inside the fracture) is small. So, the error on the well modeling between the fracture and the well is not observed. However, for a low conductivity case such as $C_f=0.1$, not all gas in fracture is immediately produced by the well, and the pressure drop inside the fracture is important. To correct errors in flow modeling from the fracture to the well, we adapt here a simple approach by introducing a skin factor for the connection between the fracture and the well (or modify the numerical PI). The skin (or modified numerical *PI*) is obtained using the inversion procedure (Ding and Renard, 2005) by comparing with a pseudo-steady-state solution from a fine grid simulation. The coarse grid simulation results with the inversed skin factor for $C_f=0.1$ (fracture permeability 5 D) are also shown in Fig. 4a. Very satisfactory result is obtained.

Fig. 3 presents also the pressure distribution at 800 days for C_f =100 with the fine and coarse grid simulations. These two pressure distributions are quite similar. Using the proposed fractured well model can correctly simulate fractured wells with a coarse grid system. The transient effect in a conventional reservoir is very small.

Now consider a tight-gas reservoir with a reservoir permeability of 0.02 mD. If we still use a fracture permeability of 5 D and 50 D, the dimensionless conductivity becomes 10 and 100 respectively, much higher than those in a conventional reservoir. Fig. 5 compares the gas production rate with the coarse and fine grid



Fig. 3. Discretization of a single fractured horizontal well (pressure distribution at 800 days with C_f =100 in a conventional low permeability reservoir): (a) fine grid system and (b) coarse grid system.



Fig. 4. Comparison between coarse grid and fine grid simulations in a conventional low permeability reservoir: (a) $C_f=0.1$, (b) $C_f=1$ and (c) $C_f=100$.



Fig. 5. Comparison between coarse grid and fine grid simulations in a tight reservoir: (a) $C_f = 1$ and (b) $C_f = 10$.

simulations. The coarse grid simulation provides generally accurate long-term production, but it remains inaccurate in early-time period due to large coarse gridblocks and lower reservoir permeability. The transient duration depends on the fracture permeability (or conductivity). In case of high fracture permeability, the transient period may last around 20 days, but the difference between coarse and fine grid simulations is not very large. Fig. 6 shows the pressure distribution at 800 days with fine and coarse grid simulations. The pressure distributions around the fracture are quite similar. Thus, we can simulate a fractured well in the tight-gas reservoir with a coarse grid with reasonable precisions, especially for long-term well production.

The transient effect seems limited in this example, because we consider only a single-phase Darcy flow. The transient effect can be much more important if other physical processes such as formation damage are considered. Fig. 7 shows a fine grid simulation by considering fracturing fluid induced formation damage for the case of K_f =5D. It is found that the errors related

to the coarse grid simulation are more important, due to the presence of fracturing fluid, and the transient regime lasts longer. In Example 2 below, we will study details of the transient effect related to the fracturing induced formation damage and show how to correctly handle this effect with a coarse grid simulation using the couple modeling technique.

For the shale-gas or ultra-tight reservoir, the tight matrix permeability is extremely low. Let us consider a reservoir matrix permeability of 0.0001 mD. In case of a hydraulic fracture permeability of 5 D, the dimensionless conductivity is as high as 2000. Fig. 8a shows the simulation results with the coarse and fine grid systems. As the matrix permeability is ultra-low, a pseudo-steadystate regime cannot be reached before several years. At the beginning, gas production is underestimated with the coarse grid system due to its large gridblock sizes, which cannot simulate correctly the gas flow in the reservoir close to the fracture. This phenomenon lasts for a very long time. Later on, after the gas near the fracture is produced with the fine grid model, the coarse grid



Fig. 6. Pressure distribution with coarse grid and fine grid simulations in a tight reservoir: (a) fine grid simulation and (b) coarse grid simulation.



Fig. 7. Transient effect considering fracturing fluid induced formation damage.

model starts to overestimate the gas production, because of the remaining gas in the near-fracture coarse blocks. Fig. 8b and c shows the simulation results with a lower fracture conductivity (C_{f} = 160 and 16 respectively). For all simulations, the durations of transient period with the coarse grid system are almost the same. Only long-term production is slightly improved with low fracture conductivity.

The transient period is too long in the above simulations, and it is not suggested to use a transient numerical *PI* or *FCI* during a very long time period. A slight reduction of fracture block size can significantly decrease the transient time. Fig. 9 presents the simulation with one level grid refinement by subdividing a fracture block into two parts: a (fracture) block with a width of 10 m and a block with a width of 40 m. This subdivision reduces the transient period by almost a factor of 10. The coupled modeling can, therefore, be conveniently applied to simulate the transient effect if necessary.

The production with a fractured well generally follows two steps: flow from matrix to the fracture and then from the fracture to the well. In a shale-gas reservoir, the production is usually limited by the matrix–fracture transfer for the flow from the reservoir to the fracture. Transient effect with a coarse grid is extremely long. It has to be mentioned that the fracture system in a shale gas reservoir is much more complex than a single hydraulic fracture. Co-existence of fracture networks and natural fractures is usually needed to be considered. To simulate fractured wells in a shale-gas reservoir with a coarse grid, it is necessary to use a dualporosity model and implement the fractured well modeling technique for dual-porosity simulations, which is our ongoing work.

In summary, the modeling technique presented in this paper can improve the simulation efficiency of fractured wells with a coarse grid system. When using a 1D grid in the fracture plane, a skin can be used to correct errors for the flow simulation from the fracture to the well. But this is not necessary if the fracture conductivity is high. For a low permeability "conventional" reservoir, transient effect can generally be neglected. For the tight gas reservoir, handling of transient period with a coarse grid simulation is an issue, especially when considering some damage processes. For the shale gas reservoir, the transient period depends greatly on the fracture block size, and the transient time might be extremely long. We should reasonably reduce the gridblock sizes for the fracture blocks in shale-gas reservoir simulations.

3.2. Example 2 – coarse grid simulation with coupled modeling

Considering a tight-gas reservoir of $1000 \text{ m} \times 1000 \text{ m} \times 50 \text{ m}$. The reservoir is water under-saturated: the initial reservoir water saturation of 0.2 is smaller than the irreducible water saturation of 0.28, which will induce water blocking in the fracturing fluid invaded zone. The relative permeability and capillary pressure curves are shown in Fig. 10. The average horizontal permeability is 0.02 mD and the vertical and horizontal permeability ratio is 0.5.

Two wells are drilled in this reservoir: one multi-fractured horizontal well with 3 transverse fractures (W1) and one singlefractured horizontal well (W2). Fig. 11a shows the coarse grid



Fig. 8. Comparison between coarse grid and fine grid simulations in a shale gas reservoir: (a) $C_f=2000$, (b) $C_f=160$, and (c) $C_f=16$.



Fig. 9. A slight refinement of fracture blocks in coarse grid simulations in the shale gas reservoir: (a) $C_f=2000$, (b) $C_f=160$, and (c) $C_f=16$.

model and the location of these two fractured wells. All fracture half lengths are 50 m and its width is 1 cm. The dimensionless fracture conductivity in the multi-fractured horizontal well (W1) is 2, and it is 20 in the single-fractured horizontal well (W2). In all

the wells, only fractures contribute directly to the gas productions. Due to water blocking, the gas relative permeability is reduced from 0.59 to 0.37 in the damaged zone (fracturing fluid invaded area). At the beginning of production, gas is produced together



Fig. 10. Relative permeability and capillary pressure: (a) hysteresis of gas relative permeability and (b) capillary pressure.



Fig. 11. Coupled modeling of a two-well system: (a) full-field coarse grid, (b) near-fracture model for well W2, and (c) near-fracture model for well W1.

with the injected fracturing fluid, which reduces the gas flow rate in the cleanup period.

For the coarse grid simulation, the reservoir is discretized by 20 gridblocks in the *x*-direction, 20 gridblocks in the *y*-direction and 5 blocks in the *z*-direction with gridblock size of $50 \text{ m} \times 50 \text{ m} \times 10 \text{ m}$ as shown in Fig. 11a. Transmissibilities around the fractures are modified by using Eq. (5) and the fracture connection indices *FCI* are given by Eq. (11) based on a constant potential steady-state flow.

Well W1 is drilled and fractured first. Well W2 is drilled and fractured one month later. A volume of 300 m³ of water-based polymer is injected to create each fracture, and the wells are shut in for two days before putting into productions. To simulate the fracturing fluid invasion and its cleanup, very fine gridblocks (sizes of several centimeters) are required around the fractures to get the reference solution, as the invasion depth is usually of the order of several to several tens centimeters. Fig. 12 presents a comparison of gas production between the coarse grid simulation with constant *PIs* and the reference solution. It is found that the coarse grid model cannot simulate the fracturing fluid invasion and the cleanup, due to large sizes of the fracture blocks. In fact, the initial reservoir saturation of 0.2 is lower than the irreducible water saturation of 0.28. Because of the large block size, all the injected



Fig. 12. Coarse grid simulation with constant numerical PI.

fracturing fluid is blocked in the coarse gridblock and cannot be reproduced. A big difference is observed in early-time, due to the cleanup process and large gridblock sizes. The transient period lasts for around 200 days for well W1. Nevertheless, the long-term simulation is quite satisfactorily simulated with the coarse grid. Although some drilling fluid is still trapped inside the reservoir, it has not much impact on long-term well productions, because the flow is linear close to the fracture face and the damage in a small zone near the fracture does not have much impact on the linear flow (perpendicular to the fracture). This example also shows that the fracturing fluid induced formation damage has little impact on long-term gas production, but the cleanup period may last several months. It may also be much longer according to the reservoirs.

To take into account the transient effects, the coupled modeling is used in the early time simulation when the well is fractured and starts the production. Fig. 11 also shows the fine grid models and their corresponding zones in the coarse grid. The coupled modeling for these two wells does not start at the same time, and the coupling periods are also different. Fig. 13 presents schematically the periods for the coupled modeling. The coarse grid full-field simulation is performed for a total of 800 days. The coupled modeling for well W1 starts with the injection of fracturing fluid at the 1st day. The duration of the coupling modeling is 200 days for this low conductivity multi-fractured well, because the cleanup time is long for a low conductivity fracture. In the same time, well W2 is fractured at the 31st day. So the coupled modeling for well W2 is started at the 31st day, and ended at the 61st day for a period of 30 days for this high conductivity fractured well. Table 1 presents the time steps for the data exchanges in the coupled modeling of these two wells. In this coupled modeling, we update the numerical PIs for the connections between the fractures and the well. Fig. 14 presents the simulation results of the coupled modeling. These results are very close to the reference solutions, which are obtained using local grid refinement with very fine gridblocks around the fractures.

This example shows that the proposed model, obtained by modifying the near-fracture transmissibility and the fracture connection factor *FCI* to adapt near-fracture flow behavior, gives very satisfactory solution for long-term well production. However, the



Table 1			

Time steps	for da	ta updatir	ng in the	e coupled	modeling.
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Coarse grid full-field model	Time steps for the data exchange			
Time (day)	Well W1 (day)	Well W2		
0–1	No coupling	No coupling		
1-21	1	No coupling		
21-31	5	No coupling		
31–51	5	1		
51-61	10	1		
61-201	10	No coupling		
201-800	No coupling	No coupling		



Fig. 14. Coarse grid simulation with time-dependent PI.

differences between using the constant numerical *PI* and transient *PI* (or the reference model) are significant during the early time. To take into account near-fracture damage process, it is essential to use a coupled modeling, where the transient affects, related to the fracturing fluid induced formation damage and gridblock sizes, can be correctly handled through time-dependent *PIs*. The transient effect is small for high conductivity fractures, while it is large for low conductivity fractures. This example shows also the importance of a high fracture conductivity, since the single fractured well W2 with a conductivity of 20 can produce almost the same volume as the 3 fractured well W1 with a conductivity of 2.

4. Discussions and conclusions

This paper studies the techniques for fractured well simulations in unconventional reservoirs with a coarse grid. A fractured well model, based on near-fracture transmissibilities and fracture connection indices determined from a steady-state flow regime, is proposed. This model is well suited for the simulation of long-term well production. If the near-well/near-fracture formation damage processes can be neglected, this model is suitable for the simulation of fractured wells in low permeability "conventional" reservoirs as well as unconventional tight-gas reservoirs. For shale-gas reservoirs, the transient period may be very long with a large coarse grid. A reasonable block size for fracture blocks (of the order of several meters) is required.

In homogeneous media, an analytical steady-state solution can be obtained using constant potential condition (equivalent to the uniform flux condition), and the fractured well modeling based on that solution is accurate enough for a wide range of fracture conductivity values. In heterogeneous media, the upscaling procedure, based on a fine grid simulation, can be used. Satisfactory simulation results make us confident to generalize this approach to more complex discrete fracture networks.

Accurate simulation of flow inside the fracture plane is also required for low conductivity fractures, especially if a fracture is simulated with a 1D grid. One practical solution is using a skin factor or modifying the numerical *PI* with an inversion procedure.

Transient flow may occur during early-time due to large gridblock, low permeability and some near-well/near-fracture physical processes. The transient period may be very long and important according to the reservoir and damage process. To handle the transient flow simulation with a coarse grid system, timedependent numerical *PIs* and/or *FCIs* are necessary. The coupled modeling is a useful technique to provide time-dependent numerical *PI* for coarse grid simulations, even if the coarse grid model does not dispose the option to simulate the near-well/nearfracture physical processes.

Appendix A

Using the boundary integral method and potential theory (Nedelec, 2001), differential equation Eq. (8) can be written with an integral representation (Fig. 2):

$$P(x) = -\frac{1}{2\pi} \int_{\Gamma_f} \varphi(y) \ln(|x-y|) d\sigma_y$$
(A1)

where $\varphi = \frac{\partial p}{\partial p}|_{\Gamma^-} - \frac{\partial p}{\partial n}|_{\Gamma^+}$ is the single-layer potential defined on fracture boundary Γ_f with Γ^+ and Γ^{--} the exterior and interior boundaries of the fracture respectively. The relation between the single-layer potential φ , which is defined on the fracture boundary, and the fracture pressure P_f on the fracture boundary, is given by the following equation:

$$P_f(x) = -\frac{1}{2\pi} \int_{\Gamma_f} \varphi(y) \ln(|x-y|) d\sigma_y, \quad \forall x \in \Gamma_f$$
(A2)

Besides, the fracture inflow $Q_f(x) = (\partial P / \partial n)(x)$ from the reservoir to the fracture is given by

$$Q_f(x) = -\frac{\varphi(x)}{2} - \frac{1}{2\pi} \int_{\Gamma_f} \varphi(y) \frac{\partial}{\partial n_x} (\ln(|x-y|)) d\sigma_y, \quad \forall x \in \Gamma_f$$
(A3)

According to the fracture boundary condition, integral equation Eq. (A2) or Eq. (3) will be solved to get the potential φ . This kind of approach is sometimes called as the semi-analytical method. For example, for an infinite conductivity fracture, the potential φ can be obtained by solving the following equation:

$$C = -\frac{1}{2\pi} \int_{\Gamma_f} \varphi(\mathbf{y}) \ln(|\mathbf{x} - \mathbf{y}|) d\sigma_{\mathbf{y}}, \quad \forall \mathbf{x} \in \Gamma_f.$$
(A4)

Once the potential φ is known, we can get the pressure distribution with Eq. (A1), the fracture pressure $P_f(x)$ with Eq. (A2) and the fracture inflow $Q_f(x)$ with Eq. (A3).

In this work, we are particularly interested in the constant potential condition, where an analytical solution can be obtained. Considering that the origin of the coordinate is at the center of the fracture, and x_1 direction is oriented along the fracture length as shown in Fig. 2. Let $\varphi \equiv C$ a constant, by neglecting the contribution from the two small rectangular surfaces of width w at the fracture extremities, the pressure field at a point x with Eq. (A1) is approximated by

$$P(x) \approx -\frac{C}{2\pi} \left(\int_{-X_f}^{X_f} \ln\left(\sqrt{(x_1 - y_1)^2 + (x_2 + \frac{w}{2})^2}\right) dy_1 + \int_{-X_f}^{X_f} \ln\left(\sqrt{(x_1 - y_1)^2 + (x_2 - \frac{w}{2})^2} dy_1\right) \right)$$
(A5)

where X_f is the fracture half length, and w is the fracture width. The fracture boundary is composed of two parts: first term corresponds to the fracture boundary below the *x*-axis (y = -w/2) and the second term corresponds to the fracture boundary above the *x*-axis (y = w/2). By neglecting the fracture width w (for *x* is not too close to the fracture), we get

$$P(x) \approx -\frac{C}{2\pi} \int_{-X_f}^{A_f} \ln((x_1 - y_1)^2 + x_2^2) dy_1$$

= $-\frac{C}{2\pi} ((X_f - x_1) \ln((X_f - x_1)^2 + x_2^2))$
+ $(X_f + x_1) \ln((X_f + x_1)^2 + x_2^2)$
+ $2x_2 \left(\operatorname{arctg} \left(\frac{X_f - x_1}{x_2} \right) + \operatorname{arctg} \left(\frac{X_f + x_1}{x_2} \right) \right) - 4X_f \right)$ (A5')

The pressure on the fracture boundary $P_f(x)$ with Eq. (A2) is approximated by (neglecting always the contribution from the two

fracture extremities)

$$P_{f}(x) \approx -\frac{C}{2\pi} \left(\int_{-X_{f}}^{X_{f}} \ln(\sqrt{(x_{1} - y_{1})^{2}}) dy_{1} + \int_{-X_{f}}^{X_{f}} \ln(\sqrt{(x_{1} - y_{1})^{2} + w^{2}}) dy_{1} \right)$$

$$= -\frac{C}{2\pi} ((X_{f} - x_{1}) \ln(X_{f} - x_{1}) + (X_{f} + x_{1}) \ln(X_{f} + x_{1}) - 2X_{f} + \frac{1}{2} (X_{f} - x_{1}) \ln((X_{f} - x_{1})^{2} + w^{2}) + \frac{1}{2} (X_{f} + x_{1}) \ln((X_{f} + x_{1})^{2} + w^{2}) + w \left(\operatorname{arctg} \left(\frac{X_{f} - x_{1}}{w} \right) + \operatorname{arctg} \left(\frac{X_{f} + x_{1}}{w} \right) \right) - 2X_{f} \right)$$
(A6)

If the fracture width w is neglected, the calculation can be simplified by

$$P_f(x) \approx -\frac{C}{\pi} ((X_f - x_1) \ln(X_f - x_1) + (X_f + x_1) \ln(X_f + x_1) - 2X_f)$$
(A6')

Moreover, the fracture inflow $Q_f(x)$ with Eq. (A3) is approximated by

$$Q_{f}(x) \approx -\frac{C}{2} - \frac{C}{2\pi} \left(\int_{-X_{f}}^{X_{f}} \frac{x_{2} + w/2}{(x_{1} - y_{1})^{2} + (x_{2} + w/2)^{2}} dy_{1} - \int_{-X_{f}}^{X_{f}} \frac{x_{2} - w/2}{(x_{1} - y_{1})^{2} + (x_{2} - w/2)^{2}} dy_{1} \right)$$
$$= -\frac{C}{2} - \frac{C}{2\pi} \left(\int_{-X_{f}}^{X_{f}} \frac{w}{(x_{1} - y_{1})^{2} + w^{2}} dy_{1} \right)$$
$$= -\frac{C}{2} - \frac{C}{2\pi} \left(\operatorname{arctg} \left(\frac{X_{f} - x_{1}}{w} \right) + \operatorname{arctg} \left(\frac{X_{f} + x_{1}}{w} \right) \right) \approx -C$$
(A7)

The last step is obtained by the fact that w is small. Eq. (A7) also shows that the constant potential is equivalent to the uniform flux condition for the fractures. With Eqs. (A5)–(A7), we can determine near-fracture transmissibility with Eq. (5) and *FCI* with Eq. (11) for the fractured well modeling.

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