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A multiple-continuum model for simulating single-phase and multiphase flow in naturally fractured vuggy reservoirs

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ABSTRACT

The existence of vugs or cavities in naturally fractured reservoirs has long been observed. Even though these vugs are known for their large attribution to reserves of oil, natural gas, or groundwater, few quantitative investigations of fractured vuggy reservoirs have been conducted. In this paper, a multiple-continuum conceptual model is presented, based on geological data and observations of core samples from carbonate formations in China, to investigate single-phase and multiphase flow behavior in such vuggy fractured reservoirs. The conceptual model has been implemented into a three-dimensional, three-phase reservoir simulator with a generalized multiple-continuum modeling approach. The conceptual model considers fractured vuggy rock as a triple- or multiple-continuum medium, consisting of (1) highly permeable and wellconnected fractures, (2) low-permeability rock matrix, and (3) various-sized vugs. The matrix system may contain a large number of small or isolated cavities, whereas vugs are larger cavities, indirectly connected to fractures through small fractures, microfractures or matrix. Similar to the conventional double-porosity model, the fracture continuum is primarily responsible for the occurrence of global flow, while vuggy and matrix continua, providing storage space, are locally connected to each other and interacting with globally connecting fractures. In addition, flow in fractured vuggy reservoirs may be further complicated by occurrence of non-Darcy's and other nonlinear flow behavior, because of large pore space and highpermeability flow channels. To account for such complicated flow regime, our model formulation includes non-Darcy flow using the multiphase extension of the Forchheimer equation as well as flow according to parallel-wall fracture and tube models, based on solutions of flow through a parallel-wall, uniform fracture and Hagen-Poiseuille tube flow.

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1. Introduction

Naturally fractured reservoirs exist throughout the world represent significant amount of the world oil and gas reserves, water and other natural resources. In the past half century, significant progress has been made towards understanding and modeling of flow processes in fractured rock (Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985). However, most studies have focused primarily on naturally fractured reservoirs without taking into consideration of large cavities. Recently, characterizing vuggy fractured rock has received attention, because a number of fractured vuggy reservoirs have been found worldwide that can significantly contribute to reserves and the production of oil and gas (Kossack and Curpine, 2001; Rivas-Gomez et al., 2001; Liu et al., 2003; Hidajat et al., 2004; Camacho-Velazquez et al., 2005; Kang et al. 2006; Wu et al. 2006).

Among the commonly used conceptual models for analyzing flow through fractured rock, dual-continuum models, i.e., double- and multiporosity, and dual-permeability, are perhaps the most popular approaches employed in fractured reservoir modeling studies. In addition to the traditional double-porosity concept, a number of triple-porosity or triple-continuum models have been proposed to describe flow through fractured rock (Closemann, 1975; Wu and ge, 1983; Abdassah and Ershaghis, 1986 Bai et al., 1993; Wu et al. 2004; Kang et al., 2006; Wu et al. 2006). In particular, Liu et al. (2003), Camacho-Velazquez et al. (2005); and Wu et al. (2007) present several new triple-continuum models for single-phase flow in a fracture-matrix system that includes cavities within the rock matrix (as an additional porous portion of the matrix). In general, these models have focused on handling different level/scaled heterogeneity of rock matrix or fractures, e.g., subdividing the rock matrix or fractures into two or more subdomains with different properties for single-phase and multiphase flow in such fractured reservoirs.

Based on these conceptual models, mathematical modeling approaches to flow through fractured reservoirs in general rely on

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continuum approaches and involve developing conceptual models, incorporating the geometrical information of a given fracture-matrix system, setting up mass and energy conservation equations for fracturematrix domains, and then solving discrete nonlinear algebraic equations. The commonly used mathematical methods for modeling flow through fractured rock include: (1) an explicit discrete-fracture and matrix model (Snow, 1965), (2) the dual-continuum method, including double- and multi-porosity, dual-permeability, or the more general "multiple interacting continua" (MINC) or the TOUGH2 method (Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985; Pruess et al., 1999), and (3) the effective-continuum method (ECM) (Wu, 2000a). Among these three commonly used approaches, the dual-continuum method has been perhaps the most used in application. This is because of its computationally less demanding than the discrete-fracture approach, and its capacity of handling fracturematrix interactions under multiphase flow, heat transfer, and chemical transport conditions in fractured reservoirs.

This paper summarizes our recent study of modeling flow in fractured vuggy reservoirs, based on an oilfield example of fractured vuggy reservoirs in western China. In particular, we discuss the issues and the physical rationale for how to conceptualize fracture–vuggy–matrix systems and how to represent them using a numerical modeling approach. The objective of this paper is (1) to present a general triple- or multiple-continuum conceptual model to include effects of different-sized vugs and cavities on single-phase and multiphase flow processes in naturally fractured vuggy reservoirs; (2) to describe a methodology for numerically implementing the proposed triple-continuum conceptual model; and (3) to present examples for verifying and applying the proposed model to analysis of transient flow behavior in fractured vuggy reservoirs.

2. Observation and conceptual model

As observed, e.g., from the carbonate formations of the Tahe Oilfield in western China, a typical fractured vuggy reservoir consists of a large and well connected fractured, lower-permeable rock matrix, as well as a large number of varying-sized cavities or vugs. As shown in outcrops of the reservoir layers in Figs. 1-(a), 2-(a), and 3-(a), those vugs and cavities are irregular in shape and very different in size from millimeter to meters in diameter. Many of the small-sized cavities appear to be isolated from fractures. In this paper, we use "cavities" for those small caves (with sizes of centimeters or millimeters in diameter), while "vugs" represent larger cavities (with sizes from centimeters to meters in diameter). Several conceptual models for vugs are shown in Figs. 1–6: (1) vugs (Figs. 1 and 4) are indirectly connected to fractures through small fractures or microfractures; (2) vugs (Fig 1 and 5) are isolated from fractures or separated from fractures by rock matrix; and (3) some of vugs are directly connected to fractures and some are isolated (Figs. 1, 2, 3, and 6). In reservoirs, there are many more vug varieties and spatial distributions than those shown in Figs. 1-(a), 2-(a), and 3-(a), some of which may be approximated by the conceptual models of Figs. 1-(b), 2-(b), 3-(b), and 4-6, or their combinations. In no circumstance, however, is there a requirement for uniform size distribution patterns for vugs and cavities in this study.

Similar to the conventional double-porosity concept (Warren and Root, 1963), large fractures or the fracture continuum, are conceptualized to serve as main pathways for global flow, while vuggy and matrix continua, mainly providing storage space as sinks or sources, are locally connected to each other, as well as directly or indirectly interacting with globally connecting fractures. Note that vugs and cavities directly connected with fractures (e.g., Fig. 3) are considered part of the fracture



Fig. 1. Schematic of conceptualizing fractured vuggy formation as a multiplecontinuum system with different-scale fractures, various vugs and cavities; (a) outcrop pictures and (b) conceptual model.





Fig. 2. Schematic of conceptualizing fractured vuggy formation as a discrete fracture system with well connected, (a) outcrop pictures and (b) conceptual model.



Fig. 3. Schematic of conceptualizing fractured vuggy formation as a fracture-vug-matrix system with well partially filled vugs, (a) core sample and (b) conceptual model.

continuum. More specifically, as shown in Figs. 4, 5, and 6, we conceptualize the fractured-vug-matrix system as consisting of (1) "large" fractures (or fractures), globally connected on the model scale to wells; (2) various-sized vugs or cavities, which are locally connected to fractures either through "small" fractures or through rock matrix; and (3) rock matrix, which may contain a number of cavities, locally connected to large fractures and/or to vugs.

In principle, the discussed triple-continuum model can be considered to be a natural extension of the generalized multi-continuum (MINC) approach (Pruess and Narasimhan, 1985; Wu and Pruess, 1988; Wu et al., 2004). In this approach, an "effective" porous medium is used to approximate the fracture, vugs, or rock matrix continua, respectively, by considering the three continua to be spatially overlapping and interacting to each other. The triple-continuum conceptual model assumes that approximate thermodynamic equilibrium exists locally within each of the three continua at all times. Based on this local equilibrium assumption, we can define thermodynamic variables, such as fluid pressures and saturation, for each continuum. Physically, fracture–vug systems, as shown in Figs. 4 and 5, cannot be approximated by a double-porosity conceptual model, i.e., the vugs cannot be lumped in the matrix system simply by upscaling or averaging for multiphase flow, because of the entirely different capillary forces in vugs and matrix.

Note that the triple-continuum model is not limited to the orthogonal idealization of the fracture system or uniform size or distribution of vugs and cavities, as those illustrated in Figs. 1, 2, and 3. Irregular and stochastic distributions of fractures and cavities can be handled numerically, as long as the actual distribution patterns are known (Pruess, 1983).

3. Mathematical model

A multiphase isothermal system in fractured vuggy reservoirs is assumed to be composed of three phases: oil, gas, and water. Each phase flowing in response to pressure, gravitational, and capillary forces according to Darcy's law. Note that in this work, single-phase flow is considered as a special case of multiphase flow. Therefore, three mass-balance equations fully describe the system in an arbitrary flow region of the porous, fractured, vuggy domain: For gas flow,

$$\frac{\partial}{\partial t} \left\{ \phi \left(S_o \overline{\rho}_{dg} + S_g \rho_g \right) \right\} = -\nabla \cdot \left(\overline{\rho}_{dg} \nu_o + \rho_g \nu_g \right) + q_g \tag{1}$$

For water flow,

$$\frac{\partial}{\partial t}(\phi S_{w}\rho_{w}) = -\nabla \cdot (\rho_{w}\nu_{w}) + q_{w}$$
⁽²⁾

For oil flow,

$$\frac{\partial}{\partial t} \left(\phi S_o \overline{\rho}_o \right) = -\nabla \cdot \left(\overline{\rho}_o v_o \right) + q_o \tag{3}$$

where the Darcy's velocity of phase β ($\beta = g$ for gas, = w for water, and = o for oil) is defined as,

$$\mathbf{v}_{\beta} = -\frac{\mathbf{k} \ k_{r\beta}}{\mu_{\beta}} \cdot \left(\nabla P_{\beta} - \rho_{\beta} g \nabla D \right). \tag{4}$$



Fig. 4. Conceptualization #1 of fractured vuggy rock as a triple-continuum system with vugs indirectly connected to fractures through small fractures.



Fig. 5. Conceptualization #2 of fractured vuggy rock as a triple-continuum system with vugs isolated from or indirectly connected to fractures through rock matrix.

In Eqs. (1)–(4), ϕ is the effective porosity of the medium; ρ_{β} is the density of phase β at reservoir conditions; $\overline{\rho}_{o}$ is the density of oil, excluding dissolved gas, at reservoir conditions; $\overline{\rho}_{dg}$ is the density of dissolved gas (dg) in oil phase at reservoir conditions; μ_{β} is the viscosity of phase β ; S_{β} is the saturation of phase β ; P_{β} is the pressure of phase β ; q_{β} is the sink/source term of component β per unit volume of the medium, representing mass exchange through injection/production wells or due to fracture–matrix–vug interactions, g is gravitational acceleration; \mathbf{k} is absolute/intrinsic permeability (tensor) of the medium; $k_{r\beta}$ is relative permeability to phase β ; and D is depth.

Eqs. (1), (2) and (3), governing mass balance for three-phase fluid flow, need to be supplemented with constitutive equations that express all the secondary variables and parameters as functions of a set of key primary thermodynamic variables. The following relationships will be used to complete the description of multiphase flow through fractured porous media:

$$S_w + S_o + S_g = 1 \tag{5}$$

In addition, capillary pressure and relative permeability relations are also needed for each continuum, which are normally expressed in terms of functions of fluid saturations. The densities of water, oil, and gas, as well as the viscosities of fluids, can in general be treated as functions of fluid pressures.

4. Numerical formulation

The governing equations, as discussed above, for multiphase flow in fractured vuggy reservoirs have been implemented into a generalpurpose, three-phase reservoir simulator, the MSFLOW code (Wu, 2000b). As implemented numerically, Eqs. (1), (2) and (3)are discretized in space using an integral finite-difference or controlvolume finite-element scheme for a porous-fractured-vuggy medium. Time discretization is carried out with a backward, first-order, finitedifference scheme. The discrete nonlinear equations for water, oil, and gas flow at Node i are written as follows:

$$\left\{\left(\phi S_{\beta}\rho_{\beta}\right)_{i}^{n+1} - \left(\phi S_{\beta}\rho_{\beta}\right)_{i}^{n}\right\}\frac{V_{i}}{\Delta t} = \sum_{j\in\eta_{i}}F_{\beta,ij}^{n+1} + Q_{\beta i}^{n+1}$$
(6)

where superscript n denotes the previous time level; n + 1 is the current time level; V_i is the volume of element i (porous or fractured block); Δt is time step size; η_i contains the set of neighboring elements (j) (porous, vuggy, or fractured) to which element i is directly connected; $F_{\beta,ii}$ is the



Fig. 6. Conceptualization #3 of fractured vuggy rock as a triple-continuum system with partial vugs isolated from.

Table 1

Table 2

Characteristic distances* for evaluating flow terms between fractures, vugs, and matrix systems.

Fracture Sets	1-D	2-D	3-D
Dimensions of Matrix Blocks (m)	А	А, В	A, B, C
Characteristic F-M Distances (m)	$l_{FM}=rac{A}{6}$	$l_{FM} = \frac{AB}{4(A + B)}$	$l_{FM} = \frac{3ABC}{10(AB + BC + CA)}$
Characteristic F-V Distances (m)	$l_{FV} = l_x$	$l_{FV} = \frac{l_x + l_y}{2}$	$l_{Ff} = \frac{l_x + l_y + l_z}{3}$
Characteristic V-M Distances ¹ (m)	$l_{VM} = \frac{a}{6}$	$l_{VM} = rac{ab}{4(a+b)}$	$l_{VM} = \frac{3abc}{10(ab + bc + ca)}$
Characteristic V-M Distances ² (m)	$l_{VM} = \frac{(A-d_c)}{2}$	$l_{VM} = \frac{A + B - 2d_c}{4}$	$l_{Ff} = \frac{A + B + C - 3d_c}{6}$

* Note in Table 1, A, B, and C are dimensions of matrix blocks along x, y, and z directions, respectively.

¹ Characteristic V-M distances are estimated for the case (Fig. 4), i.e., vuggy-matrix connections are dominated by small fractures, where dimensions a, b, and c are fracture-spacings of small fractures along x, y, and z directions, respectively.

 $^{2}\,$ Characteristic V-M distances are used for the case (Figs. 5 and 6), i.e., vugs are isolated from fractures.

mass flow term for phase β between elements i and j; and $Q_{\beta i}$ is the mass sink/source term at element i, of phase β .

The "flow" term $(F_{\beta,ij})$ in discrete Eq. (6) for multiphase flow between and among the triple-continuum media, along the connection (i, j), is given by

$$F_{\beta,ij} = \lambda_{\beta,ij+1/2} \gamma_{ij} \Big[\psi_{\beta j} - \psi_{\beta i} \Big]$$
⁽⁷⁾

where $\lambda_{\beta,i}$ $_{i+1/2}$ is the mobility term to phase β , defined as

$$\lambda_{\beta,ij+1/2} = \left(\frac{\rho_{\beta}k_{\tau\beta}}{\mu_{\beta}}\right)_{ij+1/2} \tag{8}$$

Here subscript ij + 1/2 denotes a proper averaging or weighting of properties at the interface between two elements i and j; and $k_{r\beta}$ is the relative permeability to phase β . In Eq. (7), γ_{ij} is transmissivity and is defined, if the integral finite-difference scheme (Pruess et al., 1999), as

$$\gamma_{ij} = \frac{A_{ij}k_{ij+1/2}}{d_i + d_j} \tag{9}$$

Parameters used in the single-phase flow problem in the triple-continuum, fractured



Fig. 7. Comparison between analytical and numerical solutions for single-phase transient flow through fractured vuggy formation.

where A_{ij} is the common interface area between connected blocks or nodes i and j; d_i is the distance from the center of block i to the interface between blocks i and j; and $k_{ij + 1/2}$ is an averaged (such as harmonic weighted) absolute permeability along the connection between elements i and j. The flow potential term in Eq. (4) is defined as

$$\psi_{\beta i} = P_{\beta i} - \rho_{\beta, ij + 1/2} g D_i \tag{10}$$

where D_i is the depth to the center of block i from a reference datum. The mass sink/source term at element i, $Q_{\beta i}$ for phase β , is defined as

$$Q_{\beta i} = q_{\beta i} V_i \tag{11}$$

Note that Eq. (6) has the same form regardless of the dimensionality of the model domain, i.e., it applies to one-, two-, or threedimensional analyses of multiphase flow through fractured vuggy porous media. In our numerical model, Eq. (6) is written in a residual form and is solved using Newton iteration fully implicitly.





Fig. 8. Comparison between Conceptualization#1 and Conceptualization#2 for base case fracture, vug, and matrix parameters.



Fig. 9. Sensitivity runs on vug porosity for Conceptualization #1.



Fig. 11. Sensitivity runs on vug permeability for Conceptualization #1.

5. Handling fractures and vugs

The technique used in this work for handling multiphase flow through fractured vuggy rock follows the dual- or multi-continuum methodology (Warren and Root, 1963; Pruess and Narasimhan, 1985; Wu and Pruess, 1988). With this dual-continuum concept, Eqs. (1), (2), (3), and (4) can be used to describe multiphase flow along fractures, inside matrix blocks, as well as fracture-matrix-vug interaction. However, special attention needs to be paid to treating inter-porosity flow in the fracture-matrix-vug triple continua. Flow terms between fracture-matrix, fracture-vug, and vug-matrix connections are all evaluated using Eq. (6). However, the transmissivity of Eq. (9) will be evaluated differently for different types of inter-porosity flow. For fracture-matrix flow, γ_{ij} , is given by

$$\gamma_{FM} = \frac{A_{FM}k_M}{l_{FM}} \tag{12}$$

where A_{FM} is the total interfacial area between fractures (F) and the matrix (M) elements; k_M is the matrix absolute permeability; and l_{FM} is the characteristic distance for flow crossing fracture–matrix interfaces. For fracture–vug flow, γ_{ij} is defined as

$$\gamma_{FV} = \frac{A_{FV}k_V}{l_{FV}} \tag{13}$$

where A_{FV} is the total interfacial area between the fracture (F) and vugs (V) elements; l_{VM} is a characteristic distance for flow crossing vug–matrix interfaces; and k_V is the absolute vuggy permeability,

which is the actual permeability of small fractures that control flow between vugs and fractures (Fig. 1). Note that for the case in which vugs are isolated from fractures, as shown in Figs. 5 and 6, no fracture– vug flow terms need to be calculated, because they are indirectly connected through matrix. For vug–matrix flow, γ_{ii} is evaluated as

$$\gamma_{VM} = \frac{A_{VM}k_M}{l_{VM}} \tag{14}$$

where A_{VM} is the total interfacial area between the vug (V) and matrix (M) elements; and l_{VM} is a characteristic distance for flow crossing vug–matrix interfaces.

Table 1 summarizes several simple models for estimating characteristic distances in calculating inter-porosity flow within fractures, vugs, and the matrix, in which cases we have regular one-, two-, or three-dimensional large fracture networks, each with uniformly distributed small fractures connecting vugs or isolated vugs from fractures (Figs. 4, 5, and 6). The models in Table 1 rely on the quasisteady-state flow assumption of Warren and Root (1963) to derive characteristic distances for flow between fracture-matrix and (through small fractures) fracture-vug connections. Another condition for using the formulation in Table 1 is that fractures, vug, and matrix are all represented by only one grid block. In addition, the flow distance between large fractures (F) and vugs (V), when connected through small fractures, is taken to be half the characteristic length of the small fractures within a matrix block (Fig. 4). Furthermore, the interface areas between vugs and the matrix should include the contribution of small fractures for the case of Fig. 4. Interface areas



Fig. 10. Sensitivity runs on vug porosity for Conceptualization #2.



Fig. 12. Sensitivity runs on vug permeability for Conceptualization #2.



Fig. 13. Sensitivity runs on matrix permeability for Conceptualization #1.

between fractures and the matrix, and between fractures and vugs through connecting small fractures, should be treated using the geometry of the large fractures alone. This treatment implicitly defines the permeabilities of the fractures in a continuum sense, such that bulk connection areas are needed to calculate Darcy flow between the two fracture continua.

The MINC concept (Pruess, 1983; Pruess and Narasimhan, 1985) is extended to generate a triple-continuum grid, which is a key step in modeling flow through fractured-vuggy rock. We start with a primary or single-porous medium mesh that uses bulk volume of formation and layering data. Then, we use geometric information for the corresponding fractures and vugs within each formation subdomain or each finite-difference gridblock of the primary mesh. Fractures are lumped together into the fracture continuum, while vugs with or without small fractures are lumped together into the vuggy continuum. The rest is treated as the matrix continuum. Connection distances and interface areas are then calculated accordingly, e.g., using the relations listed in Table 1 and the geometric data of fractures. Once a proper mesh for a triple-continuum system is generated, fracture, vuggy, and matrix blocks are specified, separately, to represent fracture or matrix continua.

6. Handling non-Darcy's and other complicated flow

Flow regime may be more complicated within fracture-vuggy reservoirs or through faults or fault zones, because of (1) the high permeability of fractures and (2) large pores, such as vugs and largeraperture fractures, in vug or fault zones. Several common scenarios are discussed below:



Fig. 14. Sensitivity runs on matrix permeability for Conceptualization #2.



Fig. 15. Grid of the simulation model.

Non-Darcy Flow: In addition to Darcy flow, as described in Eqs. (4) or (7), non-Darcy flow may also occur between and among the multiple continua within fault zones. A general numerical approach for modeling non-Darcy flow (Wu, 2002) can be directly extended to the multiple-continuum model of this work for flow in fractured or fault zones. Volumetric flow rate (namely Darcy velocity for Darcy flow) for non-Darcy flow of each fluid may be described using the multiphase extension of the Forchheimer equation:

$$-(\nabla P_{\mathbf{w}} - \rho_{\mathbf{w}} \mathbf{g}) = \frac{\mu}{k} \mathbf{v}_{\mathbf{w}} + \beta \rho_{\mathbf{w}} \mathbf{v}_{\mathbf{w}} |\mathbf{v}_{\mathbf{w}}|$$
(15)

 β is the non-Darcy flow coefficient, intrinsic rock property, with a unit m^{-1} under non-Darcy flow condition.

Note that no mater what type the flow, i.e., Darcy's flow, non-Darcy's flow, or the following pipe-type flow, the discrete mass balance equation of (6) is always valid. For the case of non-Darcy's flow, the flow term ($F_{\beta,ij}$) in Eq. (7) along the connection (i, j), between elements i and j, is numerically replaced by (Wu, 2002),

$$F_{\beta,ij} = \frac{A_{ij}}{2(k\beta)_{ij+1/2}} \left\{ -\frac{1}{\lambda\beta} + \left[\left(\frac{1}{\lambda\beta} \right)^2 - \overline{\gamma}_{ij} \left(\psi_{\beta j} - \psi_{\beta i} \right) \right]^{1/2} \right\}$$
(16)

$$\overline{\mathbf{y}}_{ij} = \frac{4\left(k^2 \rho_{\mathsf{w}} \beta\right)_{ij+1/2}}{D_i + D_j} \tag{17}$$

Flow in Parallel-Wall Fracture or Tube: In general, flow along connecting paths of large-aperture fractures or vugs through narrow pores or fractures may be too fast or openings are too large to describe using Darcy's law. In particular, when these large-aperture fractures vuggy connections could be approximated as a single (or parallel) fracture or tube within fault zones, solutions of flow through a



Fig. 16. The relative permeability curves of rock matrix, fracture and vug continua.



Fig. 17. The capillary pressures curve of rock matrix.

parallel-wall, uniform fracture or Hagen–Poiseuille tube-flow solution (Bird et al., 1960) may be extended to describe such flow in Eq. (9):

$$\gamma_{ij} = \frac{wb^3}{12(D_i + D_j)} \tag{18}$$

for fracture-type connection and,

$$\gamma_{ij} = \frac{\pi r^4}{8 \left(D_i + D_j \right)} \tag{19}$$

for tube-type connection. In (18) and (19), b is fracture aperture, w is fracture width, and r is tube radius. Similarly, flow solutions for both laminar and turbulent flow through simple geometry of vug–vug connections can be used for flow between these vuggy connections.

7. Comprison with analytical solution

We have examined the numerical model using analytical solutions (Liu et al., 2003; Wu et al., 2004; Wu et al., 2007). The verification problem concerns typical single-phase transient flow towards a well that fully penetrates a radially infinite, horizontal, and uniformly vuggy fractured reservoir. Numerically, a radial reservoir ($r_e = 10,000$ m) of 20 m thick is represented by a 1-D (primary) grid of 2,100 intervals. A

Table 3				
The parameters	for the	two	dimensional	model

Parameter		Unit
Matrix porosity	0.10	
Fracture porosity	0.04	
Vuggy porosity	0.10	
Matrix absolute permeability	1.0×10^{-15}	m^2
Fracture absolute permeability	1.0×10^{-13}	m^2
Vuggy absolute permeability	1.0×10^{-12}	m^2
A_{VM}/l_{VM}	1000.0	т
A_{FV}/l_{FV}	11.0	т
A_{FM}/l_{FM}	0.5	т
Oil viscosity	2.17×10^{-3}	Pa·s
STC oil density	0.80	T/m^3
Water density	1.00	T/m^3
Oil compressibility	8.0×10 ⁻⁹	1/Pa
Water compressibility	1.0×10^{-10}	1/Pa
Water injection rate q	83.0 or 28.0	m^3/day



Fig. 18. Oil saturation after water injection of 10 days. (a) vug continua; (b) fracture continua; (c) rock matrix continua.

triple-continuum mesh is then generated using a 1-D vuggy–fracture– matrix conceptual model, consisting of a horizontal large–fracture plate network with a uniform disk–shaped matrix block. Uniform spherical vugs are contained inside the matrix and connected to fractures through small fractures. Fracture, vugs and matrix parameters are given in Table 2.

Fig. 7 compares numerical-modeling results with the analytical solution for a single-phase transient flow case (in terms of dimensionless variables). Excellent agreement exists between the two solutions, which provide verification of the numerical formation and its implementation. Note that there are very small differences at very early times (t_p <10 or 0.2 s) in the two solutions in Fig. 7, which may



Fig. 19. Oil and water rate curves in the case of injection rate $q = 83.0m^3/day$.

occur because the analytical solution, which is long-time asymptotic and similar to the Warren–Root solution, may not be valid for t_D <100.

8. Model application

We present two application examples of the proposed multiplecontinuum model for simulation flow in fractured vuggy reservoirs. The first example is a set of single-phase flow simulations for insight of flow behavior in fractured vuggy reservoirs, and the second application is a multi-phase flow problem.

8.1. Single-phase flow example

To investigate flow behavior and contribution of vug parameters in triple continuum vuggy fractured reservoirs, conceptualization #1 and #2 (Figs. 4 and 5) together with varied fracture, vug, and matrix parameters were numerically modeled. An infinite radial reservoir of 10 m thick is represented by a 1-D (primary) radial grid of 1,101 grids. Single phase reservoir simulation was used to generate sets of pressure transient data. Basecase fracture, vug and matrix parameters are also given in Table 2.

Comparison of simulation results between conceptualization#1 and #2 are shown in Fig. 8. It is clear that, fracture and vug connection (F-V) plays a major role on the behavior. Conceptualization#1 which has F-V connection generate triple porosity behavior, whereas simulation results from Conceptualization#2 which does not have F-V connection yield a double porosity response. To explain this observation, one has to consider well-to-reservoir connection. In these simulation cases, an injector well only connects to fracture continuum. Consequently fluid flow starts from fracture and propagates into other connected continuum. Conceptualization#1, the start of fluid flow in vug continuum takes place before matrix continuum because vug has larger permeability. In contrast, begin of fluid flow in vugs of Conceptualization #2 cannot be clearly identified, because it is dominated by flow in matrix as vugs do not directly connect to fracture. As a result, vugs are acting like additional storage or a source to matrix.

Figs. 9 and 10 show results from sensitivity on vug porosity. It is observed that vug porosity controls appearance of inter-porosity flow in Conceptualization #1's simulation results. In contrast, it does not affect flow behavior in Conceptualization #2. This observed behavior is caused by F-V connectivity. In Conceptualization #1 which has F-V connection, changes in vug's rock properties directly influence flow behavior in vug continuum. In contrary, Conceptualization #2, which does not have F-V connection, flow from vugs is controlled by matrix properties as such changing vug porosity does not affect the flow behavior.

Figs. 11 and 12 illustrate sensitivity results of on vug permeability of Conceptualization #1 and Conceptualization #2, respectively. Vug permeability in Conceptualization #1 controls time when flow in vugs starts. The larger the permeability is, the sooner the flow in vugs is taking place. However, changes of vug rock properties influences flow behavior only in Conceptualization #1 but not in Conceptualization #2. The same reason from the previous sensitivity is also applicable for this case.

Figs. 13 and 14 show results of sensitivity on matrix permeability of Conceptualization #1 and Conceptualization #2, respectively. Both figures indicate effect of changing matrix permeability. It is observed that matrix permeability controls the beginning of fluid flow in matrix. The larger the permeability is, the sooner the flow in matrix starts.

8.2. Multi-phase flow example

A two dimensional synthetic model is used to illustrate the use of proposed method for multi-phase flow. The reservoir is considered to be a triple-continuum system in the case of Conceptualization #1



Fig. 20. Oil and water rate curves in the case of injection rate $q = 28.0m^3/day$.

(Fig. 4) and with a volume of $100 \times 10 \times 20 \text{ m}^3$. The model grid is shown in Fig. 15 with $100 \times 1 \times 20$ grid blocks and has two flowing phases, namely oil and water. Boundaries of the model domain are noflow. The initial pressure is 60*MPa*. The injected fluid is water at a rate of *q*, located at Grid (1,1). The producing well is located at Grid (1,100) and the production pressure is 10*MPa*. The relative permeability curves of rock matrix, fracture and vugs continua are shown in Fig. 16. The capillary pressures of fracture and vugs continua are assumed to be negligible and that of rock matrix continua is shown in Fig. 17. The summarized input data are given in Table 3.

Two cases with different injection rates, $q = 83.0m^3/day$ and $q = 28.0m^3/day$, are simulated. The distributions of oil saturation in triple continua after water injection of 10 days ($q = 83.0m^3/day$) are shown in Fig. 18. The results of oil rate, water rate, and cumulative oil production for both cases are given in Figs. 19–21. Dual-continuum model is also simulated to compare with the triple-continuum model. When the dual-continuum model was set up, the porosity of matrix was increased by including the vug porosity, and the effective absolute permeability of matrix was obtained by harmonic mean of absolute permeability of matrix and vugs in the triple-continuum model. Thus the reservoir flow capacity and the storage capacity are



Fig. 21. Comparison of cumulative oil production.

identical. Compared with the triple-continuum model, water breakthrough in dual-continuum model occurs more early and significantly underestimates the cumulative oil production (Fig. 21).

9. Concluding remarks

Based on observation in several oilfields of fractured vuggy reservoirs, we present a physically based conceptual and numerical model for simulating single-phase and multiphase flow in fractured vuggy rock using a triple- and multiple-continuum medium approach. The proposed multicontinuum concept is a natural extension of the classic double-porosity model, with the fracture continuum responsible for conducting global flow, while vuggy and matrix continua, providing storage space, are locally connected as well as interacting with flow through globally connecting fractures. In particular, the proposed conceptual model considers fractured vuggy rock as a triple- or multiple-continuum medium, consisting of (1) highly permeable fractures, (2) low-permeability rock matrix, and (3) various-sized vugs. In addition, our model formulation includes non-Darcy flow, using the multiphase extension of the Forchheimer equation, and the flow according to parallel-wall fracture and tube models.

The proposed conceptual model has been implemented into a general numerical reservoir simulator using a control-volume, integrated finitedifference approach, which can be used to simulate single-phase as well as multiple-phase flow in 1-D, 2-D and 3-D fractured-vuggy reservoirs. We provide a verification example for the numerical scheme by comparing numerical results against an analytical solution for single-phase flow. As application examples, we apply the proposed mathematical model (1) to perform sensitivity studies of transient triple-porosity flow behavior and parameter effects and (2) to simulate two-phase flow, waterflooding, showing very different results between triple-continuum and double-porosity models.

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