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Study on fluid flow in nonlinear elastic porous media: Experimental and modeling approaches

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ABSTRACT

A theoretical model for describing the changes in the porosity and permeability with fluid pressure in elastic deformed porous media was presented firstly. Secondly, the changes in porosity and permeability of sandstones were studied by experimental approach, and the exponential correlations between porosity, permeability and fluid pressure in porous media were obtained by regression based on these experimental data. Thirdly, based on the theory of nonlinear deformation of porous media and the continua theory on porous media and fluids, a fluid filtration mathematical model considering nonlinear elastic deformation of porous media and seveloped. To get insights into the performances of fluid flow in the media, exact analytical solutions of the nonlinear model were obtained by traveling-wave transmission. The main parameters were input to analyze the difference of the solutions and their effects on the characteristics of the fluid pressure distribution with space and time. It is found that the effect of nonlinear elastic deformation of fluid pressure in porous media at one-dimensional case. However, the distribution of fluid pressure in porous media is linear only when the pressure-dependent factors of pressure rock permeability, porosity, fluid density and viscosity satisfy a special correlation.

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1. Introduction

The fluid flow in soil, sedimentary sand formation underground or petroleum reservoir is a common phenomenon. The material such as soil, sedimentary sand formation and petroleum reservoir can be treated as a continuous porous media in space to describe in mathematical model. The effects of stress and fluid pressure on porous media are often neglected or regarded approximately as slightly compressible, therefore porosity is often regarded as a linear function of fluid pressure in porous media and permeability is approximately looked as constants. However, for some scenarios such as unconsolidated sand beds, abnormal highpressured oil formation or large deformation of porous media for pore pressure dropped greatly, the change in porosity is not a linear function of fluid pressure in porous media, and permeability can't keep a constant yet. Fluid pressure depletion in porous media leads to the decrease in the porosity and permeability, which not only leads to formation damage, production decline for oil payzone and surface subsidence, but also impacts surface facilities (Kristiansen, 2009). This phenomenon has been noticed for many oil reservoir engineers and petro-geologists. Schatz and Carroll (1982) thought long-range predictions of reservoir

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performance should take account of stress sensitivity, as porosity and permeability can change significantly as the effective stress is increased by pore pressure reduction during production.

Gorbunov (1973, 1987) have done research systematically on this type of oil field through the theoretical analysis approach. They presented an approximate correlation between fluid pressure and permeability, but they failed to give the experimental evidence supporting the correlation. Ruistuen and Teufe (1999) studied the stress-path-dependent nonlinear behavior of weakly cemented sandstone by experiments. Ju and Luan (1999) analyzed the factors to cause the deformation of unconsolidated sandstone and regarded porosity and permeability as pressure-dependant variables. A percolation mathematical model in reversible deformed formation was given in this paper. The advantage of the model is assuming that the porosity, permeability, fluid viscosity, and rock compressibility are pressure dependent. The disadvantage lies in the facts that some coefficients in the model should be identified by experiments and only numerical solution was obtained for over complex non-linear mathematical problem.

Thallak et al. (1993) studied mechanical damage linked to stress and deformation changes associated with drilling and well completion operations and associated permeability changes. Schutjens et al. (2001) studied the porosity and permeability reduction in sandstone reservoir data and presented a model for elasticity-dominated deformation. They also gave the correlations of porosity and permeability reduction and the difference of mean effective stress and pore pressure by

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experiments. However, the correlations are not easily coupled with fluid flow in porous media.

At present, the solution for the mathematical model describing fluid flow in deformable porous media often resorts to numerical methods (Cuisiat et al., 1998; Caers, 2003; Nelson, 2009) such as finite difference and finite element methods (Minkoff et al., 2003). Though it is easy to obtain the solution of the kind of non-linear equation, unfortunately, the numerical method will induce pseudo-phenomena because of the numerical diffusion and oscillation, which will influence the accuracy of the solution and even change the physical property of the solution more or less. Analytical solution (comparing to numerical solution) has some advantages in precision and physical meaning. Gorbunov (1973) gave the form of the solution of axisymmetric filtration for the case of the work of a single well with a constant output in an infinite stratum. Aadnoy (1987) gave a radial flow equation assuming laminar flow, which was used for the pressure drop in the rock. Aadnoy and Finjord (1996) derived a solution of radial flow equation, which based on the Boltzmann transformation and a first-order perturbation (by neglecting the "second-order" term) method. Three references above addressed radial flow equation in infinite spaces. The objective of this work is to obtain an exact analytical solution of filtration equation within one dimensional space and finite boundary conditions during fluid percolating in reversible elastic porous media.

2. Theoretical analysis on the deformation of porous media

As we know, sandstone is one kind of porous media that includes sand gravels and pores. The pores in sandstone are often saturated with fluids. The porous media contact with impermeable rocks at the top and bottom and the vertical profiles of porous media saturated with fluid at initial pressure and current pressure are shown in Fig. 1. According to Biot's theory (Biot, 1941), an approximate expression of average effective stress loaded on gravels can be expressed as

$$\sigma_{\rm Z}^e = \sigma_{\rm Z} - \delta p \tag{1}$$

where σ_Z is the total stress loaded on the top of the porous media, σ_Z^e is the effective stress in Z direction, p is the average pressure of fluid in the pore spaces, and δ is the Kroneker coefficient.

Similarly, the effective stresses loaded on the porous media for X, Y direction can be written as

$$\sigma_{\rm X}^e = \sigma_{\rm X} - \delta p,\tag{2}$$

$$\sigma_{\rm Y}^e = \sigma_{\rm Y} - \delta p. \tag{3}$$

The average effective stress loaded on a cubic unit of the porous media can be obtained from Eqs. (1) to (3).

$$\sigma_{ave}^{e} = \frac{\sigma_{\chi} + \sigma_{\gamma} + \sigma_{Z} - 3\delta p}{3}.$$
 (4)

Let $\delta = 1$ and we define

$$\frac{\sigma_X + \sigma_Y + \sigma_Z}{3} = \sigma_{ave},\tag{5}$$

then the average effective stress can be written as

$$\sigma_{ave}^e = \sigma_{ave} - p \tag{6}$$

The sandstone such as oil reservoir formation is buried underground with depth from hundred to thousand meters. During the fluid drainage out of the porous media, σ_{ave} can be almost kept as a constant and p will decline for the decrease in fluid volume in porous media, so it leads to the increase in average effective stress, σ_{ave}^e . If the fluid pressure (p) increases up to σ_{ave} , then $\sigma_{ave}^e = 0$ and we denote $\sigma_{ave} = p_0$. Eq. (6) is rewritten as

$$\sigma_{ave}^e = p_0 - p. \tag{7}$$

According to Eqs. (4)–(7), when the fluid pressure declines as the fluid is drawn out, it leads to the increase in the average effective stress loaded on rock framework. Therefore, the sands are compacted and pore space decreases (see Fig.1A and B). From the analysis above, the effective stress loaded on rock framework is a key parameter to describe formation deformation.

For the deformable porous media, porosity and permeability are the functions of effective stress loaded on it, that is

$$\phi = f_1(\sigma_{ave}^e),\tag{8}$$

$$K = f_2(\sigma_{ave}^e),\tag{9}$$

$$\phi = f_1(p_0 - p), \tag{10}$$

$$K = f_2(p_0 - p). (11)$$

3. Experimental study on porosity and permeability of elastic porous media

In order to estimate hydrocarbon reserves and predict fluid flow performances as accurately as possible, a reliable estimate of the porosity and permeability at the conditions loaded on stresses is



Fig. 1. Sandstone deformation induced by pressure decrease.

Table 1					
Experimental data of porosit	y and	permeability	of core	sam	oles.

	C2		C5		C7		C8	
$p-p_0$	Porosity	Permeability	Porosity	Permeability	Porosity	Permeability	Porosity	Permeability
10 ⁶ Pa	%	$10^{-3}\mu m^2$						
0.00	22.21	828.00	21.00	410.00	20.99	424.00	20.14	101.00
-5.00	22.06	813.00	20.87	404.00	20.89	418.00	20.00	99.20
-10.00	21.92	799.00	20.78	400.00	20.82	413.00	19.90	97.80
-15.00	21.81	786.00	20.72	396.00	20.76	408.00	19.82	96.60
-20.00	21.71	776.00	20.66	392.00	20.72	404.00	19.76	95.60
-25.00	21.62	768.00	20.61	388.00	20.68	400.00	19.71	94.70
-30.00	21.55	760.00	20.58	385.00	20.66	396.00	19.68	94.00
-35.00	21.50	753.00	20.56	383.00	20.64	393.00	19.64	93.40
-40.00	21.46	748.00	20.55	382.00	20.63	391.00	19.63	93.00

required. Stonecore samples provide a source for direct measurement of porosity and permeability. In principle, the stonecore should be reloaded up to the in-situ stress state in the experimental process. A steel core cell with a rubber seal holds the core and a high-pressure pump provides enough pressure to load on the core. A precision pressure gage can show the strain stress loaded on the core. The procedures to determine the porosity and permeability of sandstone cores were stated in the published works (Archer and Wall, 1986; Qin and Li, 2001).

Four sandstone core samples obtained from the oil formation of H. Z.J. Oil Field and the experimental data of porosity and permeability affected by strain stresses are shown Table 1. The porosities are from 20.14% to 22.21% and permeabilities are from 101.00 to $828.00 \times 10^{-3} \mu \text{m}^2$ of the four cores (C2, C5, C7 and C8) at the condition of $p - p_0 = 0.0$. It indicates that all the porosities and permeabilities of the cores decline with the fluid pressure (p) depletion. Let $\phi = \phi_0$ and $K = K_0$ when $p = p_0$. The trend of the changes in porosities and



Fig. 2. The relations of the porosity ratios with pressure difference $(p - p_0)$.



Fig. 3. The relations of the permeability ratios with pressure difference $(p - p_0)$.

permeabilities with fluid pressure depletion in porous media was obtained (Figs. 2 and 3).

The two sandstone core samples (C5 and C7) are drilled from the oil formation of H.Z.J. Oil Field and the depth of oil formation is from 2201.84 to 2436.37 m from the well tops. The fluid pressures (p) in the formation decrease from 24.5 MPa to 7.4 MPa, and $p - p_0$ changes from -15.5 MPa to -32.6 MPa. The relations between the ratios of porosities and permeabilities with $p - p_0$ are shown in Figs. 4 and 5 and the ratios with $p - p_0$ show a good exponential correlation.

$$\frac{\Phi(p)}{\Phi_0} = A_{\phi} e^{\alpha_{\phi}(p-p_0)},\tag{12}$$

$$\frac{K(p)}{K_0} = A_K e^{\alpha_K (p-p_0)}.$$
(13)



Fig. 4. The relations of the porosity ratios with pressure difference $(p-p_0)$ during fluid flow in sand formation.



Fig. 5. The relations of the permeability ratios with pressure difference $(p-p_0)$ during fluid flow in sand formation.

4. Model description

The sanding mathematical model was developed under the following assumptions:

- (1) The model is assumed that its single-phase flow is isothermal.
- (2) Consider the compressibility of rock and fluids.
- (3) The flows of fluids in porous media follow Darcy's law.
- (4) The fluid is Newtonian fluid.
- (5) Chemical reactions are not considered.

The governing equation of transient fluid flow in deformable porous media is

$$\frac{\partial(\phi\rho)}{\partial t} = \frac{\partial}{\partial x} \left(\frac{K_x \rho}{\mu} \frac{\partial p}{\partial x} \right). \tag{14}$$

According to references (Gorbunov, 1973; Zhang and Lei, 1998) the density and viscosity of fluid with the pressure of the fluid in porous media are written as

$$\rho(p) = \rho_0 e^{\alpha_p (p - p_0)},\tag{15}$$

$$\mu(p) = \mu_0 e^{\alpha_{\mu}(p-p_0)}.$$
(16)

We suppose the initial condition $p = p_0$ and inner and outer boundary conditions $p = p_{in}$ and $p = p_{out}$ respectively.

The initial and boundary conditions and Eqs. (12)–(16) are the mathematical model to describe fluid flow in elastic porous media. The model considered the changes not only in porosity and permeability but also in fluid density and viscosity.

5. Analytical solution

Combine Eqs. (12), (13), (15) and (16) with Eq. (14) and let

$$\alpha = \alpha_K + \alpha_\rho - \alpha_\mu, \tag{17}$$

$$\beta = \alpha_{\phi} + \alpha_{\rho},\tag{18}$$

$$\varphi = e^{\beta(p-p_0)},\tag{19}$$

and

 $D^2 = K_0 / (\alpha \varphi_0 \mu_0), \ \nu = \frac{\alpha}{\beta}, \tag{20}$

then

$$\frac{\partial \varphi}{\partial t} = D^2 \nabla^2 \varphi \nu. \tag{21}$$

Further let

$$u = \varphi v, \tag{22}$$

$$\chi = \nu D^2, \tag{23}$$

then

$$\frac{\partial u}{\partial t} = \chi u^{1 - \frac{1}{\nu}} \nabla^2 u, \tag{24}$$

where

 $u = e^{\alpha(p-p_0)}.$ (25)

Eq. (24) is a parabolic partial equation. It becomes a linear parabolic partial equation when ν is equal to 1.0.

$$\frac{\partial u}{\partial t} = \chi \nabla^2 u \tag{26}$$

The analytical solution of Eq. (30) has been provided in the paper (Gorbunov, 1973). However, when $\nu \neq 1$, Eq. (24) becomes too complicated to be obtained an analytical solution. Gorbunov (1973) gave the form of the solution of Eq. (21) for the case of the work of a single well with a constant output in an infinite stratum. The solution is obtained at the case of radial flow and an infinite place. He also obtained the analytical solution at steady-state conditions and limited case for linear filtration. For the general case, they carry out a numerical solution using an electronic computer. Aadnoy and Finjord (1996) gave an analytical solution for the transient line sink for oil reservoirs. The solution of radial flow equation is based on the Boltzmann transformation and a first-order perturbation (by neglecting the "second-order" term) method. Almost all boundary conditions of physical flow filtration in porous media in oil formation are finite, therefore the focus of this paper is to obtain the solution of Eq. (24) with finite boundary conditions. The following section demonstrates the procedure how to obtain an analytical solution of Eq. (24) by the transmission of traveling wave. Let

$$1 - \frac{1}{\nu} = \theta, \tag{27}$$

Eq. (24) is expressed as

$$\frac{\partial u}{\partial t} = \chi u^{\theta} \frac{\partial^2 u}{\partial x^2}.$$
(28)

Let $u = u(\xi)$ and $\xi = x - ct$, then

$$-c\frac{du}{d\xi} = \chi u^{\theta} \frac{d}{d\xi} \left(\frac{du}{d\xi}\right).$$
⁽²⁹⁾

then the integral form of Eq. (29) is given by:

$$\frac{du}{d\xi} = C_1 e^{-\frac{c}{\chi u^0}\xi}.$$
(30)

Let

$$\Omega = \frac{\iota}{\chi},\tag{31}$$

then

$$\frac{du}{d\xi} = C_1 e^{-\frac{\Omega}{u^0}\xi} = C_1 \frac{1}{e^{u^0}}.$$
(32)

Let

$$\sigma = e^{\frac{\Omega}{u^0}\xi},\tag{33}$$

then

$$\xi = \frac{1}{\Omega} u^{\theta} \left(\ln u^{\theta - 1} + \ln \frac{C_1}{\Omega} \right). \tag{34}$$

Substituting Eq. (25) into Eq. (34),

$$(x-ct) = \frac{1}{\Omega} \left(e^{\alpha(p-p_0)} \right)^{\theta} \left(\ln \left(e^{\alpha(p-p_0)} \right)^{\theta-1} + \ln \frac{C_1}{\Omega} \right).$$
(35)

If the boundary condition satisfies

$$\xi = (x - ct) = 0.0, \quad p = p_0, \tag{36}$$

then

$$C_1 = \Omega, \tag{37}$$

Eq. (35) becomes into the following expression

$$(x-ct) = \frac{1}{\Omega} \alpha(\theta-1)(p-p_0) e^{\alpha\theta(p-p_0)}.$$
(38)

Combining Eqs. (19), (20), (23), (26) and (33), then Eq. (38) can be written as

$$(x-ct) = \frac{\frac{\alpha}{\beta} \frac{A_K K_0}{\alpha A_{\phi} \phi_0 \mu_0}}{c} \alpha(\theta-1)(p-p_0) e^{\alpha \left(1-\frac{\beta}{\alpha}\right)(p-p_0)}.$$
(39)

Combining Eqs. (17) and (18) with Eq. (39) then

$$(x-ct) = \frac{K_0}{\beta \phi_0 \mu_0 c} (\alpha) \left(-\frac{\beta}{\alpha} \right) (p-p_0) e^{\left(\alpha_K + \alpha_p - \alpha_\mu\right) \left(1 - \frac{\alpha_\phi + \alpha_p}{\alpha_K + \alpha_p - \alpha_\mu}\right)(p-p_0)},$$
(40)

$$(x-ct) = -\frac{K_0}{\phi_0 \mu_0 c} (p-p_0) e^{(\alpha_K - \alpha_\mu - \alpha_\phi)(p-p_0)}.$$
(41)

If $\nu = \frac{\alpha}{\beta}$ is equal to 1.0 in (Eq. (40)) or $\alpha_K = \alpha_\mu = \alpha_\varphi$ is equal to 0.0 (ignoring the elasticity of porous media and compressibility of fluid), and let

$$\frac{1}{R} = -\frac{K_0}{\phi_0 \mu_0 c},\tag{42}$$

then

$$p = p_0 + R(x - ct) = p_0 + R\xi.$$
(43)

Eq. (43) shows the relation between fluid pressures, p and space, ξ are linear if the elasticity of porous media and compressibility of fluid is ignored or ν keeps 1.0.

Eq. (41) is the traveling-wave solution of Eq. (28). Though the solution is implicit in form, it gives the relations between fluid pressure in deformed porous media with space x and time t. We can get an insight into the performances of fluid flow in nonlinear elastic porous media by the solution.

6. The comparisons of previous solution and the solution in this paper

Aadnoy and Finjord (1996) gave an analytical solution of radial flow equation for the transient line sink for oil reservoirs. The solution is different from our solution in boundary conditions and flow domain. To validate the solution in this paper, we compared our solutions to Gorbunov's solutions at two cases (1) incompressible liquid in a nondeformable porous medium; and (2) steady-state conditions and completely reversible deformations. The solutions obtained in this paper are the same as Gorbunov's solutions (Figs. 6 and 7). The comparisons validate the solution obtained by this work.

7. The analysis for the effects of main parameters on fluid pressure in porous media

There are 6 parameters (K_0 , ϕ_0 , μ_0 , α_K , α_{μ} , and α_{ϕ}) except for the speed of traveling wave in Eq. (41). The following section is used to



Fig. 6. Comparison of the solutions for incompressible liquid in a nondeformable porous medium.



Fig. 7. Comparison of the solutions for steady-state conditions and completely reversible deformations. 1: $K_0 = 0.5 \times 10^{-12} \text{ m}^2$; 2: $K_0 = 1.0 \times 10^{-12} \text{ m}^2$; and 3: $K_0 = 2.0 \times 10^{-12} \text{ m}^2$; c = 0.10 m/s; $\phi_0 = 0.25$; $\mu_0 = 1.0 \times 10^{-3} \text{ Pa}^{-3}$; $\alpha_K = 4.0 \times 10^{-8} \text{ Pa}^{-1}$; $\alpha_\mu = 7.0 \times 10^{-9} \text{ Pa}^{-1}$; and $\alpha_\phi = 1.0 \times 10^{-8} \text{ Pa}^{-1}$.

demonstrate how that these parameters influence pressure distribution. We let the pressure satisfy the following boundary condition.

$$\xi = \xi_{in} = 0.0, \ p_{in} = p_0 = 1.0 \times 10^7 Pa, \tag{44}$$

$$\xi = \xi_{out}, \ p_{out} = 4.0 \times 10^7 Pa.$$
 (45)

Define dimensionless pressure \overline{p} and space $\overline{\xi}$ as

$$\overline{p} = \frac{p}{p_{out}},\tag{46}$$



Fig. 8. The effect of the initial permeability on pressure distribution. 1: $K_0 = 0.5 \times 10^{-12} m^2$; 2: $K_0 = 1.0 \times 10^{-12} m^2$; 3: $K_0 = 2.0 \times 10^{-12} m^2$; c = 0.10 m/s; $\phi_0 = 0.25$; $\mu_0 = 1.0 \times 10^{-3} Pa^{-1}$; $\alpha_k = 4.0 \times 10^{-8} Pa^{-1}$; $\alpha_{\mu} = 7.0 \times 10^{-9} Pa^{-1}$; $\alpha_{\phi} = 1.0 \times 10^{-8} Pa^{-1}$.



Fig. 9. The effect of the initial porosity on pressure distribution. 1: $\phi_0 = 0.1$; 2: $\phi_0 = 0.2$; 3: $\phi_0 = 0.3$; c = 0.10 m/s; $K_0 = 1.0 \times 10^{-12} m^2$; $\mu_0 = 1.0 \times 10^{-3} Pa \cdot s$; $\alpha_K = 4.0 \times 10^{-8} Pa^{-1}$; $\alpha_\mu = 7.0 \times 10^{-9} Pa^{-1}$; $\alpha_\phi = 1.0 \times 10^{-8} Pa^{-1}$.



Fig. 10. The effect of the initial viscosity on pressure distribution. 1: $\mu_0 = 1.0 \times 10^{-3} Pa \cdot s$ 2: $\mu_0 = 2.0 \times 10^{-3} Pa \cdot s$; 3: $\mu_0 = 3.0 \times 10^{-3} Pa \cdot s$; c = 0.10 m/s; $K_0 = 1.0 \times 10^{-12} m^2$; $\phi_0 = 0.3$; $\alpha_K = 4.0 \times 10^{-8} Pa^{-1}$; $\alpha_\mu = 7.0 \times 10^{-9} Pa^{-1}$; $\alpha_\phi = 1.0 \times 10^{-8} Pa^{-1}$.

$$\overline{\xi} = \frac{\xi}{\xi_{out}}.$$
(47)

Fig. 8 shows the dimensionless pressure curve becomes steep with the decrease in initial permeability of porous media if the other parameters keep constants. The phenomenon results from the fact that the decrease in permeability of porous media leads to the increase in flow resistance of fluid in interconnected networks.

Figs. 9 and 10 show the dimensionless pressure curves become steep with the increases in initial porosity of porous media and viscosity of the fluid in porous media if the other parameters keep constants.



Fig. 11. The effect of the modifying factor of porosity on pressure distribution. 1: $\alpha_{\phi} = 1.0 \times 10^{-8} P a^{-1}$; 2: $\alpha_{\phi} = 2.0 \times 10^{-8} P a^{-1}$; 3: $\alpha_{\phi} = 3.0 \times 10^{-8} P a^{-1}$; c= 0.10 m/s; $K_0 = 1.0 \times 10^{-12} m^2$; $\mu_0 = 1.0 \times 10^{-3} P a \cdot s$; $\phi_0 = 0.3$; $\alpha_K = 4.0 \times 10^{-8} P a^{-1}$; $\alpha_{\mu} = 7.0 \times 10^{-9} P a^{-1}$.



Fig. 12. The effect of the modifying factor of permeability on pressure distribution. 1: $\alpha_{\rm K} = 1.0 \times 10^{-8} P a^{-1}$; 2: $\alpha_{\rm K} = 2.0 \times 10^{-8} P a^{-1}$; 3: $\alpha_{\rm K} = 3.0 \times 10^{-8} P a^{-1}$; c = 0.10 m/s; $K_0 = 1.0 \times 10^{-12} m^2$; $\phi_0 = 0.3$; $\mu_0 = 1.0 \times 10^{-3} P a \cdot s$; $\alpha_{\phi} = 2.0 \times 10^{-9} P a^{-1}$; $\alpha_{\mu} = 7.0 \times 10^{-9} P a^{-1}$.



Fig. 13. The effect of the modifying factor of viscosity on pressure distribution. 1: $\alpha_{\mu} = 0.5 \times 10^{-8} P a^{-1}$; 2: $\alpha_{\mu} = 1.0 \times 10^{-8} P a^{-1}$; 3: $\alpha_{\mu} = 1.5 \times 10^{-8} P a^{-1}$; c = 0.10 m/s; $K_0 = 1.0 \times 10^{-12} m^2$; $\phi_0 = 0.3$; $\mu_0 = 1.0 \times 10^{-3} P a \cdot s$; $\alpha_{\phi} = 2.0 \times 10^{-9} P a^{-1}$; $\alpha_K = 1.0 \times 10^{-8} P a^{-1}$.

Fig. 11 indicates the dimensionless pressure curve becomes steep with the increase in the modifying factor of porosity of porous media if the other parameters keep constants. The dimensionless pressure curve becomes flat with the increase in the modifying factor of permeability (see Fig. 12). The increase in the modifying factor of permeability leads to the reduction in permeability and higher flow resistance.

Fig. 13 indicates the dimensionless pressure curve becomes steep with the increase in the modifying factor of viscosity of the fluid in porous media if the other parameters keep constants. Fig. 14 gives the effect of the factor ($v = \alpha/\beta$) on pressure distribution. When *v* is equal to 1.0, the relation between dimensionless pressure and space is linear (Curve 3). The curves move to the left side of the 3rd curve if *v* is less than 1.0, and vice versa if *v* is great than 1.0.



Fig. 14. The effect of the factor $(v = \alpha/\beta)$ on pressure distribution. 1: v = 0.50; 2: v = 0.75; 3: v = 1.0; 4: v = 1.50; 5: v = 2.00; c = 0.10m/s; $K_0 = 1.0 \times 10^{-12}m^2$; $\phi_0 = 0.3$.

μ

8. Conclusion

- (1) Based on Biot's theory on effective stress loaded on porous media, a theoretical model for describing the changes in the porosity and permeability with fluid pressure in elastic deformed porous media was presented.
- (2) The exponential correlations between porosity, permeability and fluid pressure in porous media were obtained by nonlinear regression based on the experimental data obtained by experimental approach in this work.
- (3) Combining the theory of nonlinear elastic deformation of porous media and the continua theory on porous media and fluids, the authors developed a mathematical model fully considering nonlinear elastic deformation of porous media and fluid percolating in the media.
- (4) An exact analytical solution of the nonlinear model was obtained by the transmission of traveling wave and it provides an access to get more insights for the characteristics of pressure distribution during fluid flow in this type of porous media than by numerical solution because of numerical dispersion.
- (5) The solutions obtained in this paper are validated by comparison to Gorbunov's solutions. The effects of main parameters $(K_0, \phi_0, \mu_0, \alpha_K, \alpha_\mu, \text{ and } \alpha_\phi)$ on pressure distribution of fluid in porous media during percolation were demonstrated the analytical solution.
- (6) An important discovery is that the relation between dimensionless pressure and $\overline{\xi}$ is linear when not only ignoring the elasticity of porous media and compressibility of fluid, but also keeping the ratio of α to β equal to 1.0. Otherwise the relations between dimensionless pressure and $\overline{\xi}$ become into nonlinear curves.
- (7) Both the compressibilities of rock and fluid and the pressuredependent viscosity of fluid are considered in the developed model, of which importance lies in the fact that the model is more precise than the previous model treating porosity, permeability and viscosity as constants. The analytical solution provides an exact access to insights of the effects of the all pressure-dependent variables on pressure distribution in fluid flowing direction.

Nomenclature

- A_{ϕ} coefficient for calculating porosity
- A_K coefficient for calculating permeability
- traveling-wave propaganda speed, m/s С
- C1 a constant
- D^2 $D^2 = K_0 / (\alpha \phi_0 \mu_0)$, parameter
- Κ transient absolute permeability of porous media, µm²
- pressure, Pa р
- \overline{p} dimensionless pressure
- R_K the ratio of transient permeability to initial permeability
- R_φ the ratio of transient porosity to initial porosity
- t time, s
- a new variable induced for solution и
- the ratio of α to β ν
- Space, m х
- modifying factor of permeability, Pa⁻¹ α_{K}
- modifying factor of porosity, Pa⁻ α_{ϕ}
- modifying factor of viscosity, Pa⁻¹ α_{μ}
- modifying factor of density, Pa^{-1} α_{ρ}
- modifying factor group, is equal to $\alpha_{K} + \alpha_{\rho} \alpha_{\mu}$ Pa⁻¹ α
- β modifying factor group, is equal to $\alpha_{\phi} + \alpha_{\rho}$, Pa⁻
- is equal to νD^2 χ
- transient porosity of porous media φ

- viscosity of fluid, Pa·s
- φ a new variable induced for solution
- θ a new variable induced for solution
- ρ density of fluid, mPa·s
- σ^{e} stress, Pa
- τ gradient of *u*
- a new variable induced for solution
- <u>ξ</u> ξ a dimensionless space variable

Ω $\Omega = c/\chi$

Subscripts

0 = initial value0

in = inner boundary in

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