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ABSTRACT

Non-Darcy porous media flow has been traditionally handled using the Forchheimer equation. However, recent experimental studies have shown that the Forchheimer model is unable to fit laboratory results at high flow rates. On the other hand, the non-Darcy flow model, proposed by Barree and Conway, is capable of describing the entire range of relationships between flow rate and potential gradient from low- to high-flow rates through proppant packs. In this paper, we present a numerical model by incorporating the Barree and Conway model into a general-purpose reservoir simulator for modeling single-phase and multiphase non-Darcy flow in porous and fractured media. The numerical formulation is based on the TOUGH2 methodology, i.e., spatial integral-finitedifference discretization, leading to an unstructured grid, followed by time discretization carried out with a backward, first-order, finite-difference method. The final discrete nonlinear equations are handled fully implicitly by Newton iteration. In the numerical approach, flow through fractured rock is handled using a general multi-continuum approach, applicable to both continuum and discrete fracture conceptual models. In an effort for model validation, we use analytical solutions to verify our numerical model results for both single-phase and multiphase non-Darcy flow. In addition, the numerical model is applied for well testing analysis of transient non-Darcy flow toward a well.

1. INTRODUCTION

Darcy's law, describing a linear relationship between volumetric flow rate (or Darcy velocity) and pressure (or potential) gradient, has been the fundamental principle in analyzing flow processes in reservoirs. Darcy's Law has been used exclusively in reservoir studies, however, there is considerable evidence that high-velocity non-Darcy flow occurs in oil and gas reservoirs, such as for flow in the formation near oil or gas production, groundwater pumping, and liquid waste injection wells. Effects of non-Darcy or high-velocity flow regimes in reservoirs have been observed and investigated for decades (e.g., Tek et al., 1962; Scheidegger, 1972; Katz and Lee, 1990; Wu, 2002). Studies performed on non-Darcy flow in porous media in the early time have focused mostly on single-phase-flow conditions in petroleum reservoir engineering (Tek et al., 1962; Swift and Kiel, 1962; Lee et al., 1987). Some investigations have been conducted for non-Darcy flow in fractured reservoirs (Skjetne et al., 1999) and for non-Darcy flow into highly permeable fractured wells (e.g., Guppy et al., 1981, 1982). Other studies have concentrated on finding and validating correlations of non-Darcy flow coefficients (e.g., Liu et al., 1995), and modeling efforts (e.g., Wu, 2002).

In analyzing non-Darcy flow through porous media, the Forchheimer equation (1901) has been exclusively used to describe such nonlinear flow behavior, and it has been extended to multiphase flow conditions (Evans et al., 1987; Evans and Evans, 1988; Liu et al., 1995; Wu, 2001 and 2002). In recent developments, laboratory studies and analyses have shown that the Barree and Conway model is able to describe the entire range of relationships between flow rate and potential gradient from low- to high-flow rates through porous media, including those in transitional zones (Barree and Conway, 2004 and 2007; Lopez, 2007).

This paper summarizes our continual study of single-phase and multiphase non-Darcy flow in reservoirs according to the Barree and Conway model (Lai et al., 2009; Wu et al., 2009; Wu et al. 2011). The objective of this study is to present a mathematical method for quantitative analysis of single-phase multiphase non-Darcy flow through heterogeneous porous and fractured rocks, based on the Barree and Conway's model. In this paper, we discuss a mathematical and numerical model by incorporating the Barree and Conway model into a general-purpose reservoir simulator for modeling single-phase and multiphase non-Darcy flow in porous and fractured media. The numerical formulation is based on the TOUGH2 methodology, i.e., spatial integral-finite-difference discretization, leading to an unstructured grid, followed by time discretization carried out with a backward, first-order, finite-difference method. The final discrete nonlinear equations are handled fully implicitly by Newton iteration. In the numerical approach, flow through fractured rock is handled using a general multi-continuum approach, applicable to both continuum and discrete fracture conceptual models. As a demonstration of model validation and application efforts, we use analytical solutions to verify our numerical model results for both single-phase and multiphase non-Darcy flow as well as generate type-curves for transient well testing analysis of non-Darcy flow toward a well.

2. BARREE-CONWAY MODEL AND LABORATORY RESULTS

Barree and Conway (2004) proposed a new model for non-linear flow in porous media, which does not rely on the assumptions of a constant permeability or a constant β (non-Darcy flow coefficient in the *Forchheimer* model). In their model, Darcy's Law is still assumed to apply, but the apparent permeability, as a general non-linear function of flow rate, is introduced to replace the constant, intrinsic permeability of the Darcy's law for one-dimensional linear flow as,

$$v = -\frac{k_{app}}{\mu} \frac{\partial P}{\partial L} \tag{1}$$

where v is superficial or Darcy's velocity; the apparent permeability; k_{app} , is defined as,

$$k_{app} = k_{\min} + \frac{(k_d - k_{\min})}{(1 + R_e^F)^E}$$
(2)

and Reynolds number is defined as,

$$R_e = \frac{\rho v}{\mu \tau} \tag{3}$$

The Barree and Conway model, Equations (1) and (2), is a physically based correlation. It provides a single equation to describe the entire range of flow velocities versus pressure or potential gradient from low flow-rate, Darcy to high flow-rate, non-Darcy flow regimes. At low flow rates, the Barree and Conway model collapses into Darcy's law with a constant permeability, k_{d} , and it converges to the Forchheimer analysis for the intermediate flow rate. The Barree and Conway model provides a plateau area at high rates, which indicates there is a constant permeability (or minimum permeability), consistent with laboratory and finite element modeling results. In particular, considerable experimental validation efforts have been completed using proppant packs and nitrogen gas non-Darcy flow apparatus (Lopez, 2007; Lai et al., 2009). Their experimental data are analyzed using a regression method for both Forchheimer and Barree and Conway models and some example results are shown in Figure 1 (Lai et al., 2009). As shown in Figure 1, the experimental data agree extremely well with the Barree and Conway model across the entire flow velocity range from low to high gas flow rates. The Forchheimer quadratic correlation overestimates the associated pressure drop, while the Forchheimer cubic correlation underestimates the pressure drop at high gas flow rates. All sample data taken to date show similar agreement with the Barree and Conway equation across the observed wide flow spectrum (Lai et al., 2009).



Figure 1. Results of pressure gradient versus mass flow rate for a ceramic 20/40 proppant under a confining stress of 27.5 MPa (The experimental data (blue diamonds) agree with the Barree and Conway model (red) from low to high flow rates. The Forchheimer quadratic correlation (green) overestimates the pressure drop, while the Forchheimer cubic correlation (blue) underestimates the pressure drop at high gas flow rates).

Using the Renolds number, Equation (3), and the following dimensionless variable,





Figure 2. Dimensionless plot of all tested proppants using the Barree and Conway model.

Figure 2 summarized all the test data taken to date using the dimensionless form of the Barree and Conway model (Lopez, 2007; Lai et al., 2009). Figure 2 demonstrates that all of the experimental data collapse into one single curve, which can be fitted using the Barree and Conway model in a

dimensionless form. One plateau of the log-dose equation format is clearly observed at low Reynolds numbers, representing Darcy's flow range. When converted to field units, the test data shown in Figure 2 cover field gas production rates from less than 707.2 m³/D to more than 283,168 m³/D, demonstrating that the Barree and Conway model is accurate across the intervals of interest for the industry.

Barree and Conway (Barree and Conway, 2007) have recently extended their model to multiphase non-Darcy flow. With this extension, single-phase and multiphase volumetric flow rate (namely Darcy velocity with Darcy flow) for non-Darcy flow of phase β may be described in a vector form for multidimensional flow (Wu et al., 2009),

$$-\nabla \Phi_{\beta} = \frac{\mu_{\beta} \mathbf{v}_{\beta}}{k_{d} k_{r\beta} \left(k_{mr} + \frac{(1 - k_{mr}) \mu_{\beta} \tau}{\mu_{\beta} \tau + \rho_{\beta} |\mathbf{v}_{\beta}|} \right)}$$
(5)

where $\nabla \Phi_\beta$ is the flow potential gradient,

$$\nabla \Phi_{\beta} = \left(\nabla P_{\beta} - \rho_{\beta} g \, \nabla D \right) \tag{6}$$

and ρ_{β} is the density of fluid β ; \mathbf{v}_{β} is the volumetric velocity vector of fluid β ; P_{β} is the pressure of the fluid; g is gravitational acceleration; and D is the depth from a datum. In Equation (5), \mathbf{k}_{mr} is the minimum permeability ratio ($\mathbf{k}_{min}/\mathbf{k}_d$) at high rate, relative to Darcy's permeability (fraction); $\mathbf{k}_{r\beta}$ is the relative permeability to fluid β ; and μ_{β} is viscosity of fluid β . Equation (5) is used as the Barree and Conway model in this work to replace Darcy's law in modeling single-phase and multiphase flow in reservoirs.

3. MATHEMATICAL MODEL

A multiphase system in a porous or fractured reservoir is assumed to be similar to the black oil model, composed of three phases: oil, gas, and water. For simplicity, three fluid components, water, oil, and gas are assumed to be present only in their associated phases; and single-phase flow is treated as a special case of multiphase flow in this work. Each phase flows in response to pressure, gravitational, and capillary forces according to the multiphase extension of the Barree and Conway model of (5) for non-Darcy flow. In an isothermal system containing three mass components, three mass-balance equations are needed to fully describe the system, as described in an arbitrary flow region of a porous or fractured domain for flow of phase β (β = w for water, β = 0 oil, and β = g for gas),

$$\frac{\partial}{\partial t}(\varphi \ S_{\beta} \rho_{\beta}) = -\nabla \bullet (\rho_{\beta} \mathbf{v}_{\beta}) + q_{\beta}$$
⁽⁷⁾

where S_{β} is the saturation of fluid β ; ϕ is the effective porosity of formation; t is time; and q_{β} is the sink/source term of phase (component) β per unit volume of formation, representing mass exchange through injection/production wells or due to fracture and matrix interactions.

Equation (7), the governing of mass balance for three phases, needs to be supplemented with constitutive equations, which express all the secondary variables and parameters as functions of a set of primary thermodynamic variables of interest. The following relationships will be used to complete the description of multiphase flow through porous media:

$$S_w + S_o + S_o = 1 \tag{8}$$

The capillary pressures relate pressures between the phases. The aqueous- and gas-phase pressures are related by

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$$P_{w} = P_{g} - P_{cgw}(S_{w}) \tag{9}$$

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where P_{cgw} is the gas-water capillary pressure in a three-phase system and assumed to be a function of water saturation only. The oil pressure is related to the gas phase pressure by

$$P_o = P_g - P_{cgo} \left(S_w, S_o \right) \tag{10}$$

where P_{cgo} is the gas-oil capillary pressure in a three-phase system, which is a function of both water and oil saturations. For formations, the wettability order is (1) aqueous phase, (2) oil phase, and (3) gas phase. The gas-water capillary pressure is usually stronger than the gas-oil capillary pressure. In a three-phase system, the oil-water capillary pressure, P_{cow} , may be defined as

$$P_{cow} = P_{cgw} - P_{cgo} = P_{o} - P_{w}$$
(11)

The relative permeabilities are assumed to be functions of fluid saturations only, i.e., not affected by non-Darcy flow behavior. The relative permeability to the water phase is taken to be described by

$$k_{rw} = k_{rw} \left(S_{w} \right) \tag{12}$$

to the oil phase by

$$k_{ro} = k_{ro} \left(S_{w}, S_{g} \right) \tag{13}$$

and to the gas phase by

$$k_{rg} = k_{rg} \left(S_g \right) \tag{14}$$

The densities of water, oil, and gas, as well as their viscosities can in general be treated as functions of pressure.

4. NUMERICAL MODEL

Equations (7) and (5), as described by the Barree and Conway's model, for single-phase and multiphase non-Darcy flow of gas, oil and water in porous media, are highly nonlinear and in general need to be solved numerically. In this work, the methodology for using a numerical approach of TOUGH2 (Pruess et al., 1999) to simulate the non-Darcy flow consists of the following three steps: (1) spatial discretization of the mass conservation equation; (2) time discretization; and (3) iterative approaches to solve the resulting nonlinear, discrete algebraic equations. A mass-conserving discretization scheme, based on finite or integral finite-difference or finite-element methods (Pruess et al., 1999) is used and discussed here. Specifically, non-Darcy flow equations, as discussed in Section of Mathematical Model, have been implemented into a general-purpose, three-phase reservoir simulator, the MSFLOW code (Wu, 1998) As implemented in the code, Equation (7) can be discretized in space using an integral finite-difference or control-volume finite-element scheme for a porous and/or fractured medium. The time discretization is carried out with a backward, first-order, finite-difference scheme. The discrete nonlinear equations for water, oil, and gas flow at node i are written as follows:

$$\left\{ \left(\varphi \ S_{\beta} \rho_{\beta} \right)_{i}^{n+1} - \left(\varphi \ S_{\beta} \rho_{\beta} \right)_{i}^{n} \right\} \frac{V_{i}}{\Delta t} = \sum_{j \in \eta_{i}} \left(flow_{\beta} \right)_{ij}^{n+1} + Q_{\beta i}^{n+1}$$
(15)

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where n denotes the previous time level; n+1 is the current time level; V_i is the volume of element i (i=1, 2, 3, ..., N, N being the total number of elements of the grid); Δt is the time step size; h_i contains the set of neighboring elements (j), porous or fractured block, to which element i is directly connected; and "flow_{β}" is a mass flow term between elements i and j for fluid β , defined by Equation (16) implicitly. For flow between two grid blocks, the mass flow term, "flow_{β}", for phase β can be evaluated directly (Lai et al., 2009; Wu et al., 2009) as,

$$flow_{\beta,ij} = A_{ij}\rho_{\beta}v_{\beta} =$$

$$\frac{A_{ij}}{^{2\mu}\beta} \begin{cases} -[\mu_{\beta}^{2}S_{\beta}\tau - \Delta\Phi_{\beta,ij}k_{d}k_{r}\beta^{k}rm^{\rho}\beta]^{+} \\ \sqrt{[\mu_{\beta}^{2}S_{\beta}\tau - \Delta\Phi_{\beta,ij}k_{d}k_{r}\beta^{k}rm^{\rho}\beta]^{2} + 4\mu_{\beta}\rho_{\beta}\Delta\Phi_{\beta,ij}k_{d}k_{r}\beta^{\mu}\beta^{S}\beta^{\tau}} \end{cases}$$

$$(16)$$

where A_{ij} is the common interface area between connected elements i and j; all the parameters, such as permeability, relative permeability, density, and viscosity needs a proper averaging or weighting of properties at the interface between the two elements i and j; A_{ij} is the common interface area between the connected blocks or nodes i and j; and the discrete flow potential gradient is defined in an integral finite difference as,

$$\Delta \Phi_{\beta,ij} = \frac{\left(P_{\beta,i} - \rho_{\beta,i\,j+1/2} \ g \ D_i\right) - \left(P_{\beta,j} - \rho_{\beta,i\,j+1/2} \ g \ D_j\right)}{D_i + D_j} \tag{17}$$

In Equation (15), the mass sink/source term at element i, $Q_{\beta i}$ for phase β , is defined as

$$Q_{\beta i} = q_{\beta i} V_i \tag{18}$$

In the model formulation, Darcy permeability, relative permeability, and other non-Darcy flow parameters, such as minimum permeability ratio, k_{mr} , and characteristic length, τ , are all considered as flow properties of the porous media and need to be averaged between connected elements in calculating the flow terms. In general, weighting approaches used are that absolute permeability is harmonically weighted along the connection between elements i and j, relative permeability is upstream weighted, and non-Darcy flow coefficients are arithmetically averaged.

Newton/Raphson iterations are used to solve Equation (15). For a three-phase flow system, $3 \times N$ coupled nonlinear equations must be solved, including three equations at each element for the three mass-balance equations of water, oil, and gas, respectively. The three primary variables (x_1, x_2, x_3) selected for each element are oil pressure, oil saturation, and gas saturation, respectively (Wu and Forsyth, 2001). In terms of the three primary variables, the Newton/Raphson scheme gives rise to

$$\sum_{m} \frac{\partial R_{i}^{\beta, n+1}(x_{m, p})}{\partial x_{m}} \left(\delta x_{m, p+1} \right) = -R_{i}^{\beta, n+1}(x_{m, p}) \quad \text{for } m = 1, 2, \text{ and } 3$$
(19)

where index m = 1, 2, and 3 indicates the primary variable 1, 2, or 3, respectively; p is the iteration level; and i = 1, 2, 3, ..., N, the nodal index. The primary variables are updated after each iteration,

$$x_{m,p+1} = x_{m,p} + \delta x_{m,p+1}$$
(20)

A numerical method is used to construct the Jacobian matrix for Equation (19), as outlined by Forsyth et al. (1995).

Boundary Condition: Similarly to Darcy flow handling, first-type or Dirichlet boundary conditions denote constant or time-dependent phase pressure, and saturation conditions. These types of boundary conditions can be treated using the large-volume or inactive-node method (Pruess et al., 1999), in which a constant pressure/saturation node may be specified with a huge volume while keeping all the other geometric properties of the mesh unchanged. However, caution should be taken in (1) identifying phase conditions when specifying the "initial condition" for the large-volume boundary node and (2) distinguishing upstream/injection from downstream/production nodes. Once specified, primary variables will be fixed at the big-volume boundary nodes, and the code handles these boundary nodes exactly like any other computational nodes.

Flux-type or Neuman boundary conditions are treated as sink/source terms, depending on the pumping (production) or injection condition, which can be directly added to Equation (15). This treatment of flux-type boundary conditions is especially useful for a situation where flux distribution along the boundary is known, such as dealing with a single-node well. More general treatment of multilayered well-boundary conditions is discussed in Wu et al. (1996) and Wu (2000).

5. HANDLING NON-DARCY FLOW IN FRACTURED MEDIA

The technique used in the current model for handling non-Darcy flow through fractured rock follows the dual-continuum methodology (Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985; Wu, 2002). The method treats fracture and matrix flow and interactions using a multi-continuum numerical approach, including the double- or multiporosity method (Wu and Pruess, 1988), the dual-permeability method, and the more general "multiple interacting continua" (MINC) method (Pruess and Narasimhan, 1985). As shown in Wu and Qin (2009), the generalized dual-continuum, MINC method, can handle any flow processes of fractured media with matrix size varying from as large as the model domain of interest to as small as a representative elementary volume (REV) of zero volume. In general, the fracture network can be continuous in a pattern, randomly distributed or discrete.

The non-Darcy flow formulation, Equations (5) and (7), and (15) and (16), as discussed above, is applicable to both single-continuum and multi-continua media within fracture or matrix. Using the dual-continuum concept, Equations (7) and (15) can be used to describe multiphase flow, respectively, both in fractures and inside matrix blocks when dealing with fractured reservoirs. Special attention needs to be paid to treating fracture/matrix flow terms with Equations (15) and (16) for estimation of mass exchange at fracture/matrix interfaces using a double-porosity approach. In particular, special attention should be paid to selecting characteristic length of non-Darcy flow distance between fractures and matrix crossing the interface for the double-porosity or the nested discretizations may be approximated using the results for Darcy flow (Warren and Root, 1965; Pruess, 1983; Wu, 2002). However, the flow between fractures and matrix can be still evaluated using Equation (16) and the characteristic distance for flow crossing fracture/matrix interfaces for 1-D, 2-D and 3-D dimensions of rectangular matrix blocks, characteristic distances, based on quasi-steady flow assumption(Wu, 2002).

When using the Barree and Conway model for handling non-Darcy flow through a fractured rock with the numerical formulation using the generalized TOUGH2 multicontonuum approach, the problem essentially becomes how to generate a mesh that represents both the fracture and matrix systems. Several fracture-matrix subgridding schemes exist for designing different meshes for different fracture-matrix conceptual models (Pruess, 1983). Once a proper mesh of a fracture-matrix system is generated, fracture and matrix blocks are specified to represent fracture or matrix domains, separately. Formally, they are treated in exactly the same way in the solution of the discretized model. However, physically consistent fracture and matrix properties and modeling conditions must be appropriately specified for fracture and matrix systems, respectively.

6. MODEL VERIFICATION AND APPLICATION

In this section, we use analytical solutions to verify the numerical scheme implemented for modeling single-phase and multiphase non-Darcy flow in reservoirs, and demonstrate application of the numerical model.

Comparison with the Analytical Solution for 1-D, Single-Phase Steady-State Flow: Lai et at. (2009) presented an analytical solution for steady-state incompressible fluid flow:

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$$P(x) = P_i - \frac{\mu \frac{\dot{m}}{A} (\frac{\dot{m}}{A} + \mu \tau)(L - x)}{(\mu k_d \tau + \frac{\dot{m}}{A} k_d k_{mr})\rho}$$
(21)

where \dot{m} is fluid mass flow rate at inlet; and P_i is pressure at outlet. The analytical solution (21) is used to check the numerical results of modeling 1-D, single-phase flow. In numerical discretization, a 1-D linear reservoir formation 10-m long, with a unit cross-sectional area, is represented by a 1-D uniform linear grid of 1,000 elements with $\Delta x = 0.01$ m. The parameters used for this comparison are listed in Table 1. We compare two cases with different minimum permeability and characteristic length values where Case 1: $k_{min} = 0.1$ Darcy and $\tau = 100,000$ (1/m); and Case 2: $k_{min} = 1.0$ Darcy and $\tau = 10,000$ (1/m). In the two scenarios, the pressure at the outlet boundary is maintained at 10⁷ Pa, and a constant mass production rate is proposed at x = 0 for both the analytical and numerical solutions. The numerical calculation is carried out until steady state is reached. Figure 3 shows the comparison results from the two solutions and indicates that excellent results are obtained from the numerical simulation, as compared to the analytical solution. Figure 3 also shows that the pressure distributions for the two scenarios are nearly linear along the linear flow direction, and this is because under incompressible, constant mass or volumetric flow condition, the effective permeability is essentially constant along the flow system.

Table 1. Parameters Used for Model Verification of Checking of Numerical SimulationResults against the Analytical Solution, as Shown in Figure 3.

Parameter	Value	Unit
Cross section area	A = 1	m^2
Darcy permeability	$k_{d} = 10$	Darcy
Minimum permeability	$k_{min} = 0.1, 1.0$	Darcy
Viscosity	$\mu = 0.001$	Pa∙s
Characteristic lengt	$\tau = 10,000, 100, 0$	000 1/m
Density	$\rho = 1,000$	kg/m ³
Mass production ra	m = 5	kg/s
Pressure at outer boundary	$P_i = 10^7$	Pa



Figure 3. Comparison between the analytical and numerical solutions for 1-D steady-state flow in a linear system (The analytical solution for Case 1 is shown as a solid blue line, while the numerical solution for Case 1 is shown as purple squares. The analytical solution for Case 2 is shown as a solid green line, while the numerical solution for Case 2 is shown as red triangles).

Comparison with the Buckley-Leverett Type Analytical Solution: Wu et al. (2009; 2011) presents a Buckley-Leverett type analytical solution for non-Darcy displacement according to the Barree and Conway model, and we use the analytical solution to examine the numerical model formulation as well as the correctness of its numerical implementation. The physical flow model is a one-dimensional linear porous medium, which is at first saturated uniformly with a nonwetting fluid ($S_0 = 0.8$) and a wetting fluid ($S_w = S_{wr} = 0.2$). A constant volumetric injection rate of the wetting fluid is imposed at the inlet (x = 0), starting from t = 0. The relative permeability curves used for all the calculations in this paper are shown in Figure 4. The properties of the rock and fluids used are listed in Table 2.



Figure 4. Relative-permeability curves used in analytical and numerical solutions for Barree and Conway non-Darcy displacement.

Table 2 Parameters for the non-Darcy Displacement Example.

Parameter	Value	Unit
Effective Porosity	$\phi = 0.30$	
Darcy permeability	$k_{d} = 10$	Darcy
Minimum permeability	$k_{min}=0.1, 1.0$	Darcy
Characteristic length	τ=10,000	1/m
Wetting Phase Density	$\rho_{\rm w} = 1,000$	kg/m ³
Wetting Phase Viscosity	$\mu_{\rm w} = 1.0 \times 10^{-3}$	Pa∙s
Nonwetting Phase	$\rho_{\rm n} = 800$	kg/m ³
Density		
Nonwetting Phase	$\mu_n = 5.0 \times 10^{-3}$	Pa∙s
Viscosity	•	
Injection Rate	$q=1.0\times10^{-5}$	m ³ /s

The resulting fractional flow and its derivative curves are shown in Figure 5. Note that fractional flow curves change also with the non-Darcy model parameters due to the change in pressure gradient and flow rate for different non-Darcy flow parameters under the same saturation.

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Figure 5. Fractional flow and its derivative curves with respect to wetting phase saturation in the non-Darcy displacement system.

For the comparison, to reduce the effects of discretization on numerical simulation results, very fine, uniform mesh spacing ($\Delta x = 0.01$ m) is chosen. A one-dimensional 5 m linear domain is discretized into 500 one-dimensional uniform gridblocks. The comparison between the analytical and numerical solutions is shown in Figure 6. The figure indicates that the numerical results are in excellent agreement with the analytical prediction of the non-Darcy displacement for the entire wetting-phase sweeping zone. Except at the shock, advancing saturation front, the numerical solution deviates only slightly from the analytical solution, resulting from a typical "smearing front" phenomenon of numerical dispersion effects when matching the Buckley-Leverett solution using numerical results (Aziz and Settari, 1979).



Figure 6. Comparison between displacement saturation profiles calculated from analytical and numerical solutions after 10 hours of injection.

Model Application: The application example presents a radial flow problem using the numerical model to calculate transient pressure at an injection well. The reservoir formation is a uniform, radially infinite system (approximated by $r_e = 10,000,000$ m in the numerical model) of 10 m thick, and is represented by a 1-D radial grid of 1,202 radial increments with a Δr size that increases logarithmically away from the well radius ($r_w = 0.1$ m). The formation is initially at a constant pressure of 10⁷ Pa and is subjected to a constant volumetric injection rate of 10,000 m³/d at the

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well, starting at t = 0. Parameters used for the simulation study are listed in Table 3.

Figure 7 presents the simulated transient pressure responses at the well and a comparison for the four cases with a combination of minimum permeability and characteristic length values of Case 1: Darcy flow, i.e., $k_{min} = k_d$; Case 2: $k_{min} = 0.1$ Darcy and $\tau = 10,000$ (1/m); and Case 3: $k_{min} = 0.1$ Darcy and $\tau = 100,000$ (1/m); and Case 4: $k_{min} = 1$ Darcy and $\tau = 10,000$ (1/m). In Figure 7, the lowest, solid, black curve shows the results for Case 1 or Darcy flow for comparison. The upper most, solid pink curve shows the results for Case 2, indicating the largest increase in injection pressure or the highest flow resistance caused by the non-Darcy flow, because of the smaller values of the characteristic length, τ , and minimum permeability, k_{min} . The solid-blue-circle curve, the second from the bottom, is the result of Case 3, showing a very small difference from the Darcy flow case, because of using a large τ (=100,000). The green-triangle curve, the second from the top, is for Case 4, also showing large injection pressure increase, because of a smaller τ (=10,000) used in the case.

It is very interesting to note that in all the four cases of Figure 7, the pressure responses have a linear relationship with time on the semi-log plot, except the very early time. In addition, the four semi-log straight lines are in parallel to one another. This behavior indicates (1) the impact of non-Darcy flow on pressure transients is equivalent to a constant flow resistance, superposed onto the pressure change of Darcy flow and (2) the Darcy permeability, k_d , can be estimated using the slope of semi-log straight lines from pressure draw down or buildup curves.



Figure 7. Transient pressure responses simulated at an injection well. Case 1: the bottom black solid curve for Darcy flow, i.e., $k_{min} = k_d$; Case 2: top solid pink curve, $k_{min} = 0.1$ Darcy and $\tau = 10,000$ (1/m); Case 3: solid-blue-circle curve, $k_{min} = 0.1$ Darcy and $\tau = 100,000$ (1/m); and Case 4: solid-green-triangle curve, $k_{min} = 1$ Darcy and $\tau = 10,000$ (1/m). The simulation parameters used are listed in Table 3.

Figures 8 and 9 examine the effect of wellbore storage on transient pressure behavior at the bottom hole using the numerical model, in which the dimensionless time and dimensionless pressure are defined as:

$$t_D = \frac{k_d t}{\varphi \mu C_T r_w^2} \tag{22}$$

and

$$P_{D} = \frac{P(r,t) - P_{i}}{\frac{q\mu}{2\pi k_{i}h}}$$
(23)

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The influence of wellbore storage occurs mainly in the earlier time, as shown in Figures 8 and 9, which similar to that for Darcy flow, described in the conventional well testing analysis. In addition, Figure 9 shows the period of wellbore storage dominated flow is characterized by the log-log straight lines with a unit slope in the pressure curves.



Figure 8. Semi-log plots for effect of wellbore storage on wellbore pressure responses flow. Case 1: C_D =0; Case 2: C_D =100; Case 3: C_D =1,000; and Case 4: C_D =100,000. The non-Darcy flow parameters: k_{min} = 0.1 Darcy and τ = 10,000 (1/m), and the rest of the simulation parameters used are listed in Table 3.



Figure 9. Log-log plots for effect of wellbore storage on wellbore pressure responses flow. Case 1: C_D =0;Case 2: : C_D =100; Case 3: C_D =1,000; and Case 4: C_D =100,000. The non-Darcy flow parameters: k_{min} = 0.1 Darcy and τ = 10,000 (1/m), and the rest of the simulation parameters used are listed in Table 3.

In an effort to analyze the wellbore formation damage or skin effect under non-Darcy flow condition, we use the following equation to correlate the skin factor, *S*, to skin-zone permeability and radius:

$$S = \left(\frac{k_d}{k_s} - 1\right) \ln(r_s / r_w) \tag{24}$$

where k_s and r_s are the Darcy permeability and skin-zone radius, respectively. In the numerical calculation, $k_{min} = 1\%$ of k_s and $r_s = 0.12$ m for the skin-zone. This results in a thickness of the skin-zone is 0.02 m. As shown in Figures 10 and 11, the skin effect is significant and occurs during the entirely testing period from early to later times. Even though the curves of Figures 10 and 11 appear similar to those for Darcy flow with skin effect, careful check of the results in Figures 10 and 11 indicates much stronger skin effect under non-Darcy flow condition. This is because the vertical differences between dimensionless pressure curves of Figure 10 or 11 are always larger than the difference in skin factors used for the curves. These additional pressure drops are due to non-Darcy flow effect, which is expected the strongest near the wellbore or at the skin-zone.



Figure 10. Semi-log plots for skin effect on wellbore pressure responses flow. Case 1: S = 0; Case 2: S = 1;Case 3: S = 5; and Case 4: S = 10. The non-Darcy flow parameters: $k_{min} = 0.1$ Darcy and $\tau = 10,000$ (1/m), and the rest of the simulation parameters used are listed in Table 3.



Figure 11. Log-log plots for skin effect on wellbore pressure responses flow. Case 1: S = 0; Case 2: S = 1; Case 3: S = 5; and Case 4: S = 10. The non-Darcy flow parameters: $k_{min} = 0.1$ Darcy and $\tau = 10,000$ (1/m), and the rest of the simulation parameters used are listed in Table 3.

Parameter	Value	Un
Darcy permeability	$k_{d} = 10$	Darcy
Minimum permeability	$k_{min} = 0.1$	Darcy
Viscosity	µ=0.001	Pa∙s
Reference density	ρ=1,000	kg
Volumetric Injection rate	q = 1,000	m ³ /d
Total compressibility of fluid and rock	$C_{T}=6\times10^{-1}$	1/Pa
Well radius	$r_{\rm w} = 0.1$	m
Formation thickness	h = 10	m
Initial formation gas pressure	$P_i\!=\!10^7$	Ра

7. SUMMARY AND CONCLUSIONS

This paper presents a general mathematical model and numerical approach for incorporating the Barree and Conway model to simulating single-phase and multiphase non-Darcy flow. The model formulation is implemented into a general reservoirs simulator for multidimensional non-Darcy flow in porous and fractured media. In numerical solution, the multiphase non-Darcy flow formulation is solved using an unstructured grid with regular or irregular meshes for multidimensional simulation, while flow in fractured rock is handled using a general TOUGH2 multicontinuum approach. In an effort to check the numerical scheme implemented, we use analytical solutions to verify our numerical model results for both single-phase and multiphase non-Darcy flow. As application examples, the model is applied for pressure transient analysis of single-phase well flow. The numerical model results show (1) in an infinite-acting reservoir, semi-log straight lines will develop in pressure drawn-down or buildup plots; (2) effect of wellbore storage is similar to that under Darcy flow condition; and (3) the skin effect is much stronger when compared to the Darcy flow case, because of the addition pressure drops also due to flow through formation-damaged skin zone.

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REFERENCES

Aziz, K. and A. Settari, Petroleum Reservoir Simulation, Applied Science, London, 1979.

- Barree, R.D. and, M.W. Conway, Multiphase non-Darcy flow in proppant packs, Paper SPE 109561, presented at the 2007 Annual Technical Conference and Exhibition, Anaheim, CA, 11-14 November, 2007.
- Barree, R. D. and, M. W. Conway, Beyond Beta Factors: A complete model for Darcy, Forchheimer and trans-Forchheimer flow in porous media, Paper SPE 89325 presented at the 2004 Annual Technical Conference and Exhibition, Houston, Texas 26-29 September, 2004.
- Evans, E. V. and R. D. Evans, Influence of an immobile or mobile saturation on non-Darcy compressible flow of real gases in propped fractures, J. Petroleum Technology, Vol. 40, No. 10, pp.1343-1351, 1988.
- Evans, R. D., C. S. Hudson and J. E. Greenlee, The effect of an immobile liquid saturation on the non-Darcy flow coefficient in porous media, *J. SPE Production Engineering*, *Trans.* AIME, Vol. 283, pp.331-338, 1987.
- Forchheimer, P., Wasserbewegung durch Bode, ZVDI (1901) 45, 1901.
- Forsyth, P. A., Y. S. Wu and K. Pruess, Robust numerical methods for saturated-unsaturated flow with dry initial conditions in heterogeneous media, *Advance in Water Resources*, 18, p. 25-38, 1995.
- Guppy, K. H., H. Cinco-Ley, H. J. Ramey, Jr. and F. Samaniego, Non-Darcy flow in wells with finite-conductivity vertical fractures, Soc. Pet. Eng. J., pp.681-698, 1982.

- Guppy, K. H., H. Cinco-Ley, and H. J. Ramey, Jr., Effects of non-Darcy flow on the constant-pressure production of fractured wells, Soc. Pet. Eng. J., pp.390-400, 1981.
- Katz, D. L. and R. L. Lee, Natural Gas Engineering, Production and Storage, Chemical Engineering Series, McGraw-Hill Book Co. Inc., New York, 1990.
- Kazemi, H., Pressure Transient Analysis of naturally fractured reservoirs with uniform fracture distribution, *SPEJ*, 451-62. Trans., AIME, 246, 1969.
- Lai, B, J. L. Miskimins, and Y. S. Wu, Non-Darcy Porous Media Flow According to the Barree and Conway Model: Laboratory and Numerical Modeling Studies, SPE-122611, Presented at the 2009 Rocky Mountain Petroleum Technology Conference, 14-16 April, 2009, Denver, CO, 2009.
- Lee, R. L., R. W. Logan and M. R. Tek: Effetcs of turbulence on transient flow of real gas through porous media, SPE Formation Evaluation, pp.108-120, 1987.
- Liu, X., F. Civan and R. D. Evans, Correlations of the non-Darcy flow coefficient, J. Canadian Petroleum Technology, Vol. 34, No. 10, pp.50-54, 1995.
- Lopez. H. D., Experimental analysis and macroscopic and pore-level flow simulations to compare non-Darcy flow models in porous media, PhD dissertation, Colorado School of Mines, Golden, Colorado, 2007.
- Pruess, K., C. Oldenburg, and G. Moridis, *TOUGH2 User's Guide*, Version 2.0, Report LBNL-43134, Berkeley, California: Lawrence Berkeley National Laboratory, 1999.
- Pruess, K. and Narasimhan, T. N., A practical method for modeling fluid and heat flow in fractured porous media, Soc. Pet. Eng. J., 25, pp.14-26, 1985.
- Pruess, K., GMINC A mesh generator for flow simulations in fractured reservoirs, Report LBL-15227, Berkeley, California: Lawrence Berkeley National Laboratory, 1983.
- Scheidegger, A. E. The Physics of Flow through Porous Media, University of Toronto Press, 1972.
- Skjetne, E., T. K. Statoil and J. S. Gudmundsson, Experiments and modeling of high-velocity pressure loss in sandstone fractures, SPE 56414, Presented at the 1999 SPE Annual Technical Conference and Exhibition, Houston, Texas, 3-6, October, 1999.
- Swift, G. W. and O. G. Kiel, The prediction of gas-well performance including the effects of non-Darcy flow, J. Petroleum Technology, Trans. AIME, Vol. 222, 791-798, 1962.
- Tek, M. R., K. H. Coats and D. L. Katz, The effects of turbulence on flow of natural gas through porous reservoirs, J. Petroleum Technology, Tran.s AIME, Vol. 222, pp.799-806, 1962.
- Warren, J.E., and P. J. Root, The behavior of naturally fractured reservoirs, *Soc. Pet. Eng. J., Trans., AIME*, pp.245-255, 228, 1963.
- Wu, Y. S., B. Lai, J. L. Miskimins, P. Fakcharoenphol, and Y. Di, Analysis of multiphase non-darcy flow in porous and fractured media, Journal of *Transport in Porous Media*, DOI 10.1007/s11242-011-9735-8, 2011.
- Wu, Y. S., B. Lai, and J. L. Miskimins, Simulation of multiphase non-Darcy flow in porous and fractured media, SPE-122612, presented at the 2009 SPE Annual Technical Conference and Exhibition held in New Orleans, Louisiana, USA 4-7 October 2009.
- Wu, Y. S. and G. Qin, A general numerical approach for modeling multiphase flow and transport in fractured porous media, *Communications in Computational Physics*, Vol. 6, No. 1, pp.85-108, 2009.
- Wu, Y. S., Numerical simulation of single-phase and multiphase non-darcy flow in porous and fractured reservoirs, *Transport in Porous Media* 49 (2): 209-240, 2002.
- Wu, Y. S., Non-Darcy displacement of immiscibel fluids in porous media, Water Resources Research, Vol. 37, No. 12, pp.2943-2950, 2001.
- Wu, Y. S. and P. A Forsyth, On the selection of primary variables in numerical formulation for modeling multiphase flow in porous media, *Journal of Contaminant Hydrology*, Vol. 48(3-4), pp.277-304, 2001.
- Wu, Y. S. A virtual node method for handling wellbore boundary conditions in modeling multiphase flow in porous and fractured media, LBNL-42882, *Water Resources Research*, Vol. 36, NO. 3, pp.807-814, 2000.
- Wu, Y. S., MSFLOW: Multiphase Subsurface Flow Model of Oil, Gas and Water in Porous and Fractured Media with Water Shut-off Capability, DOCUMENTATION and User's Guide, Walnut Creek, California, 1998.
- Wu, Y. S., P. A. Forsyth and H. Jiang, A consistent approach for applying numerical boundary conditions for subsurface flow, *Journal of Contaminant Hydrology* 23: 157-185. 1996.
- Wu, Y. S. and K. Pruess, and P. A. Witherspoon, Flow and displacement of Bingham non-Newtonian fluids in porous media, SPE Reservoir Engineering, 369-376, 1992.

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Wu, Y. S. and K. Pruess, A multiple-porosity method for simulation of naturally fractured petroleum reservoirs, SPE Reservoir Engineering, 3, pp. 327-336, 1988.