An approximate analytical solution for non-Darcy flow toward a well in fractured media

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[1] This paper presents an approximate analytical solution for non-Darcy flow of a slightly compressible fluid through a fractured reservoir. The analytical solution is obtained using the traditional Warren-Root solution superposed on a dimensionless non-Darcy flow coefficient. The model formulation incorporates the Forchheimer equation into the Warren-Root model for describing non-Darcy flow through fractured media. The approximate analytical solution, verified for its accuracy by comparison with numerical solutions, provides a useful tool in analyzing non-Darcy flow in fractured reservoirs for practical applications. *INDEX TERMS:* 1829 Hydrology: Groundwater hydrology; 1832 Hydrology: Groundwater transport; 1875 Hydrology: Unsaturated zone; 1719 History of Geophysics: Hydrology; *KEYWORDS:* non-Darcy flow, fractured reservoirs, double porosity

1. Introduction

[2] Darcy's law of flow (or Darcy flow), describing a linear relationship between volumetric flow rate (Darcy velocity) and pressure (head or potential) gradient, has been the fundamental principle in analyzing flow and transport processes in porous and fractured media. Any deviations from this linear relation may be defined as non-Darcy flow. In this work, I would like to focus on the non-Darcy flow in a fractured medium caused by high flow velocities. Even though Darcy's law has been used exclusively in studies of porous-medium phenomena, there is considerable evidence that high-velocity, non-Darcy flow occurs in many subsurface systems, such as in the flow near wells of oil or gas production, water pumping, and liquid waste injection.

[3] The effects of non-Darcy or high-velocity flow regimes in porous media have been observed and investigated for decades [e.g., *Tek et al.*, 1962; *Scheidegger*, 1972; *Katz and Lee*, 1990]. Theoretical, field, and experimental studies performed on non-Darcy flow in porous media have focused mostly on flow in single-porosity media that pertains to the oil and gas industry [*Tek et al.*, 1962; *Swift and Kiel*, 1962; *Lee et al.*, 1987]. Few investigations have been conducted for non-Darcy flow in fractured reservoirs [*Skjetne et al.*, 1999] or in fractured wells [*Guppy et al.*, 1981, 1982]. Other studies have concentrated on finding and validating correlation of non-Darcy flow coefficients [*Liu et al.*, 1995]. Because of insufficient studies in this area as well as the mathematical difficulty in handling nonlinear, non-Darcy flow terms in flow equations, understanding of non-Darcy flow through porous media, in particular for fractured reservoirs, is currently very limited.

[4] The objective of this work is to present an analytical solution for non-Darcy flow of slightly compressible fluids through a homogeneous fractured formation, described by the Warren-Root double-porosity model. The Forchheimer equation, the most widely used non-Darcy flow model [*Katz and Lee*, 1990], is adapted to describe flow through a fracture continuum. The new analytical solution is derived by simply superposing the Warren-Root solution [*Warren and Root*, 1963] on a dimensionless non-Darcy flow coefficient. The analytical solution is checked using a numerical method and found to be very accurate in many cases for which the Warren-Root model applies. This approximate analytical solution can be applied to analyze well tests and obtain insight into non-Darcy flow through fractured reservoirs.

2. Flow Model

[5] The non-Darcy flow model of this work is based on the same assumptions as those for the Warren-Root model, except that the non-Darcy flow effect, described by the Forchheimer equation, is included for flow through fractures only. The Warren-Root model treats fracture and matrix flow and interactions using a double-porosity concept [*Barenblatt et al.*, 1960; *Warren and Root*, 1963; *Kazemi*, 1969]. It is a physically based approach, which assumes that a flow domain is composed of matrix blocks of low permeability, embedded in a network of interconnected, more permeable fractures. Global flow in the formation occurs only through the fracture system, described as an effective continuum. The matrix behaves as spatially distributed sinks or sources to the fracture network without accounting for global matrix-matrix flow. In addition, the double-porosity model relies on a quasi-steady state flow assumption to account for fracture-matrix interporosity flow.

[6] Non-Darcy flow through fracture-matrix flow may also occur and should in general be taken into consideration. However, a recent study [*Wu*, 2002], using both analytical analyses and numerical simulations, concludes that the impact of non-Darcy fracture-matrix flow can be ignored for almost all practical purposes with a typical fracture reservoir, when compared with non-Darcy flow effects in fracture flow. In addition, that study also finds that the quasi-steady flow assumption in the Warren-Root model provides a good approximation to non-Darcy flow cases as long as the double-porosity concept applies (i.e., the fracture system has much higher permeability and much lower porosity than the matrix system).

[7] Non-Darcy flow in a double-porosity reservoir is considered in this work for fracture flow only and described by

$$-\nabla \cdot (\rho \mathbf{v}) - \phi_m C_m \rho_i \frac{\partial P_m}{\partial t} = \phi_f C_f \rho_i \frac{\partial P_f}{\partial t} \tag{1}$$

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and for fracture-matrix flow, it is

$$\phi_m C_m \frac{\partial P_m}{\partial t} = \frac{\alpha k_m}{\mu}\left(P_f - P_m\right), \tag{2}$$

where ρ is the density of the fluid, a linear function of pressure; $\mathbf{v}$ is the Darcy (or volumetric) velocity of the fluid in the fracture continuum; ϕ_m and ϕ_f and C_m and C_f are effective porosities and total compressibilities of the matrix or fracture system, respectively; P_m and P_f are fluid pressures in the matrix or fracture system, respectively; ρ_i is the initial or reference density of the fluid; t is time; μ is the viscosity of the fluid; k_m is matrix permeability; and α is a shape factor for the matrix block geometry with a unit of m^{-2}, as defined by Warren and Root, for controlling fluid exchange between fractures and matrix.

[8] The flow term in (1) for non-Darcy flow through fractures is described using the Forchheimer equation [*Katz and Lee*, 1990]:

$$-\left(\nabla P_f\right) = \frac{\mu}{k_f}\mathbf{v} + \beta\rho\mathbf{v}|\mathbf{v}|, \tag{3}$$

where k_f is the permeability of the fracture continuum and β is the non-Darcy flow coefficient with a unit of m^{-1} through fractures. Under one-dimensional radial, horizontal flow conditions the component of the flow vector in the r direction term may be solved from (3) as

$$\rho\, v = \frac{1}{2k_f\beta}\left\{-\mu + \left[(\mu)^2 + 4k_f^2\rho\beta\left(\frac{\partial P_f}{\partial r}\right)\right]^{1/2}\right\}, \tag{4}$$

where v is a component of the volumetric velocity vector along the r coordinate. With the double-porosity model the governing flow equations are (1), (2), and (4) for describing non-Darcy flow through fractured media. To complete the flow problem description, we also need the following initial and boundary conditions. The problem of interest is a one-dimensional radial flow into or from a fully penetrating well in a radially infinite, horizontal reservoir with uniform fracture and matrix properties. The well is subject to a constant volumetric flow rate of pumping or injection.

[9] The initial pressure P_i is assumed constant within both fracture and matrix systems throughout the reservoir

$$P_f(r, t = 0) = P_m(r, t = 0) = P_i, \tag{5}$$

The same constant pressure remains at the outer (infinite) boundary,

$$P_f(r = \infty, t) = P_m(r = \infty, t) = P_i. \tag{6}$$

At the well bore a constant volumetric flow rate is imposed, with well bore storage and skin effects ignored:

$$\frac{\partial P_f(r_w, t)}{\partial r} = \frac{\mu q}{2\pi r_w k_f h}, \tag{7}$$

where r_w is the well radius, h is the thickness of the radial-infinite horizontal formation, and q is the volumetric flow rate at the well.

3. Analytical Solution

[10] Before further discussing the analytical solution, let us define the following group of dimensionless variables [*Earlougher*, 1977; *Warren and Root*, 1963]. The dimensionless radius is

$$r_D = \frac{r}{r_w}, \tag{8}$$

and the dimensionless time is

$$t_D = \frac{k_f\, t}{\mu r_w^2\left(\phi_m C_m + \phi_f C_f\right)}. \tag{9}$$

The dimensionless non-Darcy flow coefficient is

$$\beta_D = \frac{k_f q_m \beta}{2\pi r_w h \mu}, \tag{10}$$

and the dimensionless fracture pressure is

$$P_D = \frac{P_i - P_f}{\frac{q\mu}{2\pi k_f h}}. \tag{11}$$

Note that in (10), q_m is a mass production or injection rate, treated as a constant.

[11] In addition, Warren and Root define two more dimensionless parameters to characterize double-porosity flow behavior. The first one is the ratio of porosity compressibility of fractures to the total system porosity compressibility product as

$$\omega = \frac{\phi_f C_f}{\phi_m C_m + \phi_f C_f}, \tag{12}$$

and the second is the interporosity flow parameter:

$$\lambda = \frac{\alpha r_w^2 k_m}{k_f}. \tag{13}$$

For Darcy or linear flow in a double-porosity, fractured reservoir, Warren and Root gave an asymptotic (long time) solution in terms of the dimensionless fracture pressure at the well ($r_D = 1$) as

$$P_{D,WR}(r_D = 1, t_D) = \frac{1}{2}\left[\ln t_D + 0.80907 + E_i\left(-\frac{\lambda t_D}{\omega(1-\omega)}\right) - E_i\left(-\frac{\lambda t_D}{(1-\omega)}\right)\right], \tag{14}$$

where the function E_i is called the exponential integral.

[12] An analytical solution for describing steady state non-Darcy flow in a one-dimensional, finite, radial fracture flow system is derived Appendix A as follows:

$$P_D(r_D) = \ln\left(\frac{r_D^e}{r_D}\right) + \beta_D\left(\frac{1}{r_D} - \frac{1}{r_D^e}\right). \tag{15}$$

Here we use the same dimensionless variables, as defined before, and the dimensionless radius of the outer boundary is defined as

$$r_D^e = \frac{r_e}{r_w}, \tag{16}$$

with r_e being the radius of the outer boundary of the finite radial formation.

Table 1. Parameters Used in the Verification Examples for Non-Darcy Flow in a Double-Porosity, Fractured Reservoir

Parameter	Value	Unit
Matrix porosity	$\phi_m = 0.30$	
Fracture porosity	$\phi_f = 0.0006$	
Reference water density	$\rho_i = 1{,}000$	kg m^{-3}
Water phase viscosity	$\mu = 1 \times 10^{-3}$	Pa s
Matrix permeability	$k_m = 1.0 \times 10^{-16}$	m^2
Fracture permeability	$k_f = 9.869 \times 10^{-13}$	m^2
Water production rate	$q_m = 0.1$	kg s^{-1}
Rock compressibility	$C_r = 1.0 \times 10^{-9}$	1 Pa^{-1}
Water compressibility	$C_w = 5.0 \times 10^{-10}$	1 Pa^{-1}
Dimensionless non-Darcy flow coefficient for fracture	$\beta_{D,f} = 1 \times 10^{-4}$, 1, 5, 10, 15, and 20	
Dimensionless non-Darcy flow coefficient for matrix	$\beta_{D,m} = \beta_{D,f} \times 10$	
Shape factor	$\alpha = 60$	m^{-2}
Well bore radius	$r_w = 0.1$	m

[13] The steady state solution, (15), for non-Darcy flow in fracture consists of two terms on the right-hand side. The first term, $\ln(r_D^e/r_D)$, is the precise steady state solution for Darcy flow, and the second term, $\beta_D\,(1/r_D - 1/r_D^e)$, represents non-Darcy flow effect. From (15), as long as $r_e \gg r_w$, which is almost always true, at the well ($r_D = 1$), we will have

$$P_D(r_D = 1) \approx \ln\left(r_D^e\right) + \beta_D. \tag{17}$$

Equation (17) indicates that a steady state solution for non-Darcy flow in the fracture medium is simply a superposition of the Darcy flow solution and a non-Darcy flow coefficient. For transient non-Darcy flow in a double-porosity reservoir the flow may be quick to approach a quasi-steady state within a region near the well because of the low storage capacity and high permeability of the fracture continuum. This encourages us to try an approximate analytical solution for transient non-Darcy flow in a fractured medium as a superposition of the Warren-Root solution, (14), and the non-Darcy flow coefficient:

$$P_D(r_D = 1, t_D) = \beta_D + P_{D,WR} = \beta_D + \frac{1}{2}\left[\ln t_D + 0.80907 + E_i\left(-\frac{\lambda t_D}{\omega(1-\omega)}\right) - E_i\left(-\frac{\lambda t_D}{(1-\omega)}\right)\right]. \tag{18}$$

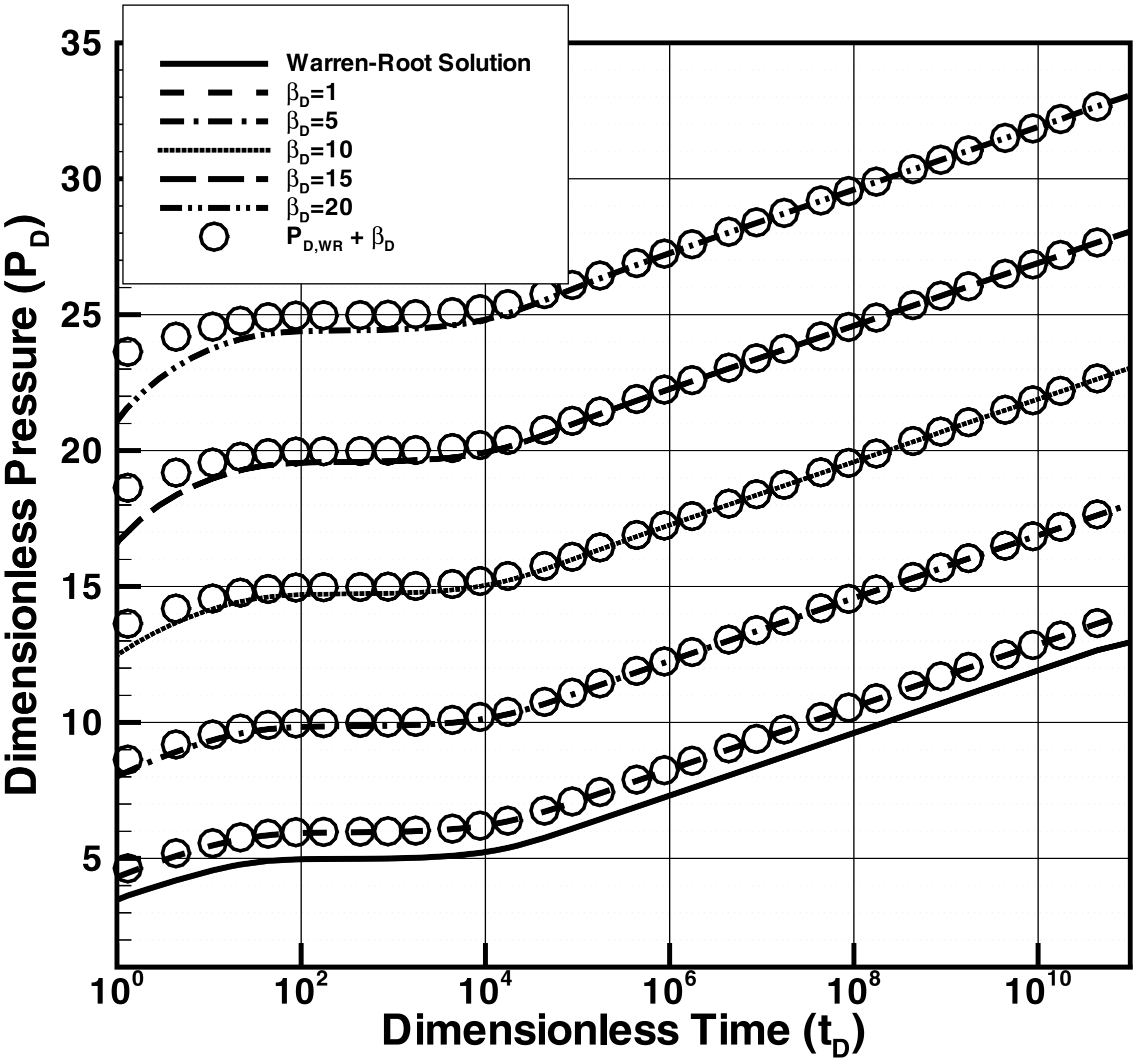

Figure 1. Comparison of dimensionless pressures calculated from the approximate analytical solution (circled symbol curves, denoted as $P_{D,WR} + \beta_D$) and numerical solutions (solid curves) for transient non-Darcy flow in double-porosity, fractured rock with different non-Darcy flow coefficients.

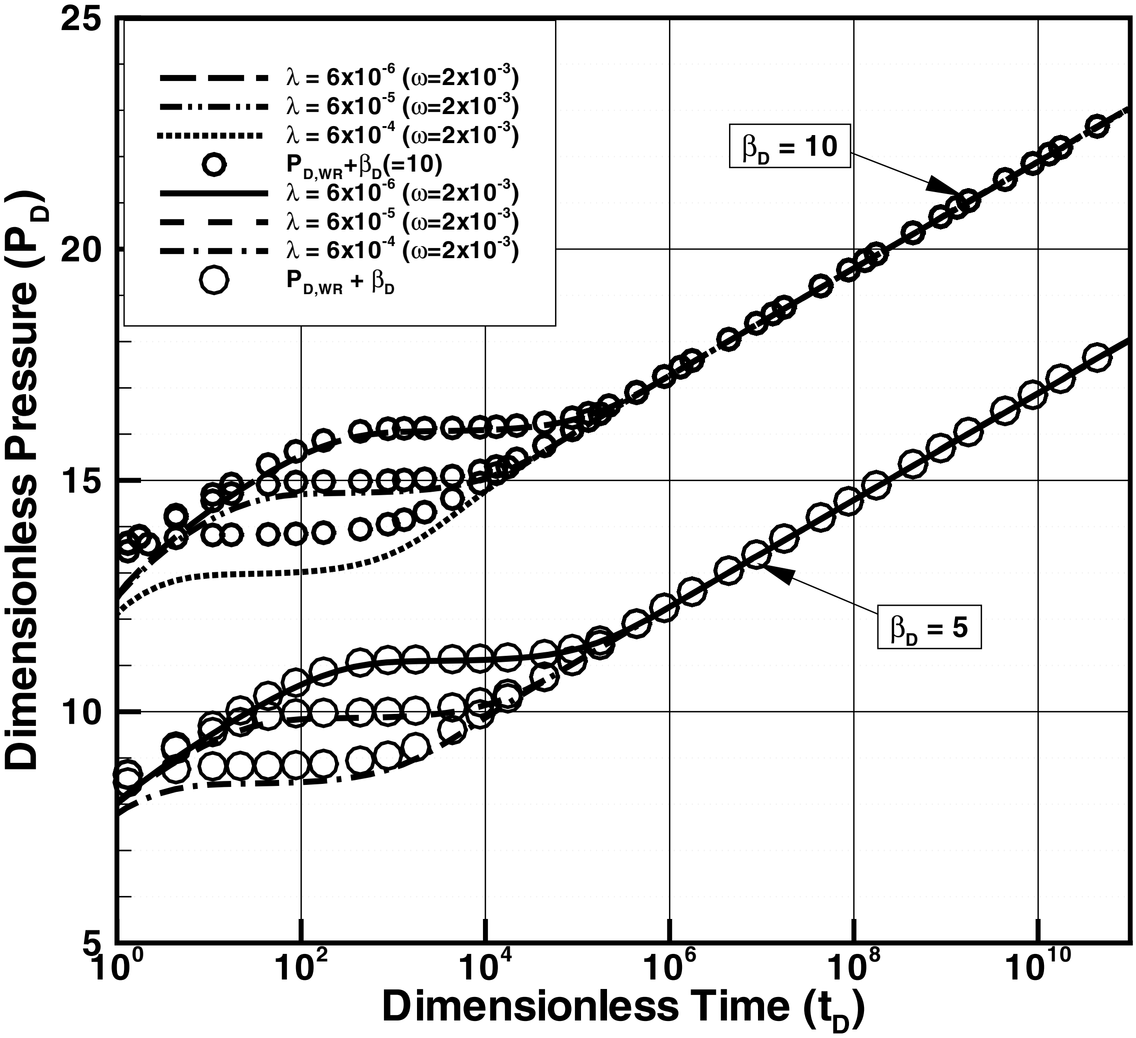

Figure 2. Comparison of dimensionless pressures calculated from the approximate analytical solution (circled symbol curves, denoted as $P_{D,WR} + \beta_D$) and numerical solutions (solid curves) for transient non-Darcy flow in double-porosity, fractured rock with different λ parameters and non-Darcy flow coefficients.

It is easy to examine that the proposed solution, (18), satisfies the boundary condition, (7), at the well but does not satisfy for the initial condition (5) or the boundary condition (6) at the infinite, in general, differing by a constant, β_D. This indicates that the solution may not be accurate in very early transient times at wells.

4. Verification

[14] In this section, we will check the proposed solution (18) for its accuracy and applicability to non-Darcy flow in fractured reservoirs. To evaluate the approximate analytical solution (18), we have to use an exact analytical or numeric solution. However, because of the extra nonlinear, non-Darcy flow term in the governing equation, (1), the exact analytical solution to the non-Darcy flow problem, as defined by (1), (2), and (4), may not be traceable without several dramatic linearized assumptions. Instead of using an exact solution, here we use a numerical simulator [*Wu*, 2002] to verify the approximate solution and examine the conditions under which the solution may apply. The numerical simulator has the capability to model single-phase and multiphase non-Darcy flow through multidimensional fractured reservoirs.

[15] The verification problems below concern typical transient flow toward a well that fully penetrates a radially infinite horizontal, uniform, fractured reservoir. In numerical modeling for comparison, a radially finite reservoir ($r_e = 5 \times 10^6$ m) is used and discretized into a one-dimensional (primary) grid. The r distance of 5×10^6 m is subdivided into 3100 intervals in a logarithmic scale. A double-porosity mesh is generated from the primary grid, in which a three-dimensional fracture network and cubic matrix blocks are used. The uniform matrix block size is $1 \times 1 \times 1$ m, and fracture permeability and aperture are correlated by the cubic law. Input parameters are given in Table 1. In addition, the numerical simulations also include non-Darcy effects on flow between fracture and matrix, which has been found to have insignificant, negligible impact on non-Darcy flow through fractured rocks [*Wu*, 2002]. Here we use 10 times larger non-Darcy flow coefficients for the matrix than those for fractures for each case to account for low matrix permeability effects on non-Darcy fracture-matrix flow. A fully penetrating pumping well is represented by a well element with a specified constant water-pumping rate. Note that many numerical tests and analyses have been performed to confirm

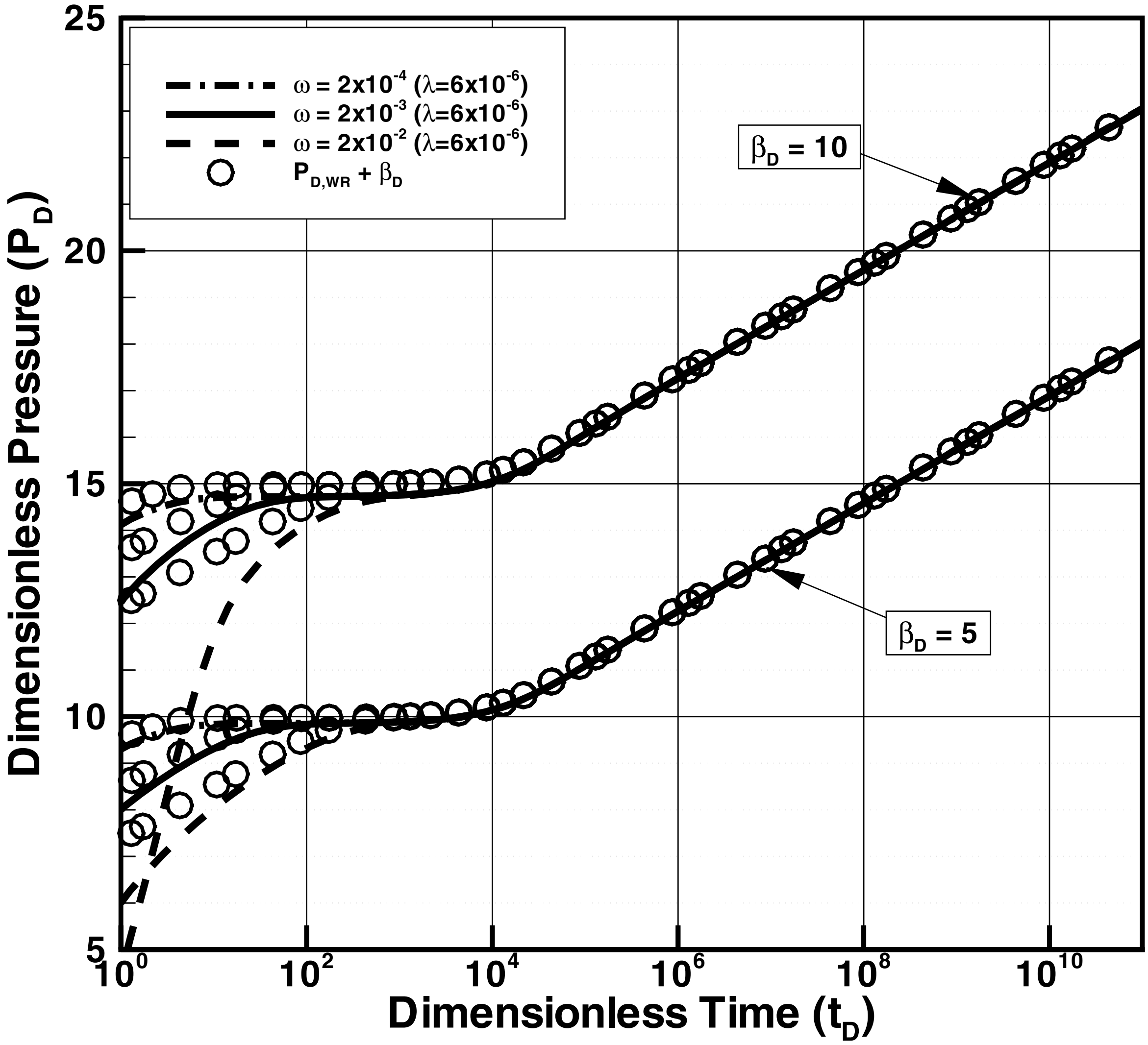

Figure 3. Comparison of dimensionless pressures calculated from the approximate analytical solution (circled symbol curves, denoted as $P_{D,WR} + \beta_D$) and numerical solutions (solid curves) for transient non-Darcy flow in double-porosity, fractured rock with different ω parameters and non-Darcy flow coefficients.

the accuracy of these numerical simulation results, and the numerical solutions are considered to be "exact" in the following verification examples.

[16] Figure 1 shows a comparison of the numerical modeling results and the approximate analytical solution (18) with different dimensionless non-Darcy flow coefficients (defined in (10)). The two characteristic parameters for these cases are: $\lambda = 6 \times 10^{-5}$ and $\omega = 2 \times 10^{-3}$ from the parameters used as listed in Table 1. Note that the values of these two parameters are within a typical range of double-porosity flow behavior as discussed by Warren and Root.

[17] Figure 1 shows an excellent agreement between the analytical (circled symbol curves, labeled as $P_{D,WR} + \beta_D$) and numerical (solid curves) solutions, except at earlier times ($t_D <$ 100) or for large non-Darcy flow coefficients ($\beta_D > 10$). Note that a dimensionless time ($t_D = 219$) corresponds to 1 s of actual time for the parameters listed in Table 1. In addition, the Warren-Root solution itself may not be valid for $t_D < 100$. On the other hand, values of dimensionless non-Darcy flow coefficients >10 are considered very large in fields (e.g., Figure 1 shows that at $t_D = 10^4$, the dimensionless pressure for $\beta_D = 20$ is 5 times higher than that predicted by the Warren-Root solution). This non-Darcy flow effect may be too large for a normal well flow problem. Therefore, early time errors with the proposed analytical solution may not limit its practical applications, considering that during very early times (i.e., the first several seconds), no accurate well testing data can normally be measured because of well bore storage or other well bore conditions effects.

[18] Figure 2 examines possible effects of the λ parameter. Here we increase and decrease matrix permeability k_m to vary the λ parameter by one order of magnitude lower and higher, respectively. Figure 2 indicates that for the cases with the lower matrix permeability scenario ($\lambda = 6 \times 10^{-6}$), the approximate analytical solution becomes more accurate than for the high matrix permeability cases ($\lambda = 6 \times 10^{-4}$) overall. For a lower value of the non-Darcy flow coefficient ($\beta_D = 5$) the approximate analytical solution is good even for the very early times. In the early times up to $t_D = 10^4$ or 50 s of real times for the problem, large errors are observed to occur only with the high matrix permeability case ($\lambda = 6 \times 10^{-4}$).

[19] Figure 3 presents the results for checking the effect of the ω parameter ($\omega = 2 \times 10^{-4}$, 2×10^{-3}, and 2×10^{-2} by varying fracture porosity). Figure 3 shows that the smaller ω is, the more accurate the analytical solution becomes because non-Darcy flow effect develops slower with large fracture porosity near the well. For example, a larger value, $\omega = 2 \times 10^{-2}$, introduces significant errors in earlier times up to $t_D = 10^3$ (or 5 s real time) for the analytical solution. However, as shown in Figure 3, the "late" time (more than several seconds in real times) solutions are all very accurate when compared with those of the numerical solutions.

[20] The λ parameter increases as matrix permeability increases, and the ω parameter increases with the increase in fracture porosity. In both cases, the formation is said to be approaching a single-porosity medium or getting further away from the definition of a typical fractured reservoir. A typical double-porosity, fractured reservoir is characteristic of two features: (1) $k_f \gg k_m$ and (2) $\phi_m \gg \phi_f$. Under these two conditions, both λ and ω parameters are small. In addition, Figures 1–3 show that errors in the analytical solutions occur mainly at an earlier time within 100 s of flow, during which an accurate pressure measurement is very difficult to make. Therefore, for a wide range of practical applications, the analytical solution (18) may provide a good approximation for analyzing non-Darcy flow in a double-porosity reservoir.

5. Concluding Remarks

[21] This work presents an approximate analytical solution for analyzing non-Darcy flow through double-porosity fractured media. The analytical solution, a direct extension of the Warren-Root solution to non-Darcy flow situation, has been examined using a numerical method for its accuracy and applicability. It has been shown that the analytical solution provides good approximations for many practical applications in non-Darcy flow in fractured reservoirs, as long as the two conditions, $k_f \gg k_m$ and $\phi_m \gg \phi_f$, are satisfied. In general, the approximate analytical solution is found to be accurate for the flat or transitional portion as well as the late time (second straight line portion of pressure versus time curves. However, the solution will introduce errors in early transient times of well flow.

[22] We can draw several conclusions from this work. The behavior of non-Darcy flow through a fractured reservoir is characteristic of the three parameters: λ, ω, and β_D. The very form of the approximate analytical solution (equation (18)) itself indicates that the non-Darcy effect is similar to a skin effect on Darcy flow into a well in a fractured reservoir. These two factors (non-Darcy flow and skin effect) are inseparable from a single non-Darcy flow testing result. This observation also indicates that non-Darcy flow in a fractured reservoir is controlled by near-well flow regimes because of the much higher flow velocities there. Since skin and non-Darcy flow effects cannot be separated from a single well test under non-Darcy flow condition, we recommend that skin effects be estimated using a low flow rate or Darcy flow test first. Then, non-Darcy flow coefficients can be effectively determined using type curve fitting methods with the approximate analytical solution. Furthermore, similarity between the Warren-Root solution and the non-Darcy flow analytical solution makes it possible to use the well-testing analyses developed from the Warren-Root solution to determine formation parameters for non-Darcy flow in fractured reservoirs.

Appendix A: Derivation of Steady State Solution

[23] At steady state, fracture-matrix flow under the double-porosity conceptualization is no longer occurring, and the flow through a double-porosity medium becomes flow through the fracture system only. The steady state flow solution can then be derived for a problem of fluid flow from a fully penetrating well in a finite, radial system, subject to a constant outer boundary pressure.

$$\frac{\partial}{\partial r}\left[\rho\left(P_f\right) r\, v\right] = 0, \qquad \text{(A1)}$$

where v is volumetric flow rate along the r direction. At the outer boundary ($r = r_e$),

$$P_f(r = r_e) = P_i \quad \text{(constant)} \qquad \text{(A2)}$$

and at the inner boundary of the well bore, $r = r_w$, the fluid is produced at a constant mass (not volumetric) rate,

$$2\pi r_w h [\rho\, v_r]_{r=r_w} = q_m \qquad \text{(constant)}. \qquad \text{(A3)}$$

Integrating (A1) and using the well boundary condition (A3) and the following relation

$$v = \frac{1}{2k_f\rho\beta}\left[-\mu + \left(\mu^2 + 4k_f^2\rho\beta\frac{\partial P_f}{\partial r}\right)^{1/2}\right], \qquad \text{(A4)}$$

we will have

$$4k_f^2\rho\beta\frac{\partial P_f}{\partial r} = 2\mu\frac{k_f q_m\beta}{\pi h}\frac{1}{r} + \left(\frac{k_f q_m\beta}{\pi h}\frac{1}{r}\right)^2. \qquad \text{(A5)}$$

Assuming a constant density, integrating (A5), and using the outer boundary condition, we will have the steady state solution of (15) (in terms of dimensionless variables). For a general case that the density is a function of pressure, the steady state solution is given by *Wu* [2002].

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