WATER RESOURCES RESEARCH, VOL. 37, NO. 12, PAGES 2943–2950, DECEMBER 2001

Non-Darcy displacement of immiscible fluids in porous media

Yu-Shu Wu

Earth Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California, USA

Abstract. This paper presents a Buckley-Leverett analytical solution for non-Darcy displacement of two immiscible fluids in porous media. The multiphase non-Darcy displacement is described using a Forchheimer equation or other non-Darcy flow correlations under multiphase flow conditions. The analytical solution is used to obtain some insight into the physics of displacement involving non-Darcy flow effects in porous media. The solution reveals how non-Darcy displacement is controlled not only by relative permeability curves but also by non-Darcy flow coefficients as well as injection or flow rates. This analytical solution is then applied to verify a numerical simulator for modeling multiphase non-Darcy flow.

1. Introduction

Flow and displacement of multiphase fluids through porous media occur in many subsurface systems in the areas of oil, gas, and geothermal reservoir engineering, vadose zone hydrology, and soil sciences. *Buckley and Leverett* [1942] established the fundamental principle for flow and displacement of immiscible fluids through porous media in their classic study of fractional flow theory. Their solution involves the noncapillary displacement process of two incompressible, immiscible fluids in a one-dimensional, homogeneous system. Several forms of analytical solutions with capillary effects have also been presented in the literature [*Yortsos and Fokas*, 1983; *Chen*, 1988; *McWhorter and Sunada*, 1990]. In addition, the Buckley-Leverett solution has been extended to flow in a composite, one-dimensional heterogeneous system [*Wu et al*., 1993].

The Buckley-Leverett fractional flow theory has been applied and generalized to study enhanced oil recovery problems [*Pope*, 1980], surfactant flooding [*Larson and Hirasaki*, 1978; *Hirasaki*, 1981], polymer flooding [*Patton et al*., 1971; *Hirasaki and Pope*, 1974], the mechanisms of chemical methods [*Larson et al*., 1982], detergent flooding [*Fayers and Perrine*, 1959], displacement of oil and water by alcohol [*Wachmann*, 1964; *Taber et al*., 1961], displacement of viscous oil by hot water and chemical additive [*Karakas et al*., 1986], and alkaline flooding [*de Zabala et al*., 1982]. An extension to more than two immiscible phases dubbed "coherence theory" was described by *Helfferich* [1981]. The more recent example in the development of the Buckley-Leverett theory is the extension to non-Newtonian fluid flow and displacement [*Wu et al*., 1991, 1992]. However, studies of multiphase non-Darcy flow have received little attention in the literature [*Evans and Evans*, 1988; *Wang and Mohanty*, 1999].

The effects of non-Darcy or high-velocity flow regimes in porous media have long been noticed and investigated for porous media flow [e.g., *Tek et al*., 1962; *Scheidegger*, 1972; *Katz and Lee*, 1990]. However, theoretical, field, and experimental studies performed so far on non-Darcy flow in porous media have focused mostly on single-phase flow conditions that pertain to the oil and gas industry [*Tek et al*., 1962; *Swift and Kiel*, 1962; *Lee et al*., 1987]. Some investigations have been conducted for non-Darcy flow in fractured reservoirs [*Skjetne et al*., 1999] and for non-Darcy flow into highly permeable fractured wells [*Guppy et al*., 1981, 1982]. Those studies have concentrated on finding and validating correlations of non-Darcy flow coefficients [*Liu et al*., 1995].

There are several studies reported in the literature that extend the Forchheimer equation to multiphase flow and provide equations for correlating non-Darcy flow coefficients under multiphase conditions [*Evans et al*., 1987; *Evans and Evans*, 1988; *Liu et al*., 1995]. A recent study [*Wang and Mohanty*, 1999] has discussed the importance of multiphase non-Darcy flow in gas-condensate reservoirs and presents a pore-scale network model for describing non-Darcy gas-condensate flow. Because of the insufficient studies as well as the mathematical difficulty in handling highly nonlinear, non-Darcy flow terms in multiphase flow equations, our knowledge is very limited in the area of non-Darcy multiphase flow through porous media. In general, compared with multiphase Darcy flow, the mechanism of immiscible displacement involving non-Darcy porous media flow is not well understood.

This paper presents an analytical solution describing the displacement mechanism of non-Darcy multiphase flow in porous media. The analysis approach follows upon the work for multiphase non-Newtonian fluid flow and displacement in porous media [*Wu et al*., 1991, 1992] and results in an analytical solution that includes effects of non-Darcy multiphase displacement. A practical procedure for evaluating the behavior of the analytical solution that is similar to the graphic method by *Welge* [1952] for solving the Buckley-Leverett problem is also provided. The analytical solution and the resulting procedure can be regarded as an extension of the Buckley-Leverett theory to the non-Darcy flow problem of two-phase immiscible fluids in porous media.

The analytical results reveal that the saturation profile and displacement efficiency are controlled not only by relative permeabilities, as in the Buckley-Leverett solution, but also by the non-Darcy flow equations and parameters as well as injection rates. Using the new analytical solution, this paper discusses how immiscible displacement of two fluids is affected by non-Darcy flow. In addition, the analytical solution is used to check numerical solutions from a reservoir simulator for simulating multiphase non-Darcy flow.

Paper number 2001WR000389.
0043-1397/01/2001WR000389$09.00

2. Governing Equations

Consider the flow of two immiscible fluids (one wetting and one nonwetting phase) in a homogeneous, isothermal, and isotropic porous medium. Assume that no interphase mass transfer occurs between the two fluids and ignore dispersion and adsorption effects. The governing equation for fluid f is then given by the mass conservation equation,

$$-\nabla \cdot (\rho_f \mathbf{v}_f) = \frac{\partial}{\partial t}(\rho_f S_f \phi) \tag{1}$$

where ρ_f is the density of fluid f ($f = w$ for the wetting phase and $f = n$ for the nonwetting phase), $\mathbf{v}_f$ is the Darcy (or volumetric flow) velocity of fluid f, S_f is the saturation of fluid f, t is time, and ϕ is the effective porosity of formation.

The Darcy velocity (volumetric flow rate) in non-Darcy flow for each fluid and its dependence on other parameters need to be defined before the governing equation (2) can be solved. On the basis of experimental data and analyses several recent studies [e.g., *Evans and Evans*, 1988; *Liu et al.*, 1995] extended the Forchheimer-type equation to two-phase non-Darcy flow condition:

$$-(\nabla P_f - \rho_f \mathbf{g}) = \frac{\mu_f}{k k_{rf}} \mathbf{v}_f + \beta_f \rho_f \mathbf{v}_f |\mathbf{v}_f| \tag{2}$$

where k is the absolute permeability of the porous media, $\mathbf{g}$ is the gravitational constant, k_{rf} is the relative permeability to fluid f, μ_f is the dynamic viscosity of fluid f, and β_f is the effective non-Darcy flow coefficient (per meter) for fluid f under multiphase flow conditions [*Evans and Evans*, 1988]. Under single-phase flow condition this coefficient is traditionally called the turbulence coefficient or the inertial resistance coefficient [*Tek et al.*, 1962; *Lee et al.*, 1987]. Note that to include multiphase effects on non-Darcy flow, (2) is modified by the following: (1) Pressure gradient is replaced by flow potential gradient (the left-hand side term of (2)) to include gravity effects. (2) Absolute permeability is replaced by an effective permeability term (kk_{rf}). (3) β_f is described as the effective non-Darcy flow coefficient (per meter) for a flowing phase under multiphase flow conditions.

Note that the reasoning for this extension of the Forchheimer-type equation to two-phase non-Darcy flow is based on very limited laboratory testing and analysis only [*Evans et al.*, 1987; *Evans and Evans*, 1988; *Liu et al.*, 1995]. The physical model has not been thoroughly established, contrary to the viscous control case, i.e., the Darcy flow. The validity of such an extension needs further investigation.

Darcy law describes a linear relationship between volumetric flow rate and pressure (head or potential) gradient in porous media. Any deviations from this linear relation may be defined as non-Darcy flow. In this work the only concern is with the non-Darcy flow caused by high flow velocities. The linear term, the first term $[(\mu_f/kk_{rf})\mathbf{v}_f]$ on the right-hand side of (2), represents viscous flow, and it is dominant at low flow rates. Additional pressure drop or energy assumption due to non-Darcy or high flow velocities is described by the second term $(\beta_f \rho_f \mathbf{v}_f |\mathbf{v}_f|)$ on the right-hand side of (2) for the extra friction or inertial effects [*Katz and Lee*, 1990]. Equation (2) indicates that the non-Darcy flow equation reduces to the multiphase Darcy law if the non-Darcy term $(\beta_f \rho_f \mathbf{v}_f |\mathbf{v}_f|)$ can be ignored, when compared with the first term $[(\mu_f/kk_{rf})\mathbf{v}_f]$ for low flow velocity. For high velocities, however, the second term becomes dominant, and it must be included. Therefore Darcy flow can generally be considered as a special case of non-Darcy flow as described by (2).

On the basis of their experimental study of single-phase flow, *Tek et al.* [1962] correlated the non-Darcy flow coefficient to porosity and permeability for a given porous formation:

$$\beta = \frac{C_\beta}{k^{5/4}\phi^{3/4}}, \tag{3}$$

where C_β is a non-Darcy flow constant with a unit of meters$^{3/2}$ if converted to SI units. A recent study [*Liu et al.*, 1995] indicates that the β coefficient may be also correlated to tortuosity or the representative length of tortuous flow paths in pore structures of a porous material.

The empirical correlations, e.g., (3), for estimating the non-Darcy flow coefficient, β, were originally determined experimentally for single-phase flow. Some recent work has extended the single-phase non-Darcy flow correlations, such as (3), to incorporate the multiphase effect in terms of the saturation and effective permeability correction [*Liu et al.*, 1995; *Evans and Evans*, 1988]. These studies show that non-Darcy flow coefficients are dependent on relative permeability functions as well as fluid saturations under multiphase flow conditions. Non-Darcy flow coefficients may change significantly as mobile or immobile phase saturation changes. Using a pore-scale, microscope model, *Wang and Mohanty* [1999] show that non-Darcy flow coefficients for gas-condensate fluid flow depend not only on wetting phase saturation but also on capillary numbers or pressure gradients to a certain extent.

Note that a capillary number is defined as [*Willhite*, 1986]

$$N_c = \frac{\mu_f v}{\sigma} \tag{4}$$

where v is the magnitude of Darcy velocity for a fluid and σ is the interface tension between the two phases. For non-Darcy flow the velocity, as shown in (2), depends on pressure gradients and relative permeability among other factors and is interrelated with non-Darcy flow coefficient. Therefore it may be logical and reasonable to assume that the non-Darcy flow coefficient in (2) for multiphase non-Darcy flow can be, in general, expressed as a function of fluid saturation, relative permeability, and pressure gradients:

$$\beta_f = \beta_f(S_f, k_{rf}, \nabla P_f) \tag{5}$$

for a given porous medium.

Equation (2) implicitly defines the Darcy velocity as a function of pressure gradient as well as saturation, relative permeability, and effective non-Darcy flow coefficient. A more general relation for the Darcy velocity in multiphase non-Darcy flow may be proposed as follows:

$$\mathbf{v}_f = \mathbf{v}_f(\nabla P_f, S_f, k_{rf}) \tag{6}$$

as a function of pressure gradient, saturation, and relative permeability functions. It should be mentioned that (6) implicitly includes effects of non-Darcy flow coefficient with the definition of (5). With (6) many other kinds of equations for non-Darcy flow, in addition to the Forchheimer equation [e.g., *Scheidegger*, 1972], can be selected, if available, to describe multiphase non-Darcy flow.

Similar to multiphase Darcy flow, k_{rf} and capillary pressure (P_c) may be assumed to be functions only of saturations under non-Darcy flow conditions. However, some recent studies

[*Henderson et al.*, 1997; *Wang and Mohanty*, 1999; *Pope et al.*, 1998] indicate that in addition to dependence on saturation the relative permeability for multiphase non-Darcy flow may also depend on capillary number, which, in turn, can be expressed as a function of pressure gradients, saturation, etc., as discussed above. We may assume the following general function:

$$k_{rf} = k_{rf}(S_w, \nabla P_f) \tag{7}$$

to include non-Darcy flow effects on relative permeability.

Note that relative permeability functions of (6) should be determined using the selected non-Darcy flow equation, such as (2), under a two-phase, non-Darcy flow condition instead of using a linear Darcy law as in the case of Darcy flow. For capillary pressure we still use

$$P_c = P_n - P_w = P_c(S_w) \tag{8}$$

$$S_w + S_n = 1. \tag{9}$$

3. Analytical Solution

For the derivation of the analytical solution we assume the following Buckley-Leverett flow conditions [*Wu et al.*, 1991]: (1) Both fluids and the porous medium are incompressible. (2) Capillary pressure gradient is negligible. (3) Gravity segregation effect is negligible (i.e., stable displacement exists near the displacement front). (4) One-dimensional flow and displacement is along the x coordinate of a semi-infinite linear flow system with a constant cross-sectional area (A).

Among these assumptions, incompressibility of fluids and formation is critical to deriving the Buckley-Leverett solution. This assumption provides a good approximation to displacement processes of two liquids (e.g., oil and water) through porous media, because of the small compressibilities of the two fluids. For gas and liquid displacement, however, this assumption may pose certain limitation to the resulting solution, when large pressure gradients build up in a flow system, such as in the case of non-Darcy gas-water flow near a well in a low-permeability formation. In many cases, however, this assumption may still provide acceptable approximations because the viscosity of the gas (air) phase in normal conditions is ~2 orders of magnitude lower than the liquid phase. This tends to prevent high-pressure gradients from building up, as in the case of the Buckley-Leverett solution, which was also derived for oil and gas displacement originally.

Equation (1) can then be changed to read as follows:

$$-\frac{\partial v_f}{\partial x} = \phi \frac{\partial S_f}{\partial t}, \tag{10}$$

where v_f is the Darcy velocity component or volumetric flow rate (m/s) per unit area of formation for fluid f. For the one-dimensional flow, v_f can be determined from (2) or (6). When using (2), we have

$$v_f = \frac{1}{2k\rho_f\beta_f}\left\{-\frac{\mu_f}{k_{rf}} + \left[\left(\frac{\mu_f}{k_{rf}}\right)^2 - 4k^2\rho_f\beta_f\left(\frac{\partial P}{\partial x} + \rho_f g \sin(\alpha)\right)\right]^{1/2}\right\}, \tag{11}$$

where $(\partial P/\partial x)$ is a component of the pressure gradient along the x coordinate, the same for the wetting or nonwetting phase (since there is no difference in capillary gradients of the two phases), g is the gravitational acceleration constant, and α is the angle between the horizontal plane and the flow direction (the x coordinate).

To complete the mathematical description of the physical problem, the initial and boundary conditions must be specified. For simplicity in derivation the system is initially assumed to be uniformly saturated with both wetting and nonwetting fluids. The wetting phase is at its residual saturation, and a nonwetting fluid, such as oil or gas, is at its maximum saturation in the system as follows:

$$S_n(x, t = 0) = 1 - S_{wr}, \tag{12}$$

where S_{wr} is the initial, residual wetting phase saturation. Wetting fluid, such as water, is continuously being injected at a known rate $q(t)$, generally a function of injection time t. Therefore the boundary conditions at the inlet ($x = 0$) are

$$v_w(x = 0, t) = q(t)/A \tag{13}$$

$$v_n(x = 0, t) = 0. \tag{14}$$

The derivation of the analytical solution follows the work by *Wu et al.* [1991], in which the fractional flow concept is used to simplify the governing equation (10) in terms of saturation only. The fractional flow of a fluid phase is defined as a volume fraction of the phase flowing at a location x and time t to the total volume of the flowing phases [*Willhite*, 1986]. The fractional flow can be written as

$$f_f = \frac{v_f}{v_w + v_n} = \frac{v_f}{v(t)}, \tag{15}$$

where the total flow is

$$v(t) = v_w + v_n. \tag{16}$$

From volume balance due to incompressibility of the system we have

$$f_w + f_n = 1. \tag{17}$$

The fractional flow function for the wetting phase may be written in the following form:

$$f_w = \frac{1}{1 + \dfrac{\rho_w\beta_w}{\rho_n\beta_n}\left\{\dfrac{-\dfrac{\mu_n}{k_{rn}} + \left[\left(\dfrac{\mu_n}{k_{rn}}\right)^2 - 4k^2\rho_n\beta_n\left(\dfrac{\partial P}{\partial x} + \rho_n g \sin(\alpha)\right)\right]^{1/2}}{-\dfrac{\mu_w}{k_{rw}} + \left[\left(\dfrac{\mu_w}{k_{rw}}\right)^2 - 4k^2\rho_w\beta_w\left(\dfrac{\partial P}{\partial x} + \rho_w g \sin(\alpha)\right)\right]^{1/2}}\right\}} \tag{18}$$

when the Forchheimer equation, (11) is used for non-Darcy flow.

In general, relative permeability functions and effective non-Darcy flow coefficients, as discussed in section 2, are functions of saturation and pressure gradients, and (18) indicates that the fractional flow f_w of the wetting phase is also a function of both saturation and pressure gradient. However, for a given injection rate at a time and for given fluid and rock properties of a porous material the pressure gradient at a given time can be shown by the following to be a function of saturation only under the Buckley-Leverett flow condition:

$$q(t) - \frac{A}{2k\rho_w\beta_w}\left\{-\frac{\mu_w}{k_{rw}} + \left[\left(\frac{\mu_w}{k_{rw}}\right)^2 - 4k^2\rho_w\beta_w\left(\frac{\partial P}{\partial x} + \rho_w g \sin(\alpha)\right)\right]^{1/2}\right\} - \frac{A}{2k\rho_n\beta_n}\left\{-\frac{\mu_n}{k_{rn}} + \left[\left(\frac{\mu_n}{k_{rn}}\right)^2 - 4k^2\rho_n\beta_n\left(\frac{\partial P}{\partial x} + \rho_n g \sin(\alpha)\right)\right]^{1/2}\right\} = 0 \tag{19}$$

Equation (19) shows that the pressure gradient and the saturation are interdependent on each other for this particular displacement system. Therefore, (19) implicitly defines the pressure gradient in the system as a function of saturation. When another non-Darcy flow equation, other than the Forchheimer equation, is chosen, a similar correlation between saturation and pressure gradient can be easily derived from Equation (16) directly, instead of (19).

The governing equation, (10), subject to the boundary and initial conditions described in (12)–(14) can be solved as follows [*Wu et al.*, 1991]:

$$\left(\frac{dx}{dt}\right)_{S_w} = \frac{q(t)}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_t. \tag{20}$$

This is the frontal advance equation for the non-Darcy displacement of two immiscible fluids, and it has the same form as the Buckley-Leverett equation. However, the dependence of the fractional flow f_w for the non-Darcy displacement on saturation is different. The fractional flow, f_w, is related to saturation not only through the relative permeability functions but also through the non-Darcy flow relation, as described by (5).

Equation (20) shows that for a given time and a given injection rate a particular wetting fluid saturation profile propagates through the porous medium at a constant velocity. As in the Buckley-Leverett theory, the saturation for a vanishing capillary pressure gradient will, in general, become a triple-valued function of distance near the displacement front [*Cardwell*, 1959]. Equation (20) will then fail to describe the velocity of the shock saturation front since $(\partial f_w/\partial S_w)$ does not exist on the front because of the discontinuity in S_w at that point. Consideration of material balance across the shock front [*Sheldon et al.*, 1959] provides the velocity of the front:

$$\left(\frac{dx}{dt}\right)_{S_F} = \frac{q(t)}{\phi A}\left(\frac{f_w^+ - f_w^-}{S_w^+ - S_w^-}\right)_t, \tag{21}$$

where S_F is the displacement front saturation of the displacing, wetting phase. The plus and minus superscripts refer to values immediately ahead of and behind the front, respectively.

The location x_{S_w} of any saturation S_w traveling from the inlet at time t can be determined by integrating (20) with respect to time, which yields

$$x_{S_w} = \frac{1}{\phi A}\int_0^t q(\tau)\left(\frac{\partial f_w}{\partial S_w}\right)_{S_w} d\tau. \tag{22}$$

This shows that for a general, time-varying injection rate, $q(t)$, the derivative, $(\partial f_w/\partial S_w)$, of fractional flow with respect to saturation within the integral is also a time-dependent function (see (19)). Therefore the solution (22) for non-Darcy displacement differs from the Buckley-Leverett solution.

If a constant injection rate is proposed, then (22) becomes

$$x_{S_w} = \frac{qt}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_{S_w}, \tag{23}$$

where q is the constant injection rate and qt is the cumulative volume of the injected fluid.

Direct use of (23), given x and t, will result in a multiple-valued saturation distribution, which can be handled by a mass balance calculation, as in the Buckley-Leverett solution. An alternative graphic method of *Welge* [1952] can be shown [*Wu et al.*, 1991] to apply to calculating the above solution in this case. The only additional step in applying this method is to take into account the contribution of the pressure gradient dependence on the non-Darcy flow coefficient, using a fractional flow curve. Therefore the wetting phase saturation at the displacement saturation front may be determined by

$$\left(\frac{\partial f_w}{\partial S_w}\right)_{S_F} = \frac{(f_w)_{S_F} - (f_w)_{S_{wr}}}{S_F - S_{wr}}. \tag{24}$$

The average saturation in the displaced zone is given by

$$\left(\frac{\partial f_w}{\partial S_w}\right)_{S_F} = \frac{1}{\bar{S}_w - S_{wr}} \tag{25}$$

where $\bar{S}_w$ is the average saturation of the wetting phase in the swept zone behind the sharp displacement front. Then, the complete saturation profile can be determined using (23) for a given non-Darcy displacement problem with constant injection rate.

4. Discussion

In this section the analytical solution presented above is used to give us some insight into non-Darcy flow and displacement phenomena. The physical flow model is a one-dimensional linear porous medium, which is at first saturated uniformly with a nonwetting fluid ($S_n = 0.8$) and a wetting fluid ($S_w = S_{wr} = 0.2$). A constant volumetric injection rate of the wetting fluid is imposed at the inlet ($x = 0$), starting from $t = 0$. The relative permeability curves used for all the calculations in this chapter are shown in Figure 1, and their pressure gradient dependence is ignored in the following problems, because there are no explicit form expressions for such dependence available in the literature. The properties of the rock and fluids used are listed in Table 1. The solution (23) is used to obtain the saturation profiles, with the sharp front saturation determined by (24). The solution evaluation procedure, as discussed in section 3, is computer programmed for the analysis.

In the literature, there are several correlations for single-phase non-Darcy flow coefficients, which have been extended for analyzing experimental multiphase non-Darcy flow [*Evans and Evans*, 1988]. We select one of those extended relations for use in the demonstration examples. The selected expression is similar to that defined by (3), and the multiphase extension is given by

$$\beta_f(S_w, k_{rf}) = \frac{C_\beta}{(kk_{rf})^{5/4}[\phi(S_f - S_{fr})]^{3/4}}, \tag{26}$$

where S_{fr} is residual saturation of fluid f. Equation (26) is incorporated into the calculation of the fractional flow to solve pressure gradients and then fractional flow corresponding to saturations under different flow conditions. It should be mentioned that pressure gradient dependence in determining non-

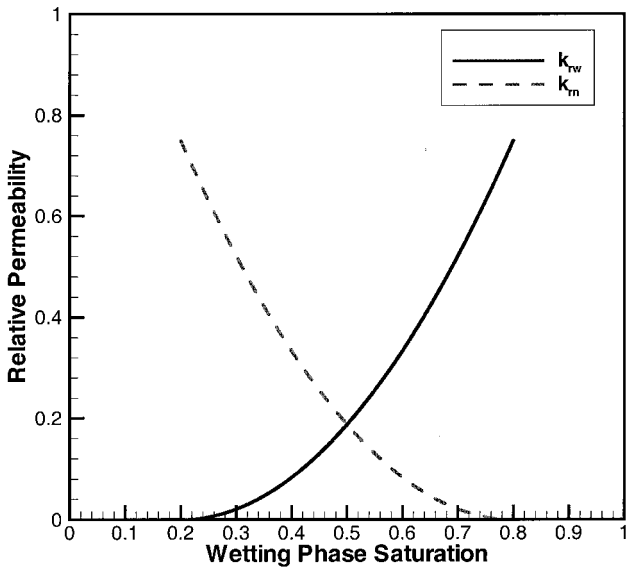

Figure 1. Relative permeability curves used in analytical and numerical solutions for non-Darcy displacement.

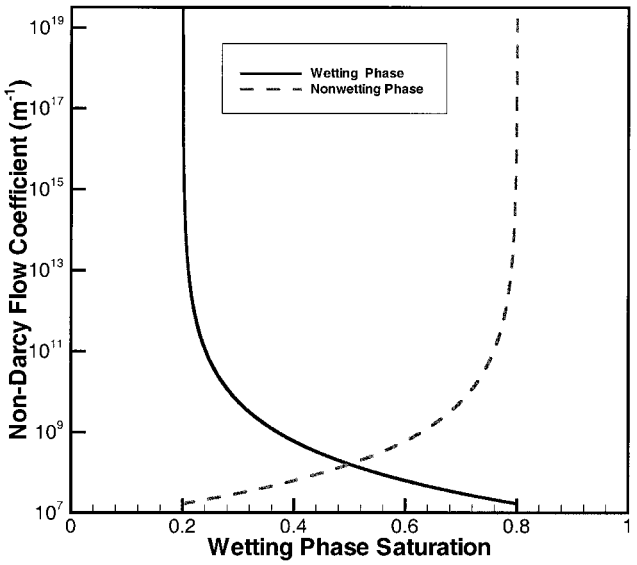

Figure 2. Non-Darcy flow coefficients as a function of displacing phase saturation ($C_\beta = 3.2 \times 10^{-9}$ $m^{3/2}$).

Darcy flow coefficients is also ignored because of the lack of available functions.

For a given operating condition of constant injection rate the solution (23) shows that non-Darcy fluid displacement in a porous medium is characterized not only by relative permeability data, as in Buckley-Leverett displacement, but also by non-Darcy flow coefficients of the two fluids. Using the results from the analytical solution, some fundamental aspects of non-Darcy fluid displacement will be established. Figure 2 shows the relationship between non-Darcy flow coefficients versus saturation, as described by (26), with the relative permeability functions of Figure 1. The large (or infinite) values of non-Darcy flow coefficients at both ends of the wetting phase saturation of Figure 2 are a result of one phase becoming nearly immobile. This is equivalent to that when approaching the two ends; decrease in effective permeability and porosity to the disappearing phase causes an increase in non-Darcy flow coefficients [*Evans et al.*, 1987]. This behavior is also consistent with observations in the laboratory tests under low saturation of a phase [*Evans and Evans*, 1988]. Physically, for any flow to occur under near residual saturation or zero effective permeability condition, the corresponding non-Darcy flow coefficient must tend toward infinity.

Figure 3, determined using (19) for the flow system, shows that pressure gradients change significantly as a function of saturation for different non-Darcy flow constants. At both high and low values for the wetting phase saturation the pressure gradients become relatively smaller because the total flow resistance decreases as the flow is close to single-phase flow condition. In addition, Figure 3 shows that as the non-Darcy flow constant increases, the pressure gradient increases at the same saturation value under the same injection rate, and this is due to a larger non-Darcy flow term or a large second term on the right-hand side of (2).

The resulting fractional flow curves and their derivatives with the three non-Darcy flow constants are shown in Figure 4.

Table 1. Parameters for the Non-Darcy Displacement Examples

Parameter	Value	Unit
Effective porosity	$\phi = 0.30$	
Permeability	$k_m = 9.869 \times 10^{-13}$	m^2
Wetting phase density	$\rho_w = 1000$	kg/m^3
Wetting phase viscosity	$\mu_w = 1.0 \times 10^{-3}$	Pa s
Nonwetting phase density	$\rho_n = 800$	kg/m^3
Nonwetting phase viscosity	$\mu_n = 5.0 \times 10^{-3}$	Pa s
Non-Darcy flow constants	$C_\beta = 3.2 \times 10^{-9}$	$m^{3/2}$
	$C_\beta = 3.2 \times 10^{-6}$	
	$C_\beta = 3.2 \times 10^{-5}$	
Injection rates	$q = 1.0 \times 10^{-5}$	m^3/s
	$q = 10 \times 10^{-4}$	
	$q = 1.0 \times 10^{-3}$	
Directional angle	$\alpha = 0$	

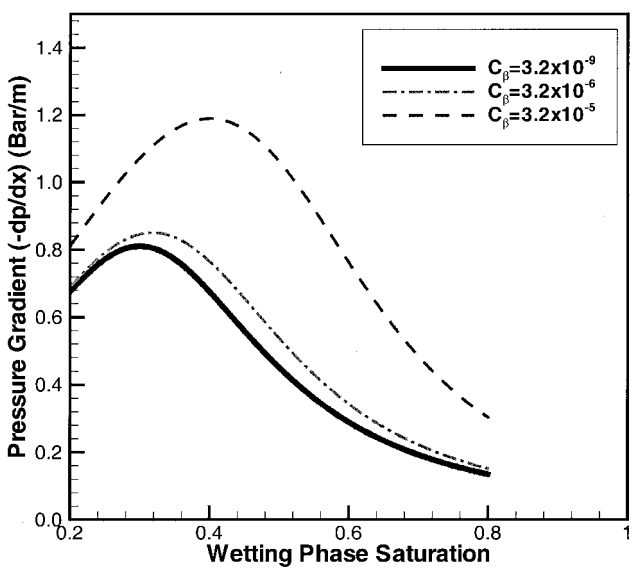

Figure 3. Pressure gradients versus displacing wetting phase saturation for different non-Darcy flow constants.

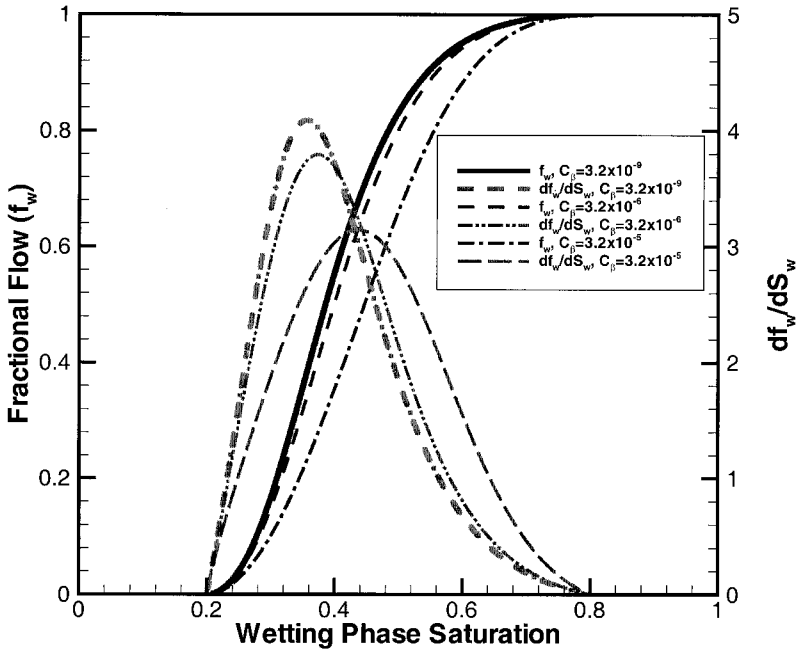

Figure 4. Fractional flow curves and their derivatives with respect to wetting phase saturation for different non-Darcy flow constants.

Note that fractional flow curves change with the non-Darcy flow constants because of the change in pressure gradient for different non-Darcy flow constants under the same saturation. Saturation profiles of displacement after a 10-hour injection period are plotted in Figure 5. In terms of higher sweeping efficiency or shorter displacement front travel distance a larger non-Darcy flow constant or coefficient gives lower wetting phase flow rates. This results in a better displacement efficiency: More nonwetting phase is displaced from the swept zone. On the other hand, the displacement becomes the Buckley-Leverett process as the non-Darcy constant, C_β, becomes small or tends to zero.

Effect of injection rates on non-Darcy displacement is shown in Figure 6, in which a constant non-Darcy flow constant, $C_\beta = 3.2 \times 10^{-6}$ m$^{3/2}$, is used with all three injection rates. Figure 6 indicates that non-Darcy displacement may be very sensitive to injection or flow rates. This rate-dependent displacement behavior is entirely different from a Buckley-Leverett or Darcy displacement, because the latter is independent of injection or flow rates. Under non-Darcy flow condition, Figure 6 shows

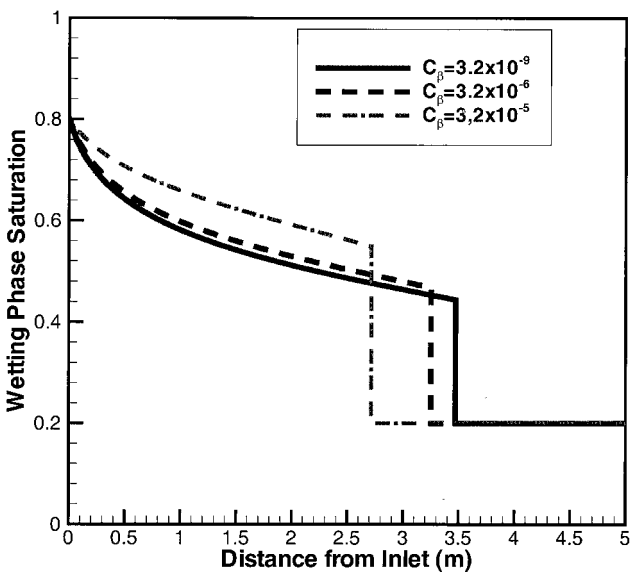

Figure 5. Saturation profiles of the non-Darcy displacement for different non-Darcy flow constants after 10 hours of injection.

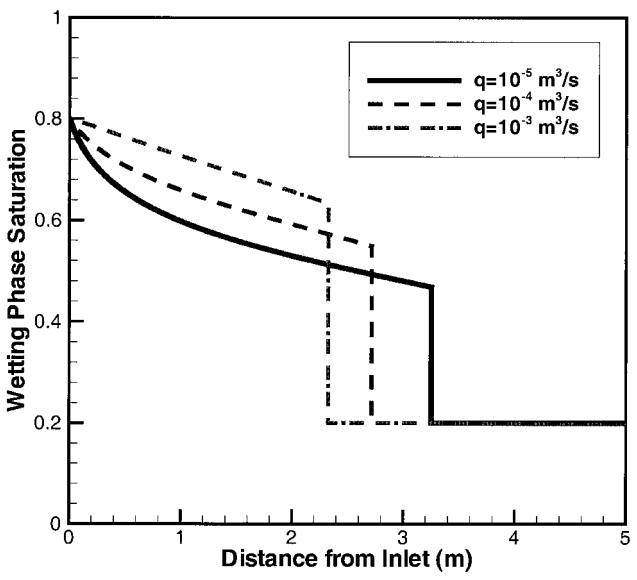

Figure 6. Saturation profiles of the non-Darcy displacement for different injection rates after injection of 0.36 (m^3) water ($C_\beta = 3.2 \times 10^{-6}$ m$^{3/2}$).

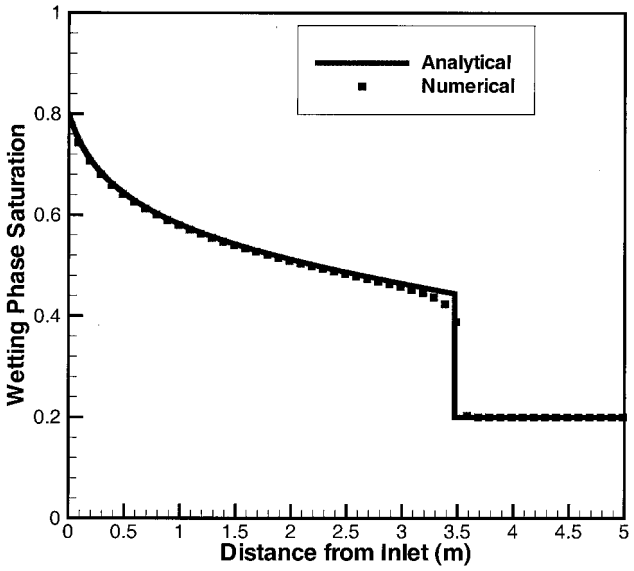

Figure 7. Comparison between saturation profiles calculated from analytical and numerical solutions after 10 hours of injection ($C_\beta = 3.2 \times 10^{-9}$ m$^{3/2}$).

that for the same volume of water injected with the three injection rates, saturation profiles in the system are very different. Larger injection rates display better sweeping efficiency overall. This is because higher injection rates create larger flow resistance to the displacing phase because of the non-Darcy effect, and as a result this will lower flow velocity of the displacing phase, relative to that of the displaced phase, resulting in a better displacement performance.

5. Application Example

In this section the analytical solution is used to examine the validity of the numerical method implemented in a general purpose, three-phase reservoir simulator, the MSFLOW code [*Wu*, 1998], for modeling multiphase non-Darcy flow and displacement processes. The Forchheimer equation is also used for the comparison.

To reduce the effects of discretization on numerical simulation results, very fine, uniform mesh spacing ($\Delta x = 0.01$ m) is chosen. A one-dimensional 5-m linear domain is discretized into 500 one-dimensional uniform grid blocks. The flow description and the parameters for this problem are identical to those in section 4 for the case of $C_\beta = 3.2 \times 10^{-9}$ (m$^{3/2}$). In the numerical simulation the non-Darcy flow coefficient, (26), is treated as a flow property and is estimated using a full upstream weighting scheme as that for the relative permeability function.

The comparison between the analytical and numerical solutions is shown in Figure 7. Figure 7 indicates that the numerical results are in excellent agreement with the analytical prediction of the non-Darcy displacement for the entire wetting phase sweeping zone. Except at the shock, advancing saturation front, the numerical solution deviates only slightly from the analytical solution, resulting from a typical "smearing front" phenomenon of numerical dispersion effects when matching the Buckley-Leverett solution using numerical results [*Aziz and Settari*, 1979].

6. Summary and Conclusions

This paper presents a Buckley-Leverett analytical solution and a theoretical study for non-Darcy displacement of two immiscible fluids through porous media. The multiphase non-Darcy flow can be described using an extended Forchheimer equation or other correlation describing multiphase non-Darcy flow. In addition, a procedure for evaluating the analytical solution is developed and its use is demonstrated.

The analytical solution, derived in this work for non-Darcy displacement, is based on the same assumptions as those used for the Buckley-Leverett solution. In addition, the present solution relies on one more critical assumption, that is, that relative permeability can be expressed a function of both saturation and pressure gradient to include possible effects of capillary number. If the Forchheimer equation is used, its effective non-Darcy flow coefficient is also treated as a function of saturation, relative permeability, and pressure gradient in this work. Because of the lack in published correlations for pressure gradient–dependent relative permeability or non-Darcy flow functions in the literature, applications of the analytical solution to a general case with pressure gradient–dependent effective permeability and non-Darcy flow coefficient are not examined. The physical model for governing multiphase non-Darcy flow is not well established in the literature, as compared with multiphase Darcy flow. Therefore the validity and reasonableness of the present solution and its applicability to an actual porous medium flow problem need further investigation.

The new analytical solution is used to obtain some insight into the physics of displacement involving non-Darcy flow when the effects of non-Darcy flow coefficients in porous media are included. The solution reveals that non-Darcy displacement is a more complicated process than the Darcy displacement described by the Buckley-Leverett solution. Multiphase non-Darcy flow and displacement are controlled not only by relative permeability curves, such as in Darcy displacement, but also by non-Darcy flow relations and parameters as well as injection or flow rates. As an example of application the analytical solution is applied to verify the numerical formulation of a numerical simulator for modeling multiphase non-Darcy flow.

The analytical solution of this work can be easily extended to other one-dimensional geometries, such as cylindrical, radial flow for a well flow problem, using the same procedure for Darcy displacement [*Willhite*, 1986]. In addition, the analytical solution is also applicable to displacement processes involving Darcy flow for one phase and non-Darcy flow for the other.

Acknowledgment. The author is indebted to C. Oldenburg for his critical review of this manuscript. The author would like to thank the two anonymous reviewers of WRR for their insightful, critical, and constructive comments and suggestions for improving this work. This work was supported in part by the Assistant Secretary for Energy Efficiency and Renewable Energy, Office of Geothermal and Wind Technologies of the U.S. Department of Energy under contract DE-AC03-76SF00098.

References

Aziz, K., and A. Settari, *Petroleum Reservoir Simulation*, Appl. Sci., London, 1979.

Buckley, S. E., and M. C. Leverett, Mechanism of fluid displacement in sands, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *146*, 107–116, 1942.

Cardwell, W. T. Jr., The meaning of the triple value in noncapillary

Buckley-Leverett theory, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *216*, 271–276, 1959.

Chen, Z. X., Some invariant solutions to two-phase fluid displacement problems including capillary effect, *SPE Reservoir Eng.*, *28*, 691–700, 1988.

de Zabala, E. F., J. M. Vislocky, E. Rubin, and C. J. Radke, A chemical theory for linear alkaline flooding, *Soc. Pet. Eng. J.*, *22*, 245–258, 1982.

Evans, E. V., and R. D. Evans, Influence of an immobile or mobile saturation on non-Darcy compressible flow of real gases in propped fractures, *JPT J. Pet. Technol.*, *40*(10), 1343–1351, 1988.

Evans, R. D., C. S. Hudson, and J. E. Greenlee, The effect of an immobile liquid saturation on the non-Darcy flow coefficient in porous media, *SPE Prod. Eng.*, *283*, 331–338, 1987.

Fayers, F. J., and R. L. Perrine, Mathematical description of detergent flooding in oil reservoirs, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *216*, 277–283, 1959.

Guppy, K. H., H. Cinco-Ley, and H. J. Ramey Jr., Effects of non-Darcy flow on the constant-pressure production of fractured wells, *SPEJ Soc. Pet. Eng. J.*, *21*, 390–400, 1981.

Guppy, K. H., H. Cinco-Ley, H. J. Ramey Jr., and F. Samaniego, Non-Darcy flow in wells with finite-conductivity vertical fractures, *SPEJ Soc. Pet. Eng. J.*, *22*, 681–698, 1982.

Helfferich, F. G., Theory of multicomponent, multiphase displacement in porous media, *SPEJ Soc. Pet. Eng. J.*, *21*, 51–62, 1981.

Henderson, G. D., A. Danesh, and D. H. Tehrani, The effect of velocity and interfacial tension on relative permeability of gas condensate fluids in the wellbore region, *J. Pet. Sci. Eng.*, *17*, 256–273, 1997.

Hirasaki, G. J., Application of the theory of multicomponent, multiphase displacement to three-component, two-phase surfactant flooding, *SPEJ Soc. Pet. Eng. J.*, *21*, 191–204, 1981.

Hirasaki, G. J., and G. A. Pope, Analysis of factors influencing mobility and adsorption in the flow of polymer solution through porous media, *Soc. Pet. Eng. J.*, *14*, 337–346, 1974.

Karakas, N., S. Saneie, and Y. Yortsos, Displacement of a viscous oil by the combined injection of hot water and chemical additive, *SPE Reservoir Eng.*, *1*, 391–402, 1986.

Katz, D. L., and R. L. Lee, *Natural Gas Engineering, Production and Storage*, McGraw-Hill, New York, 1990.

Larson, R. G., and G. J. Hirasaki, Analysis of the physical mechanisms in surfactant flooding, *Soc. Pet. Eng. J.*, *18*, 42–58, 1978.

Larson, R. G., H. T. Davis, and L. E. Scriven, Elementary mechanisms of oil recovery by chemical methods, *JPT J. Pet. Technol.*, *34*(2), 243–258, 1982.

Lee, R. L., R. W. Logan, and M. R. Tek, Effects of turbulence on transient flow of real gas through porous media, *SPE Form. Eval.*, 108–120, 1987.

Liu, X., F. Civan, and R. D. Evans, Correlations of the non-Darcy flow coefficient, *J. Can. Pet. Technol.*, *34*(10), 50–54, 1995.

McWhorter, D. B., and D. K. Sunada, Exact integral solutions for two-phase flow, *Water Resour. Res.*, *26*(3), 399–413, 1990.

Patton, J. T., K. H. Coats, and G. T. Colegrove, Prediction of polymer flood performance, *Soc. Pet. Eng. J.*, *251*, 72–84, 1971.

Pope, G. A., The application of fractional flow theory to enhanced oil recovery, *SPEJ Soc. Pet. Eng. J.*, *20*, 191–205, 1980.

Pope, G. A., W. Wu, G. Narayanaswamy, M. Delshad, M. Sharma, and P. Wang, Modeling relative permeability effects in gas-condensate reservoirs, paper SPE 49266 presented at 1998 SPE Annual Technical Conference and Exhibition, Soc. of Pet. Eng., New Orleans, La., Sept. 27–30, 1998.

Scheidegger, A. E., *The Physics of Flow Through Porous Media*, Univ. of Toronto Press, Toronto, Ont., Canada, 1972.

Sheldon, J. W., B. Zondek, and W. T. Cardwell Jr., One-dimensional, incompressible, noncapillary, two-phase fluid flow in a porous medium, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *216*, 290–296, 1959.

Skjetne, E., T. K. Statoil, and J. S. Gudmundsson, Experiments and modeling of high-velocity pressure loss in sandstone fractures, paper SPE 56414 presented at 1999 SPE Annual Technical Conference and Exhibition, Soc. of Pet. Eng., Houston, Tex., Oct. 3–6, 1999.

Swift, G. W., and O. G. Kiel, The prediction of gas-well performance including the effects of non-Darcy flow, *J. Pet. Technol.*, *222*, 791–798, 1962.

Taber, J. J., I. S. K. Kamath, and R. L. Reed, Mechanism of alcohol displacement of oil from porous media, *Soc. Pet. Eng. J.*, *1*, 195–212, 1961.

Tek, M. R., K. H. Coats, and D. L. Katz, The effects of turbulence on flow of natural gas through porous reservoirs, *J. Pet. Technol.*, *222*, 799–806, 1962.

Wachmann, C., A mathematical theory for the displacement of oil and water by alcohol, *Soc. Pet. Eng. J.*, *4*, 250–266, 1964.

Wang, X., and K. K. Mohanty, Multiphase non-Darcy flow in gas-condensate reservoirs, paper SPE 56486 presented at 1999 SPE Annual Technical Conference and Exhibition, Soc. of Pet. Eng., Houston, Tex., Oct. 3–6, 1999.

Welge, H. J., A simplified method for computing oil recovery by gas or water drive, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *195*, 91–98, 1952.

Willhite, G. P., *Waterflooding*, *SPE Textbk. Ser.*, vol. 3, Soc. of Pet. Eng., Richardson, Tex., 1986.

Wu, Y. S., *MSFLOW*: *Multiphase Subsurface Flow Model of Oil, Gas and Water in Porous and Fractured Media With Water Shut-off Capability, Documentation and User's Guide*, Twange Int., Houston, Tex., Walnut Creek, Calif., 1998.

Wu, Y. S., K. Pruess, and P. A. Witherspoon, Displacement of a Newtonian fluid by a non-Newtonian fluid in a porous medium, *Transp. Porous Media*, *6*, 115–142, 1991.

Wu, Y. S., K. Pruess, and P. A. Witherspoon, Flow and displacement of Bingham non-Newtonian fluids in porous media, *SPE Reservoir Eng.*, *7*, 369–376, 1992.

Wu, Y. S., K. Pruess, and Z. X. Chen, Buckley-Leverett flow in composite porous media, *SPE Adv. Technol. Ser.*, *1*(2), 36–39, 1993.

Yortsos, Y. C., and A. S. Fokas, An analytical solution for linear waterflood including the effects of capillary pressure, *SPEJ Soc. Pet. Eng. J.*, *23*, 115–124, 1983.

Y.-S. Wu, Earth Sciences Division, Lawrence Berkeley National Laboratory, MS 90-1116, One Cyclotron Road, Berkeley, CA 94720, USA. (yswu@lbl.gov)

(Received February 23, 2000; revised February 12, 2001; accepted July 1, 2001.)