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A virtual node method for handling well bore boundary conditions in modeling multiphase flow in porous and fractured media

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Abstract. How to treat well boundary conditions is a difficult issue when formulating and coding a multiphase numerical reservoir simulator. The difficulty arises because the partial differential equation governing multiphase subsurface flow is of a mixed parabolic-hyperbolic type. Maximum changes in primary variables and mass/heat fluxes occur at boundaries or well nodes. Consequently, these well nodes with small volume tend to be singular, leading to computational convergence problems. The conventional method of well treatment in geothermal or oil reservoir simulators is to use a sink/source term approach and to distribute flow rates by a potential or mobility allocation scheme for a multilayered well. However, this traditional method cannot handle a backflow problem, which may occur in a multilayered well in heterogeneous formations. This paper presents a "virtual node" method to handle a well bore either as a single node or several computational nodes screened and connected to many neighboring nodes for a multilayered well. The well bore can be vertical, inclined, or horizontal, and the well borehole node is treated in the same way as any other nonwell node for flow calculations. The solution at the well is then obtained by solving mass balance equations for the well node. It is shown that the new method provides a natural, physically consistent, and numerically efficient approach to handling well flow problems. Implementation of this new method for a three-phase flow reservoir simulator is discussed, and three examples are provided.

1. Introduction

Even with the continual progress made in both computational algorithms and computer hardware, efficient and rigorous well treatment in numerical simulation of multiphase subsurface flow remains a challenge. A well node is, in general, the most difficult, time-consuming part of a multiphase flow simulation because of the nature of the partial differential equations governing multiphase subsurface flow, which are of a mixed parabolic-hyperbolic type. Maximum changes in primary variables and mass fluxes occur at boundary or well nodes. Therefore well nodes are often singular and are the most difficult to converge during a simulation.

Numerical modeling studies play an important role in developing and evaluating economically feasible remediation schemes to clean up contamination in unsaturated and saturated aquifers [*Falta et al*., 1992]. Typical contaminants in shallow aquifers include volatile organic chemicals and nonaqueous phase liquids (NAPLs), which may be effectively cleaned up using in situ remediation techniques, such as soil-vapor extraction, air sparging, steam injection, or pump and treat. In many cases, efficient and rigorous treatment of well conditions for multiphase flow is critical to successful modeling investigations of a large-scale field site with a large number of remediation wells.

Strictly speaking, a boundary condition in modeling multiphase flow is a constraint condition rather than a rigorous "boundary condition" as used in the mathematical sense in solving partial differential equations [*Wu et al*., 1996]. The conventional method of well treatment in groundwater, geothermal, and oil reservoir simulators for multiphase flow is to use a sink/source term approach and distribute flow rates by a potential or mobility allocation scheme for a multilayered well [*Aziz and Settari*, 1979]. The potential allocation scheme, which is probably the most common method used in the geothermal and petroleum industries [*Thomas*, 1982], estimates correct total fluid injection/production rates as long as the maximum and minimum pressure constraints are not violated. However, this traditional method cannot handle a backflow problem, which may occur at a multilayered well with a thick, heterogeneous formation or a long horizontal well. Any potentially backflowing layers have their backflow set to zero by this method, and therefore the solution may not always be physically correct. In addition, solving for borehole pressure explicitly in order to eliminate the well bore mass balance equation from the entire equation system will introduce additional nonlinearity and increase convergence difficulties. The mobility allocation method, however, distributes grid layer fluxes along a well, based on a mobility ratio and without considering effects of pressure or potential gradients. This method is easy to implement but may result in physically incorrect solutions and poorer numerical performance as well [*Wu et al*., 1996].

This paper presents the mathematical development, program implementation, and test of a "virtual node" method for treatment of well boundary conditions. This method was previously discussed [*Wu et al*., 1996]; however, no formulation or numerical testing was presented. It will be shown that in this method, any type of well boundary condition is treated as a

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computational or "virtual" node, included in the mass balance calculations with the rest of the nodes. This method handles a well bore either as a single node or several computational nodes screened and connected to many neighboring nodes for a multilayered well. The well bore can be vertical, inclined, or horizontal, and the well borehole node is treated in the same way as any other nonwell node. Various production/injection conditions can be accounted for using this approach. The solution for the well node is then obtained by solving mass balance equations for the well node. This new scheme will provide a natural, physically consistent, and numerically efficient approach to handling well flow problems. In addition, implementation of the new method to a three-phase, three-dimensional flow reservoir simulator will be discussed, and three verification and application examples will be provided.

2. Governing Equations

A multiphase system in a porous and/or fractured aquifer is assumed to be composed of three phases: NAPL (oil), gas (air), and water. Although each of the three phases may contain several components, they are treated here as a single "pseudocomponent" with averaged properties of the fluids. For simplicity, the three fluid components, water, NAPL, and gas, are assumed to be present only in their associated phases. Each phase flows in response to pressure, gravitational, and capillary forces according to the multiphase extension of Darcy's law. In an isothermal system containing three mass components, three mass balance equations are needed to fully describe the system, as described in an arbitrary flow region of a porous or fractured domain.

For flow of phase β ($\beta = w$ for water, $\beta = n$ for NAPL, and $\beta = g$ for gas),

$$\frac{\partial}{\partial t}(\phi S_\beta \rho_\beta) = -\nabla \cdot (\rho_\beta \mathbf{V}_\beta) + q_\beta, \tag{1}$$

where the Darcy velocity of phase β is defined by

$$\mathbf{V}_\beta = -\frac{k k_{r\beta}}{\mu_\beta}(\nabla P_\beta - \rho_\beta g \nabla D). \tag{2}$$

In (1) and (2), ρ_β is the density of phase β under reservoir conditions; ϕ is the effective porosity of the formation; μ_β is the viscosity of phase β; S_β is the saturation of phase β; P_β is the pressure of phase β; q_β is the sink/source term of phase (component) β per unit volume of formation; g is gravitational constant; k is the absolute/intrinsic permeability (tensor) of the formation; $k_{r\beta}$ is relative permeability to phase β; and D is depth.

The governing equation (1) of mass balance for three phases needs to be supplemented with constitutive equations, which express all the secondary variables and parameters as functions of a set of primary thermodynamic variables of interest. The following relationships will be used to complete the description of multiphase flow through porous media:

$$S_w + S_n + S_g = 1. \tag{3}$$

The capillary pressures relate pressures between the phases. The aqueous and gas phase pressures are related by

$$P_w = P_g - P_{cgw}(S_w), \tag{4}$$

where P_{cgw} is the gas-water capillary pressure in a three-phase system; and it is assumed to be a function of water saturation only. The NAPL pressure is related to the gas-phase pressure by

$$P_n = P_g - P_{cgn}(S_w, S_n), \tag{5}$$

where P_{cgn} is the gas-NAPL capillary pressure in a three-phase system, which is a function of both water and NAPL saturations. For many aquifer formations the wettability order is (1) aqueous phase, (2) NAPL phase, and (3) gas phase. The gas-water capillary pressure is usually stronger than the gas-NAPL capillary pressure. The NAPL-water capillary pressure, P_{cnw}, in a three-phase system, may be defined as

$$P_{cnw} = P_{cgw} - P_{cgn} = P_n - P_w. \tag{6}$$

The relative permeabilities are assumed to be functions of fluid saturations only. The relative permeability to the water phase is described by

$$k_{rw} = k_{rw}(S_w), \tag{7}$$

the relative permeability to the NAPL phase is described by

$$k_{rn} = k_{rn}(S_w, S_g), \tag{8}$$

and the relative permeability to the gas phase is described by

$$k_{rg} = k_{rg}(S_g). \tag{9}$$

In these capillary pressure and relative permeability functions, hystereses can also be included for normal formation nodes except for well or virtual nodes, for which no hysteresis effects are needed. The densities of water, NAPL, and gas, as well as the viscosities of fluids can, in general, be treated as functions of fluid pressures.

3. Numerical Formulation

The virtual node method of this paper has been implemented into a general-purpose, three-phase reservoir simulator, the multiphase subsurface flow (MSFLOW) code (Y. S. Wu, MSFLOW: Multiphase subsurface flow model of oil, gas and water in porous and fractured media with water shut-off capability: Documentation and user's guide, research report 003, 841 Leroy Lane, Walnut Creek, California, 1998). As implemented in the MSFLOW code, (1) can be discretized in space using an integral finite difference or control-volume finite element scheme for a porous and/or fractured medium. The time discretization is carried out with a backward, first-order finite difference scheme. The discrete nonlinear equations for water, NAPL, and gas flow at node i are as follows:

$$\{(\phi S_\beta \rho_\beta)_i^{n+1} - (\phi S_\beta \rho_\beta)_i^n\}\frac{V_i}{\Delta t} = \sum_{j \in \eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2}^{n+1} \gamma_{ij} [\psi_{\beta j}^{n+1} - \psi_{\beta i}^{n+1}] + Q_{\beta i}^{n+1}, \tag{10}$$

where n denotes the previous time level; $n + 1$ is the current time level; V_i is the volume of element i (porous or fractured block); Δt is time step size; and η_i contains the set of neighboring elements (j) (porous or fractured block) to which element i is directly connected. Subscript $ij + 1/2$ denotes a proper averaging of properties at the interface between two elements i and j. The mobility of phase β is defined as

$$\lambda_\beta = k_{r\beta}/\mu_\beta. \tag{11}$$

Mobilities are upstream weighted. The flow potential term in (10) is

$$\psi_{\beta i}^{n+1} = P_{\beta i}^{n+1} - \rho_{\beta, ij+1/2}^{n+1} g D_i, \tag{12}$$

and the transmissivity of flow terms is defined, if the integral finite difference scheme is used [*Pruess*, 1991], as

$$\gamma_{ij} = \frac{A_{ij} k_{ij+1/2}}{d_i + d_j}, \tag{13}$$

where A_{ij} is the common interface area between connected elements i and j, d_i is the distance from the center of element i to the interface between elements i and j, $k_{ij+1/2}$ is an averaged (such as harmonic weighted) absolute permeability along the connection between elements i and j, and D_i is the depth to the center of element i. The mass sink/source term at element i, $Q_{\beta i}$ for phase β, is defined as

$$Q_{\beta i}^{n+1} = q_{\beta i}^{n+1} V_i. \tag{14}$$

If element i is a well bore node, $Q_{\beta i}$ will be determined as described in section 4.

Newton/Raphson iterations are used to solve (10). For a three-phase flow system, $3 \times N$ coupled nonlinear equations must be solved (N being the total number of elements of the grid), including three equations at each element for the three mass balance equations of water, NAPL, and gas, respectively. Three primary variables (x_1, x_2, and x_3) selected for each element are gas pressure, gas saturation, and NAPL saturation, respectively, in this work. In terms of the three primary variables the Newton/Raphson scheme gives rise to

$$\sum_m \frac{\partial R_i^{\beta, n+1}(x_{m,p})}{\partial x_m} (\delta x_{m,p+1}) = -R_i^{\beta, n+1}(x_{m,p}) \tag{15}$$

$$m = 1, 2, \text{ and } 3,$$

where index $m = 1$, 2, and 3 indicates the primary variable 1, 2, and 3, respectively; p is the iteration level; and $i = 1, 2, 3, \ldots, N$. The primary variables are updated after each iteration:

$$x_{m,p+1} = x_{m,p} + \delta x_{m,p+1}. \tag{16}$$

A numerical method is used to construct the Jacobian matrix for (15). For a fully implicit element the Jacobian is evaluated using numerical differentiation, as outlined by *Forsyth et al.* [1995], and for an implicit pressure and explicit saturation (IMPES) node a simpler, semianalytical approach is used for Jacobian calculations [*Forsyth and Sammon*, 1986].

Flow along the well bore is fully coupled with flow in the reservoir. Instead of using Darcy's law, the flow equation along boreholes for horizontal wells [*Dikken*, 1990] is extended here to describe multiphase flow in horizontal, inclined, or vertical boreholes as

$$q_{ij,\beta}^w = \gamma_{ij}^w \tau_\beta^w (\psi_{\beta j} - \psi_{\beta i}), \tag{17}$$

where $q_{ij,\beta}^w$ is mass flux of phase β between two connected well bore nodes i and j. The transmissivity of the well bore, when comparing (17) with the well bore flow equation of *Dikken* [1990], may be defined as

$$\gamma_{ij}^w = \frac{1.97588 \times d^{5/2}}{d_i + d_j}, \tag{18}$$

where d is the diameter of the well bore. The well bore mobility is given by

$$\tau_\beta^w = \frac{(S_\beta \rho_\beta)_{ij+1/2}}{(\bar{\rho})^{1/2}} \left| \frac{(\psi_{\beta j} - \psi_{\beta i})}{d_i + d_j} \right|^{-1/2}, \tag{19}$$

where the averaged fluid density is defined as

$$\bar{\rho} = \tfrac{1}{2} \{(S_g \rho_g)_i + (S_o \rho_o)_i + (S_w \rho_w)_i + (S_g \rho_g)_j + (S_o \rho_o)_j + (S_w \rho_w)_j\} \tag{20}$$

4. Well Treatment

The virtual node method handles a well bore as a single node or as several computational nodes. Figure 1 illustrates a virtual node representation of a well and its association with formation layers and model grids for a multilayered, vertical well bore. The mass balance and discrete equations (1) and (10) are still applicable to well node i. However, the coefficients for flow terms are evaluated differently. In this case a productivity (or injectivity) index is adapted for well bore–formation flow, and well bore mobility (19) and transmissivity (18) are used for well bore–well bore flow. Therefore, (10), for well node i, is rewritten as

$$R_i^{\beta, n+1} = \{(\phi S_\beta \rho_\beta)_i^{n+1} - (\phi S_\beta \rho_\beta)_i^n\} \frac{V_i}{\Delta t} - Q_{\beta,w}^{n+1} - \sum_{j \in \eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2}^{n+1} PI_{ij} [\psi_{\beta j}^{n+1} - \psi_{\beta i}^{n+1}] - \sum_{k \in \eta_w} (\tau_\beta^w)_{ik+1/2}^{n+1} \gamma_{ik}^w [\psi_{\beta k}^{n+1} - \psi_{\beta i}^{n+1}], \tag{21}$$

where PI_{ij} is a well productivity or injectivity index for the connection between well node i and neighboring formation node j; $Q_{\beta,w}^{n+1}$ is the total mass rate of pumping or injection at the well, to be determined in sections 4.1 and 4.2 for different pumping or injection specifications; j is the index of a neighboring formation node, connected to well node i; η_w is a set of well bore nodes, connected with well node i along the borehole; and k is the index of the neighboring well bore node to well node i.

In general, two types of flow terms appear in the left-hand side of the well flow equation (21) that are not described by Darcy's law. The first accounts for radial flow between the well bore and the formation, for which we need a productivity (injectivity) index or a well function. The other accounts for the flow between well bore nodes, as described by (17). If a well bore is represented by more than one computational node, that is, well bore flow is coupled with flow in the formation, the sink/source term for the well needs to be added to only one of those well nodes.

There are many methods and equations, especially in petroleum literature, for evaluating the productivity index. For a vertical borehole, *Thomas* [1982] proposed the following well index formulation:

$$PI_{ij} = \frac{2\pi k \Delta z_j}{\ln\left(\frac{r_e}{r_w}\right) + s - 1/2}, \tag{22}$$

where Δz_j is the thickness of layer j, r_e is an equivalent radius of grid block j, r_w is the well bore radius, and s is the skin factor. Various well indexes exist in the literature for vertical, inclined, or horizontal wells [*Peaceman*, 1978, 1982, 1991, 1995; *Lee and Milliken*, 1993; *Fung et al.*, 1991].

The production rate of phase β at a production well is evaluated as follows:

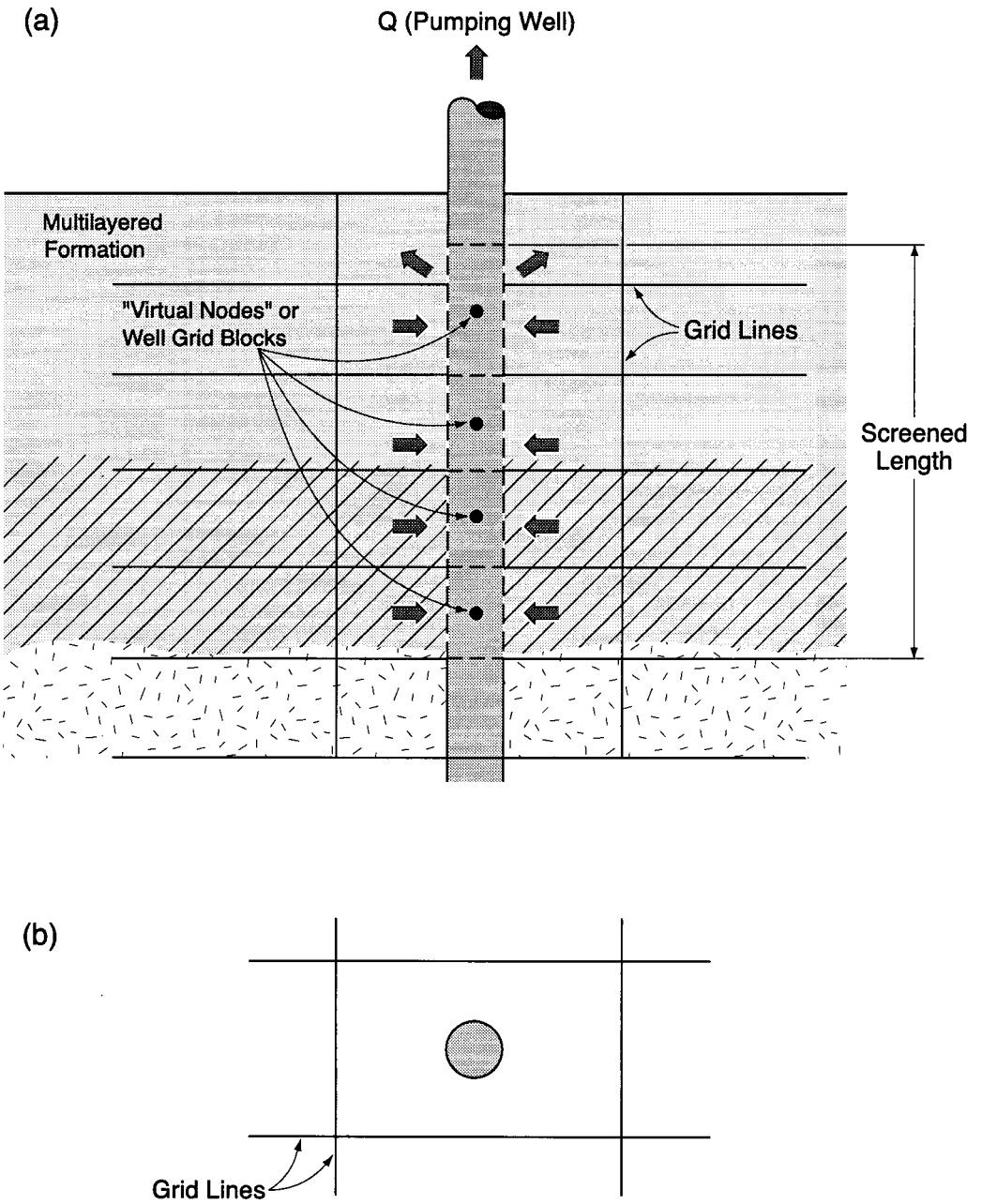

Figure 1. Schematic illustration of virtual node representation for a vertical, multigrid well in a multilayered formation. (a) Vertical profile of well bore, virtual nodes, model grids, and formation layers. (b) Overview of well bore and model grids.

$$Q_{\beta,w} = -\sum_{j\in\eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2} PI_{ij}[P_{\beta,j} - P_w - \rho_\beta g(D_j - D_w)] - \sum_{k\in\eta_w} (\tau_\beta^w)_{ik+1/2}^{n+1} \gamma_{ik}^w [P_{\beta,k} - P_w - \rho_\beta g(D_k - D_w)]. \quad (23)$$

The injection rate for phase β at an injection well is evaluated by

$$Q_{\beta,w} = \sum_{j\in\eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2} PI_{ij}[P_{\beta,j} - P_w - \rho_\beta g(D_j - D_w)] + \sum_{k\in\eta_w} (\tau_\beta^w)_{ik+1/2}^{n+1} \gamma_{ik}^w [P_{\beta,k} - P_w - \rho_\beta g(D_k - D_w)]. \quad (24)$$

In (23) and (24) the total mass rate is calculated from summation of the flow terms between well node i and all its neighbors, j and k; P_w is the well pressure, determined using an additional constraint equation; and D_w is the depth at which the pump is located inside the well bore.

The mobility terms in the well flow equation (21) are evaluated using the upstream weighting schemes. In other words, the well node is treated as a virtual node, like any other node in the grid, except that the sink/source term is determined by (23) or (24). For pressure-specified pumping or injection wells the specified well pressure is directly substituted into (23) or (24) to calculate pumping or injection rates. Different pumping

and injection scenarios with rates specified are discussed in sections 4.1 and 4.2.

4.1. Rate-Specified Pumping Well

In general, there are two types of rate-specified pumping wells: (1) In one type the total liquid (water and NAPL) volumetric production rate is specified. (2) In the other type one-phase volumetric rate is specified; that is, the pumping rate is given or fixed for only one individual phase of water, NAPL, or gas. Physically, the phase(s) of nonspecified fluids may also flow out and should be accounted for. The two rate-specified pumping scenarios are treated differently. The following approach allows backflow to occur, which may be encountered at certain layers in a pumping well that penetrates multiple layers.

4.1.1. Total liquid rate specification. With a total volumetric pumping rate Q_L^v (>0) of liquid (water plus oil) specified at the well, the well flowing pressure is evaluated by

$$P_w = \left\{ -Q_L^v + \sum_{j\in\eta_i} \sum_{\beta} (\rho_\beta \lambda_\beta / \rho_\beta^o)_{ij+1/2} PI_{ij} [P_{\beta,j} - \rho_\beta g(D_j - D_w)] + \sum_{k\in\eta_w} \sum_{\beta} (\tau_\beta^w / \rho_\beta^o)_{ik+1/2} \gamma_{ik}^w [P_{\beta,k} - \rho_\beta g(D_k - D_w)] \right\} \div \left\{ \sum_{j\in\eta_i} \sum_{\beta} (\rho_\beta \lambda_\beta / \rho_\beta^o)_{ij+1/2} PI_{ij} + \sum_{k\in\eta_w} \sum_{\beta} (\tau_\beta^w / \rho_\beta^o)_{ik+1/2} \gamma_{ik}^w \right\}, \tag{25}$$

where $\beta = o$ and w, and ρ_β^o is the density of phase β at standard conditions. The well pressure determined by (25) is subject to a constraint,

$$P_w \geq P_{w,\min}, \tag{26}$$

where $P_{w,\min}$ is the minimum well pressure allowed. Equation (26) enforces the physical constraint that it is not always possible to produce liquid at the specified rate. If pressure drop exceeds $P_{w,\min}$, the pumping well will be switched to a pressure-specified pumping operation.

The actual pumping rate, to be added to (21) as a sink term, is evaluated using (23) for water and NAPL, respectively, with the well pressure determined by (25) and (26). In addition, gas may flow out simultaneously with liquids, which cannot be controlled, even though a liquid rate is specified, and the gas production rate is also determined by (23).

In this approach the well pressure P_w from (25) and (26) should approach the well nodal pressure P_i from a simulation when the solution is converged and if no capillary forces are specified for the well node.

4.1.2. One-phase rate specification. With a single-phase fluid volumetric pumping rate Q_β^v (>0) of oil, water, or gas, specified at the well, the well pressure is evaluated as follows:

$$P_w = \left\{ -\rho_\beta^o Q_\beta^v + \sum_{j\in\eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2} PI_{ij} [P_{\beta,j} - \rho_\beta g(D_j - D_w)] + \sum_{k\in\eta_w} (\tau_\beta^w) \gamma_{ik}^w [P_{\beta,k} - \rho_\beta g(D_k - D_w)] \right\} \div \left\{ \sum_{j\in\eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2} PI_{ij} + \sum_{k\in\eta_w} (\tau_\beta^w)_{ik+1/2} \gamma_{ik}^w \right\}, \tag{27}$$

where $\beta = w$, o, or g.

The well pressure from (27) is also physically subject to the constraint condition of (26). The actual pumping rate for the phase is evaluated using (23) and added to (21). Again, nonspecified phases may be pumped out, and their flow terms are calculated accordingly, subject to the same well pressure.

4.2. Rate-Specified Injection Well

Injection rates are known in practice. The injected phase can be water, oil, or gas. With a total mass injection rate Q_β (>0) of phase β specified at the well, the well injection pressure is evaluated by

$$P_w = \{ Q_\beta + \sum_{j\in\eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2} PI_{ij} [P_{\beta,j} - \rho_\beta g(D_j - D_w)] + \sum_{k\in\eta_w} (\tau_\beta^w) \gamma_{ik}^w [P_{\beta,k} - \rho_\beta g(D_k - D_w)] \} \div \{ \sum_{j\in\eta_i} (\rho_\beta \lambda_\beta)_{ij+1/2} PI_{ij} + \sum_{k\in\eta_w} (\tau_\beta^w)_{ik+1/2} \gamma_{ik}^w \} \tag{28}$$

with $\beta = w$, n, or g, and the well injection pressure is subject to the following constraint:

$$P_w \leq P_{w,\max}, \tag{29}$$

where $P_{w,\max}$ is the maximum well injection pressure allowed. Equation (29) is to check the condition that the specified injection rate may be too high for the well condition. In this situation the injection well is switched to a pressure-specified injection operation. The actual injection rate for the specified phase is evaluated using (24), with the well injection pressure determined by (28) and (29).

4.3. Special Considerations

As discussed above, the advantages of the virtual node scheme, as compared with the conventional potential or mobility allocation method, are that it naturally includes "backflow" and incorporates contributions from all the connections to a well into the Jacobian matrix, coupling well bore flow with flow in the formation. The full Jacobian matrix and the full implicit scheme make the method very robust and stable in solving a multilayered well problem. One potential problem, however, is that since the well bore node has a very small volume and high flow rates, it may cause numerical difficulties during a Newton iteration. This problem can be alleviated by increasing the volume of the well bore nodes by a factor of 10^2–10^3. This has the effect of adding a pseudo well bore storage, which dampens out oscillations in the Newton iteration [*Wu et al.*, 1996]. Except for the case of very small-scale transient behavior of wells, numerical tests indicate that adding pseudo well bore storage has almost no effect on the converged solution.

In addition, when applying the virtual node method, special attention is needed in specifying the "rock properties" and initial conditions for the virtual node. Capillary pressure and relative permeability functions are needed for all the well nodes and should be specified differently for a particular phase of pumping or injection, since the well node is regarded as a normal grid block in the solution.

5. Verification and Demonstration Examples

Three examples are presented to examine and demonstrate the application of the proposed well treatment scheme. The

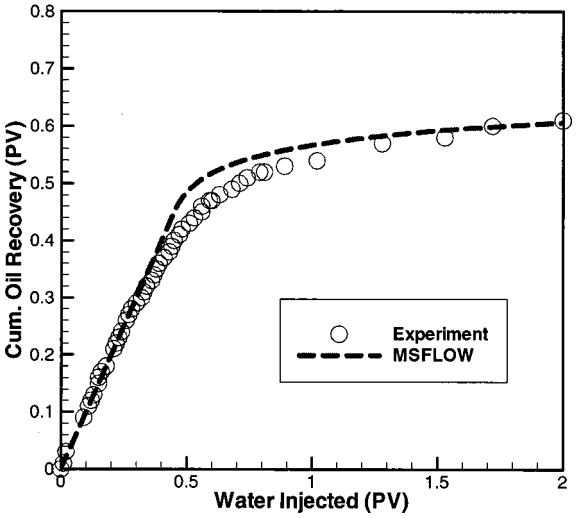

Figure 2. Cumulative oil recovery for the two-phase, five-spot well-flow problem. PV is pore volume.

first considers a three-dimensional flow of oil and water in a five-spot problem of injection and production. The second example examines single-phase flow in a fractured reservoir against an analytical solution, and the final example is to handle a "backflow" well of a two-dimensional oil and water flow problem. All the three problems are simulated using a three-phase flow mode, in which two- or single-phase flow is treated as a special case of three-phase flow.

5.1. Five-Spot Well-Flow Problem

This sample problem is used to examine (1) the numerical scheme described in this work for handling pumping and injection wells under multiphase flow conditions and (2) numerical performance of the proposed method by comparing it to the traditional method. The example is a well-known test case for which laboratory and numerical simulation results are known [*Gaucher and Lindley*, 1960; *Wu et al*., 1994]. The model domain consists of a quarter of a five-spot well pattern. A three-dimensional $10 \times 10 \times 5$ brick-type grid was used for this problem, with $\Delta x = \Delta y = 14.23$ m and $\Delta z = 1.22$ m. The formation was treated as homogeneous and isotropic. Detailed input parameters were given by *Coats et al*. [1967] and *Wu et al*. [1994].

A comparison of cumulative oil recovery versus injected water in terms of total pore volume (PV) is shown in Figure 2, indicating that the modeling results from the current work are in reasonable agreement with the laboratory results. However, the current model predicts a little higher oil recovery over the range of 0.4–0.9 PV of water injection. This is probably due to the differences and uncertainties in rock characteristic curves used in the present model [*Wu et al*., 1994].

Table 1 lists the statistics for numerical performances and comparisons with the traditional, potential allocation scheme, performed using a different simulator [*Wu et al*., 1996] with identical convergence tolerance and time-stepping scheme specified. As shown in Table 1, two pumping scenarios, one pressure-specified and one rate-specified production, were examined, and all four simulations gave almost identical results (see Figure 2). Note that Table 1 indicates that performance of the virtual node method is much more efficient than the potential allocation scheme for this test problem, with over 3 to 30 times improvement in both CPU times and numbers of Newton iterations required, respectively.

Table 1. Comparison of Numerical Results for the Five-Spot Well-Flow Problem

Method	Scenario	Newton Iteration	Normalized CPU Times
Virtual node	Pressure-specified pumping	428	1.00
	Rate-specified pumping	1,938	4.38
Potential allocation	Pressure-specified pumping	16,175	32.47
	Rate-specified pumping	6,401	13.73

5.2. Single-Phase Fractured-Medium Flow Problem

This problem is used to test the virtual node method in simulating transient flow in fractured media by comparison with an analytical solution. The example concerns transient flow of water injection through a well, which fully penetrates a horizontal, uniform, fractured, radially infinite reservoir. *Warren and Root* [1963] provided an analytical solution using a double-porosity approach.

A radially symmetrical reservoir ($R \times Z = 100{,}000 \times 10$ m) is discretized into a two-dimensional (*RZ*), primary grid, the *r* distance of 100,000 m is subdivided into 200 intervals in logarithmic scale, and the uniform spacing, $\Delta z = 2.0$ m, is set for the vertical discretization. A double-porosity mesh is generated from the primary *RZ* grid, in which a three-dimensional fracture network and cubic matrix blocks are used. The matrix block size is $1 \times 1 \times 1$ m uniformly, and fracture permeability and aperture are correlated by the cubic law. The input parameters are given in Table 2. A fully penetrating injection well is represented by a well element with a constant water-injection rate.

A comparison of the numerical modeling results and the *Warren and Root* [1963] solution is shown in Figure 3 for the pressure response at the well. Figure 3 shows that the simulated pressures at the well are in excellent agreement with the analytical solution, with a typical double-porosity behavior of two-parallel semilog straight lines developed on the plot.

5.3. Backflow Problem

This problem is designed to demonstrate the capability of the virtual node method in handling backflow in a pumping well, which may occur during production operation. The model domain is a two-dimensional, vertical cross section of an unconfined aquifer, 200 (x) m long and 30 (z) m tall with an impermeable layer near the water table, as shown in Figure 4. There are two partially penetrating wells, one injector and one

Table 2. Parameters for the Single-Phase Fractured-Medium Flow Problem

Parameter	Value	Unit
Matrix porosity	$\phi_m = 0.30$	
Fracture porosity	$\phi_f = 0.0006$	
Water density	$\rho_w = 997.6$	kg/m^3
Water phase viscosity	$\mu_w = 0.898 \times 10^{-3}$	Pa s
Matrix permeability	$k_m = 1.579 \times 10^{-16}$	m^2
Fracture permeability	$k_f = 1.579 \times 10^{-12}$	m^2
Water injection rate	$q = 864$	m^3/d
Rock compressibility	$C_r = 0.000$	1/Pa
Water compressibility	$C_w = 4.487 \times 10^{-10}$	1/Pa
Well bore radius	$r_w = 0.1$	m

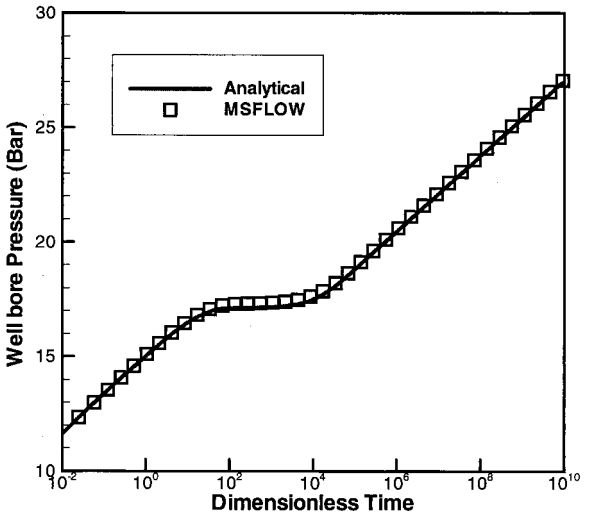

Figure 3. Comparison of multiphase subsurface flow (MSFLOW) results with the *Warren and Root* [1963] solution.

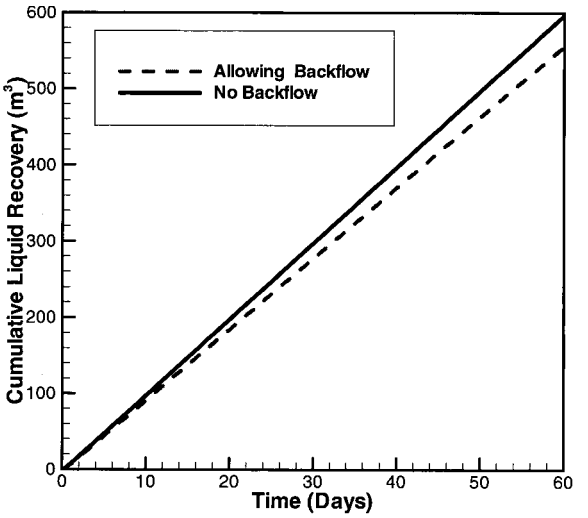

Figure 5. Comparison of cumulative liquid (oil plus water) pumping recovery, simulated with and without backflow options for the backflow problem.

producer with different screened elevations. The formation and fluid properties used are the same as those for the five-spot well-flow problem, except that the permeability value is increased to 1.0×10^{-12} (m^2). A two-dimensional 40×20 rectangular grid, $\Delta x = 5$ m and $\Delta z = 1.5$ m, was used for this problem. On the top model boundary, the water table, a constant pressure (P_{atm}) of the atmosphere is described, and the initial condition of the system corresponds to gravity-capillary equilibrium, generated with $S_n = 0.7$ and $S_w = 0.3$ specified. The injection rate is at 10 (m^3/d), and the producer is specified at a constant well pressure of 1.5 (bars) for pumping.

The simulation results indicate that the backflow does occur at the pumping well, leaking from the opening portion of the screen, above the impermeable layer (Figure 4), to the constant-pressure water table. The injection and pumping operation were simulated for 60 days, and a comparison of the cumulative liquid (oil plus water) volumes of the producer with and without allowing backflow is given in Figure 5. Figure 5 shows that when the backflow is not considered, the model overpredicts the liquid recovery by 7.4%, which is physically incorrect. In general, backflow occurrence cannot be controlled on the ground and should be handled by the well-flow model accordingly.

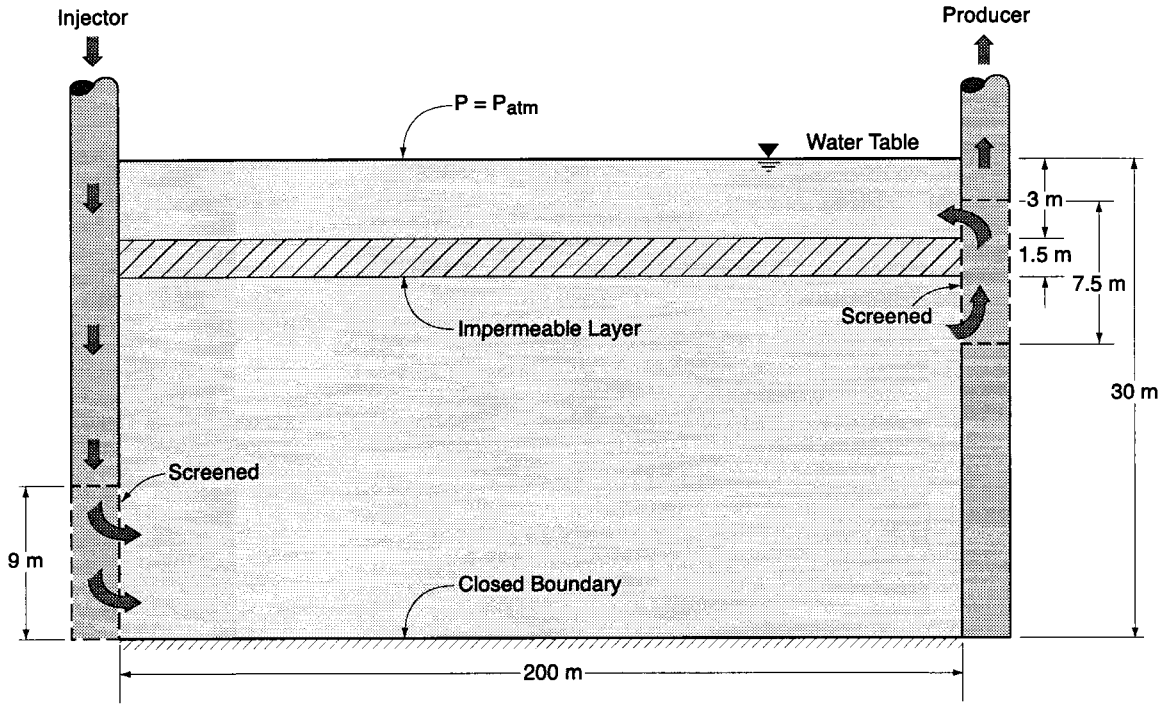

Figure 4. Schematic of two-dimensional cross section of an unconfined aquifer with an injector and a producer for demonstrating backflow occurrence.

6. Conclusions

This paper presents a virtual node method for handling well boundary conditions in simulation of multiphase subsurface flow, including the formulation and numerical implementation of the methodology and discussions of applications. The virtual node scheme is a general, physically consistent approach that treats a well bore as a single node or several computational nodes. The main advantages of this approach, compared with the conventional methods of well treatment, are that the virtual node method can handle (1) backflow; (2) coupling reservoir flow with well bore flow for long, screened boreholes; and (3) various types of vertical, inclined, or horizontal wells. The numerical tests indicate that this method provides a natural, accurate, and numerically efficient approach to handling well multiphase flow problems.

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