PERGAMON

International Journal of Rock Mechanics and Mining Sciences 37 (2000) 51–61

International Journal of
Rock Mechanics
and Mining Sciences

www.elsevier.com/locate/ijrmms

Integral solutions for transient fluid flow through a porous medium with pressure-dependent permeability

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Accepted 7 October 1999

Abstract

This paper presents an integral method for analyzing transient fluid flow through a porous medium, which has pressure-dependent permeability. Approximate analytical solutions have been obtained for one-dimensional linear and radial flow by an integral-solution technique, in which the density of the fluid, and the porosity and permeability of the formation, are treated as arbitrary functions of pressure. The integral solutions have been checked by comparison with exact solutions for constant-permeability cases and with numerical simulation results for general non-linear flow problems, and good agreement has been obtained for both cases.

In the study of transient flow of fluids through porous media, intrinsic or absolute permeability of the formation has often been treated as a constant in order to avoid solving a non-linear problem. The present work shows that the assumption of a pressure-independent permeability may introduce significant errors for flow in certain pressure sensitive media. Application of the integral solutions to slightly compressible fluid flow in a horizontal fracture set is illustrated. The calculations show that neglect of changes in fracture permeability leads to large errors under the condition of high injection pressure. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Fluid flow in porous media is determined by a coupled process between fluid movement and rock deformation. This process is in general described by the flow potential gradient and permeability of the formation according to Darcy's law. When fluid flow occurs in porous media, the flow potential will change spatially and temporally, and so will the fluid pressure. Therefore, the effective stress acting on the rock changes accordingly and will cause the solid skeletons of the rock to deform. The deformation of the solids in turn changes the fluid flow channels and geometries and this will affect the potential field.

Permeability of porous media is dependent on pore pressures and/or the stress field. In most studies of transient fluid flow in porous media, however, it has been assumed that the influence of the rock deformation on permeability is negligible, i.e., only fluid density and rock porosity are treated as functions of pressure. This assumption may be reasonable for slightly compressible fluid flow in certain purely porous media, such as sandstone, since the pore compressibility of sandstone is usually very small. Nevertheless, even for flow in fractured media, the same assumption of constant permeability is often made. The permeability for pre-existing fractures may be enhanced significantly due to the deformation of fractures in response to changes in stress fields, pore pressure and temperature as well [4]. Neglect of effects of rock deformation on fluid mobility in fractures may introduce large errors, because the flow in fractures is very sensitive to changes in apertures, which directly correlates with permeability and porosity of the fractured media. Apertures may change significantly under high fluid pressure conditions.

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PII: S1365-1609(99)00091-X

Nomenclature

A	cross-sectional area (m^2)
b	aperture of fracture (m)
b_i	initial aperture of fracture (m)
C_f	fluid compressibility (Pa^{-1})
c_k	slope of the void ratio $[\phi/(1-\phi)]$ plotted against log (k)
C_r	rock compressibility (Pa^{-1})
C_t	total compressibility (Pa^{-1})
D	half-spacing between fractures (m)
E	Young's modulus (Pa)
F	formation factor of Brace's permeability model
h	thickness of formation (m)
k	absolute permeability (m^2)
k_{eff}	effective permeability, in Eq. (42) (m^2)
k_f	fracture permeability (m^2)
k_i	coefficient of pore-geometry model (m^2)
k_0	shape factor of Brace's permeability model
m	hydraulic radius (m)
P	pressure (Pa)
P_0	pressure at inlet, $x = 0$, or $r = r_w$ (Pa)
P_i	initial formation pressure (Pa)
P_{inj}	injection pressure (Pa)
$P_n(r)$	nth-degree polynomial in r
q_m	mass injection rate (kg/s)
$Q_m(r)$	mth polynomial in r
q_{inj}	volumetric injection rate (m^3/s)
x	distance to inlet (m)
r	radial distance (m)
r_w	wellbore radius (m)
t	time (s)
T	reservoir temperature (°C)
t_D	dimensionless time, Eq. (39)
$\mathbf{u}$	Darcy's velocity (m/s)
V	volume of fluid (m^3)

Greek symbols

α	constant, Eq. (3)
β	exponential of Brace's permeability model
$\delta(t)$	pressure penetration depth (m)
μ	fluid viscosity (Pa·s)
ξ	defined in Eq. (37)
ρ	fluid density (kg/m^3)
ρ_i	initial fluid density (kg/m^3)
σ	stress (Pa)
σ'	effective stress (Pa)
ϕ	formation porosity
ϕ_i	initial formation porosity

In one of the earlier studies, Raghavan et al. [21] developed a numerical method for transient pressure responses in a well flow test, which included effects of changes in rock and fluid properties with pressure. They defined a "pseudopressure" to represent the fluid and rock properties in the flow equation. Their model was studied analytically by Samaniego et al. [24] after applying a linear approximation to the non-linear problem, and was used for drawdown, buildup, injection, and falloff testing analyses. In addition, Pedrosa [18] and Kikani et al. [10] applied the perturbation analysis technique to look at pressure transient response in stress-sensitive formations, in which permeability was represented by a special, exponential function of pressure.

More recently, Berumen and Tiab [1] presented a new numerical approach for interpreting the effect of pore pressure on the conductivity and permeability of artificially fractured rock. The non-linear effects were included in their mathematical model and new type curves for pressure-sensitive fractured formation were generated to interpret pressure data from fractured wells. They concluded that use of the conventional techniques to evaluate fractured wells may lead to incorrect estimates of fracture-formation properties.

The effects of confining or overburden pressure on the permeability of porous media were studied experimentally by a number of authors [6–8,12,13]. The major conclusions of these studies are that the rock properties are dependent only on the effective stress, and that the relationship between rock properties and effective stress is history dependent. If total or external stress is kept constant, the absolute permeability of a porous medium can be expressed as a function of the difference between the confining pressure and the pore pressure. Some theoretical models have, therefore, been proposed to relate rock permeability and confining pressure [9,25,27].

The influence of fluid injection in a fractured porous medium was investigated numerically by Noorishad et al. [15], using a finite element code. Noorishad and Tsang [16] present a numerical model (ROCMAS) for two-dimensional coupled flow and stress analysis in deformable, saturated, fractured rock media. More recently, coupled hydromechanical processes of fluid injection tests into deformable rocks were investigated using the numerical and experimental approaches [22,23].

There are few studies on analytical solutions for the general coupled process of fluid flow and rock deformation without using linearization approximations, in particular for fluid flow in fractured reservoirs. This paper presents an analytical method for analyzing the non-linear coupled rock permeability variation and fluid flow problem. Approximate analytical solutions for one-dimensional linear and radial flow are obtained by an integral method, which is widely used in the study of steady and unsteady heat conduction problems [17]. The integral method, as applied to heat transfer problems, utilizes a parametric representation of the temperature profile by means of low-order polynomials, which is based on physical concepts such as a time-dependent penetration distance. An approximate solution to the heat transfer problem is then obtained from simple principles of continuity of temperature and heat flux, and energy conservation. This solution satisfies the governing partial differential equations only in an average, integral sense. However, the accuracy of integral solutions in heat transfer problems is generally acceptable for engineering applications. When applied to fluid flow problems in porous media, the integral method consists of assuming a pressure profile in the pressure-disturbance zone and determining the coefficients of the profile by making use of the integral mass balance equation [29,30].

The integral solutions obtained in this paper are very general. Fluid density and formation porosity and permeability may be treated as arbitrary functions of pressure under isothermal conditions. The integral solutions are checked by comparison with the solutions for special linear flow cases when the exact solutions are available. Furthermore, a numerical simulator is used to check the integral solution for general non-linear flow problems. It is found that the accuracy of the integral solutions for both linear and radial flow is very good when compared with the exact solution and with the numerical results for fluid flow through a semi-infinite system. We have also found that the shape of the pressure profile for radial flow in a permeability-dependent medium can be quite different from temperature profiles typically recommended for heat conduction in radial flow systems [11]. Several analytical expressions for pressure profiles are proposed which result in very accurate results for transient fluid flow in a radial system.

As an example of application, the integral solution is applied to discuss the effects of fluid pressure on fracture permeability during slightly compressible fluid flow through a horizontal fracture system. The analytical results show that changes in fracture permeability due to changes in pressure can have a dominant influence on the flow field for high-pressure injection. Neglecting effects of pressure on fracture permeability may introduce large errors in the flow behavior prediction.

The approximate integral solutions for transient fluid flow through a permeability-dependent porous medium derived in this paper will find their applications in the following fields: (i) to obtain certain physical insight into the phenomenon of coupled fluid flow and rock permeability variations; (ii) to design and analyze well tests to determine formation and fluid properties; (iii) to verify numerical simulators which include pressure-dependent fluid and formation properties.

2. Mathematical formulation

To formulate the flow model, the basic assumptions used for fluid flow in porous media are as follows:

- isothermal, isotropic and homogeneous formation;
- single phase horizontal flow without gravity effects;
- Darcy's law applies; and
- physical properties of fluid and rock are purely elastic, depending only on stress (no hysteresis).

The governing equation is derived by combining (a) the mass conservation law, (b) Darcy's law, and (c) equations of state of the fluid and rock,

$$\nabla[\rho\mathbf{u}] = -\frac{\partial}{\partial t}(\phi\rho) \tag{1}$$

where the volumetric flux $\mathbf{u}$ is described by Darcy's law as

$$\mathbf{u} = -\frac{k}{\mu}\nabla P \tag{2}$$

and k, ρ, and μ are formation permeability, fluid density and dynamic viscosity (constant), respectively; P is fluid phase pressure, t is time and ϕ is formation porosity.

It has been shown [6,8,26] that hydrologic properties may be functions only of the effective stress, defined as

$$\sigma' = \sigma - \alpha P \tag{3}$$

where σ and σ' are the total (external) stress and effective stress, respectively; α is a parameter which depends on the mechanical properties of the rock and the geometry of the rock grains. For a particular reservoir, the total stress σ is essentially a constant, depending on the overburden weight of the formation. Therefore, the effective stress σ' is a function of the fluid pressure only. Therefore, we assume the following constitutive relations for fluid and rock:

$$\rho = \rho(P) \tag{4}$$

$$\phi = \phi(P) \tag{5}$$

and

$$k = k(P) \tag{6}$$

These correlations are often called equations of state for fluid and rock.

Isothermal compressibilities are defined as:

$$C_{\rm f} = \frac{1}{\rho}\left[\frac{\partial \rho}{\partial P}\right]_T = -\frac{1}{V}\left[\frac{\partial V}{\partial P}\right]_T \quad (7)$$

for fluid and

$$C_{\rm r} = \frac{1}{\phi}\left[\frac{\partial \phi}{\partial P}\right]_T \quad (8)$$

for rock pores. In Eqs. (7) and (8), V is volume of fluid, and T is reservoir temperature. The compressibilities $C_{\rm f}$ and $C_{\rm r}$ may or may not be constants.

Introducing Eqs. (2), and (4)–(6) into (1), we have the flow equation

$$\nabla\left[\rho(P)\frac{k(P)}{\mu}\nabla P\right] = \frac{\partial}{\partial t}(\phi(P)\rho(P)) \quad (9)$$

By using Eqs. (7) and (8) in (9), another form of the flow equation can be obtained:

$$\nabla\left[\rho(P)\frac{k(P)}{\mu}\nabla P\right] = C_{\rm t}\rho\phi\frac{\partial P}{\partial t} \quad (10)$$

where

$$C_{\rm t} = C_{\rm f} + C_{\rm r} \quad (11)$$

is the total compressibility. Again, $C_{\rm t}$ is not necessarily a constant. Eqs. (9) and (10) are generally non-linear and will be solved directly using the integral method with appropriate boundary and initial conditions in the following two sections.

3. Integral solution for one-dimensional linear flow

The integral method, which has been widely used in the heat transfer literature [17], is applied here to obtain an analytical solution for the non-linear coupled fluid flow and rock permeability varying problem. The flow system of interest is a semi-infinite linear reservoir with a constant cross-sectional area A, initially fully saturated with a single-phase fluid. The same fluid is injected (or produced) at a given constant mass rate, $q_{\rm m}$. Then the problem to be solved is as follows:

$$\frac{\partial}{\partial x}\left[\rho(P)\frac{k(P)}{\mu}\frac{\partial P}{\partial x}\right] = \frac{\partial}{\partial t}(\phi(P)\rho(P)) \quad (12)$$

The initial condition is

$$P(x, t=0) = P_{\rm i} \quad \text{(constant)} \quad (13)$$

The boundary conditions are

$$-A\left[\rho(P)\frac{k(P)}{\mu}\frac{\partial P}{\partial x}\right]_{x=0} = q_{\rm m} \quad (14)$$

and

$$\lim_{x\to\infty} P(x, t) = P_{\rm i} \quad \text{(constant)} \quad (15)$$

The integral solution for the pressure profile in the pressure penetration zone is given by [30]

$$P(x, t) = P_{\rm i} + \frac{\delta(t)}{3}\left[\frac{q_{\rm m}\mu}{A\rho(P_0)k(P_0)}\right]\left[1 - \frac{x}{\delta(t)}\right]^3 \quad (16)$$

where the pressure penetration distance, $\delta(t)$, and the injection pressure, $P_0 = P_0(t)$ at $x = 0$, are treated as unknowns, to be determined by the two following equations,

$$\int_0^{\delta(t)} A\rho(P)\phi(P){\rm d}x = A\rho_{\rm i}\phi_{\rm i}\delta(t) + q_{\rm m}t \quad (17)$$

and

$$P_0 = P_{\rm i} + \frac{\delta(t)}{3}\left[\frac{q_{\rm m}\mu}{A\rho(P_0)k(P_0)}\right] \quad (18)$$

Simultaneous solution of Eqs. (17) and (18) will determine the two unknowns, P_0 and $\delta(t)$, and substituting them into Eq. (16) yields the final, closed-form solution for pressure and distribution of the problem.

It should be mentioned that Eq. (17) is simply a mass balance equation for the fluid in the pressure penetration region $[0 < x < \delta(t)]$ of the system, namely

$$\text{mass in disturbed zone} = \text{initial mass} + \text{mass injected (or} - \text{mass produced)} \quad (19)$$

The "slightly compressible" fluid flow can be treated as a special case of the above solution. In this case, total compressibility is small and constant, and both fluid density and formation porosity are approximated as linear functions of fluid pressure. Substituting the two functions for density and porosity into (17) and performing integration analytically will give the pressure penetration distance [30] as

$$\delta(t) = \left[\frac{12\rho(P_0)k(P_0)t}{\rho_{\rm i}\phi_{\rm i}C_{\rm t}\mu}\right]^{1/2} \quad (20)$$

Introducing Eq. (20) into (18), will have one equation for one unknown, $P_0(t)$. Solving $P_0(t)$ from the resulting equation for time t and substituting it back into Eq. (20), the penetration distance, $\delta(t)$, is obtained. Then using the $P_0(t)$ and $\delta(t)$ in Eq. (16), a final solution for the pressure profile will be determined for the slightly compressible flow system.

4. Integral solution for one-dimensional radial flow

The problem considered is fluid injection into a fully penetrating well in an infinite horizontal reservoir of constant thickness, and the formation is initially saturated with the same fluid. The governing equation (9) can be expressed in a radial coordinate system as

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\rho(P)\frac{k(P)}{\mu}\frac{\partial P}{\partial r}\right] = \frac{\partial}{\partial t}(\phi(P)\rho(P)) \tag{21}$$

The initial condition is

$$P(r, t=0) = P_{\rm i} \quad \text{(constant)} \tag{22}$$

At the inner boundary at the wellbore, $r = r_{\rm w}$, the fluid is injected at a given mass injection rate, $q_{\rm m}$, i.e.

$$-\frac{2\pi r_{\rm w} h}{\mu}\left[\rho(P)k(P)\frac{\partial P}{\partial r}\right]_{r=r_{\rm w}} = q_{\rm m} \tag{23}$$

Using two different functional forms for $P(r, t)$ as trial functions, the integral solutions for radial flow under a given mass injection rate, $q_{\rm m}$, are obtained in the following two alternative forms (for more analyses with different solutions, see [30].

1. The temperature profile recommended for radial heat conduction [11,17] is

$$P(r, t) = [P_n(r)]\ln(r) \tag{24}$$

where $P_n(r)$ is an nth-degree polynomial in r. Using a pressure trial function of the form (24), we have

$$P(r, t) = P_{\rm i} - \left[\frac{q_{\rm m}\mu}{2\pi h\rho(P_0)k(P_0)}\right] \times [1+\eta-r_{\rm D}]\frac{\ln\left(\dfrac{r_{\rm D}}{1+\eta}\right)}{\eta+\ln(1+\eta)} \tag{25}$$

for a first-degree polynomial $P_1(r)$, where $\eta = \delta(t)/r_{\rm w}$ and $r_{\rm D} = r/r_{\rm w}$. It has been found [29] that Eq. (25) cannot match well the Theis solution for a linear flow case.

2. From the Theis solution for radial flow with constant permeability, we know that the pressure at a given injection time is distributed as a logarithm in (t/r^2). Thus, for $t_{\rm D} > 100$ [defined in Eq. (39)], the line-source Theis solution is simplified to [5]:

$$P(r, t) = P_{\rm i} + \left[\frac{q_{\rm inj}\mu}{4\pi kh}\right]\left[\ln\frac{kt}{\phi\mu C_{\rm t} r^2} + 0.80907\right] \tag{26}$$

This approximate solution is very accurate except near the pressure penetration front. Therefore, a pressure profile for flow in a porous medium with pressure-dependent properties can be obtained in the form [29]

$$P(r, t) = P_{\rm i} + \text{constant} \times \ln[P_n(r)] \tag{27}$$

Using this profile, we find

$$P(r, t) = P_{\rm i} - \left[\frac{q_{\rm m}\mu}{2\pi h\rho(P_0)k(P_0)}\right]\left[1+\frac{1}{2\eta}\right] \times \ln\left[\left(\frac{2r_{\rm D}}{1+\eta}\right) - \left(\frac{r_{\rm D}}{1+\eta}\right)^2\right] \tag{28}$$

It should be mentioned that the expressions (25) and (28) apply only for $r_{\rm w} \le r \le r_{\rm w} + \delta(t)$, while $P(r, t) = P_{\rm i}$ for $r \ge r_{\rm w} + \delta(t)$.

Similar to one-dimensional linear flow, the two unknowns, P_0, the wellbore pressure, and, $\delta(t)$, the pressure penetration distance, are determined by using either of the Eqs. (25) or (28) at $r = r_{\rm w}$, together with the following mass balance equation in the pressure disturbance region:

$$\int_{r_{\rm w}}^{r_{\rm w}+\delta(t)} 2\pi rh\rho(P)\phi(P)\,{\rm d}r = \pi h\rho_{\rm i}\phi_{\rm i}[(r_{\rm w}+\delta(t))^2 - r_{\rm w}^2] + q_{\rm m}t \tag{29}$$

The applicability and accuracy of the two solutions, given by Eqs. (25) and (28), will be discussed in the next section.

For slightly compressible fluid flow, we obtain the following explicit expressions of the integral mass balance equation for the different pressure profiles of Eqs. (25) and (28), respectively.

For the pressure profile, Eq. (25),

$$q_{\rm m}t + \frac{r_{\rm w}^2\rho_{\rm i}\phi_{\rm i}C_{\rm t}\mu q_{\rm m}}{\rho(P_0)k(P_0)[\eta+\ln(1+\mu)]} \times \left[-\frac{5}{36}(1+\eta)^3 + \frac{1}{4}\eta + \frac{5}{36} + \left(\frac{1}{2}\eta+\frac{1}{6}\right)\ln(1+\eta)\right] = 0 \tag{30}$$

For the pressure profile, Eq. (28),

$$q_{\rm m}t + \frac{r_{\rm w}^2\rho_{\rm i}\phi_{\rm i}C_{\rm t}\mu q_{\rm m}}{\rho(P_0)k(P_0)}\left(\frac{1+2\eta}{2\eta}\right) \times \left[-\frac{3}{2}(1+\eta)^2 + (1+\eta) + \frac{1}{2} + 2(1+\eta)^2 \times \ln(1+\eta) - \frac{1}{2}[1-4(1+\eta)^2]\ln\left[\frac{1+2\eta}{(1+\eta)^2}\right]\right] = 0 \tag{31}$$

Solving either pair of Eqs. (30) and (25) or (31) and (28) for $r = r_w$, simultaneously for the wellbore pressure, P_0, and the pressure penetration distance, $\delta(t)$, and then substituting them into (25) or (28), give the final solution for the corresponding flow problem.

5. Discussion on accuracy of integral solutions

The solutions from the integral method are approximate, and their accuracy needs to be confirmed by comparison with an exact solution or with numerical results in general. In this section, the integral solutions obtained in Sections 3 and 4 are checked by comparison with exact solutions and numerical calculations for the one-dimensional linear and radial flow of slightly compressible fluid through a horizontal formation. The numerical code used is a modified version of MULKOM-GWF [19], which includes a pressure-dependent permeability. This is a fully implicit integral finite difference code for three-phase flow of gas, water and foam, which belongs to the "MULKOM" family developed by Pruess [20].

The accuracy of integral solutions depends on the choice of pressure profiles, and on the nature of permeability dependence upon pressure as well as other variables [29]. We consider that pressure changes cause changes in porosity, which in turn affects permeability. For the permeability-porosity relationship, we use two alternative empirical models. One is a resistivity and pore shape model [2], which relates permeability to electrical resistivity by

$$k = (m^2/k_0)F^{-2}\phi^{-1} \tag{32}$$

where m is the hydraulic radius, the volume of the interconnected pores divided by their surface areas; k_0 is a shape factor; F is the formation factor, the ratio of the resistivity of fluid-saturated rock to the resistivity of fluid alone, described by

$$F = \phi^{-\beta} \tag{33}$$

where β is a constant close to 2. Thus, the permeability-porosity relationship is

$$k = (m^2/k_0)\phi^{2\beta-1} \tag{34}$$

We also consider a pore-geometry model often used in soil mechanics [14],

$$k = k_i \ \exp\left(\frac{2.303[\phi/(1-\phi) - \phi_i/(1-\phi_i)]}{c_k}\right) \tag{35}$$

where c_k is the slope of the void ratio $[\phi/(1-\phi)]$ plotted against log (k). Fig. 1 shows the relationships between the normalized permeability and the normalized porosity from these two models.

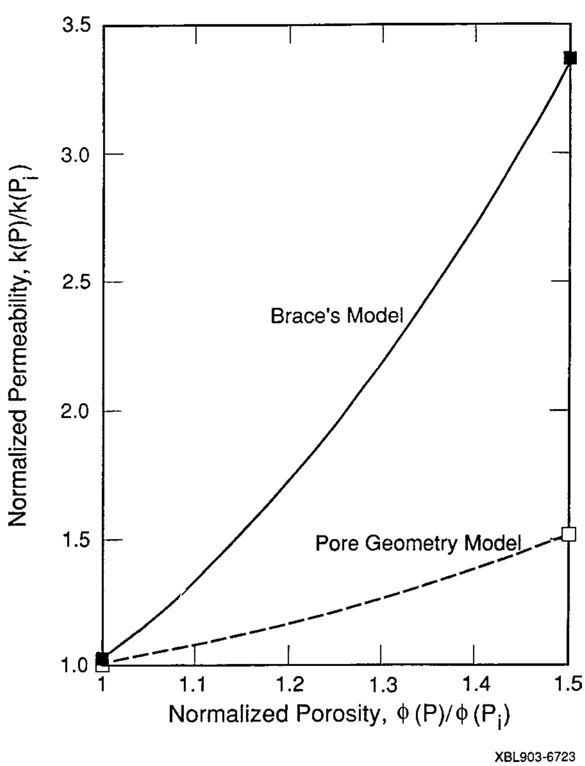

Fig. 1. Permeability functions for checking integral solutions.

5.1. Check on linear flow integral solution

5.1.1. Comparison with exact solution

The exact solution of linear flow of a slightly compressible fluid in a semi-infinite system, with constant permeability and constant injection rate at inlet $x = 0$, is given by [3]:

Table 1
Parameters for checking integral solution

Parameter	Value	Unit
Initial pressure	$P_i = 10^7$	Pa
Initial porosity	$\phi_i = 0.20$	
Initial fluid density	$\rho_w = 975.9$	kg/m^3
Cross-section area	$A = 1.0$	m^2
Formation thickness	$h = 1.0$	m
Fluid viscosity	$\mu = 0.35132 \times 10^{-3}$	Pa s
Fluid compressibility	$C_f = 4.556 \times 10^{-10}$	Pa^{-1}
Rock compressibility	$C_r = 2,\ 5 \times 10^{-9},\ 5 \times 10^{-10}$	Pa^{-1}
Initial permeability	$k = 9.869 \times 10^{-13}$	m^2
Water injection rate	$q_m = 0.01$, 0.1, 1.0 and 10	kg/s
Wellbore radius	$r_w = 0.1$	m
Hydraulic radius	$m = 10^{-5}$	m
Shape factor	$k_0 = 0.25$	
Exponential index	$\beta = 2.0$	
Coefficient, Eq. (35)	$k_i = 3.2 \times 10^{-15}$	m^2

$$P(r,\, t) = P_{\rm i} + \left[\frac{q_{\rm inj}\mu x}{kA}\right]\left[\frac{1}{\pi^{1/2}\xi}{\rm e}^{-\xi^2} - {\rm erfc}(\xi)\right] \tag{36}$$

where $q_{\rm inj}$ is the volumetric injection rate, and

$$\xi = \frac{x}{2\left(\frac{kt}{\phi_{\rm i} C_{\rm t}\mu}\right)^{1/2}} \tag{37}$$

The parameters as shown in Table 1 are used to evaluate both the exact solution (36) and the integral solution (16)–(18).

A comparison of injection pressures calculated from the integral and the exact solutions is shown in Fig. 2. The agreement between the two solutions is excellent for the entire transient period.

5.1.2. Comparison with numerical solution

The above example is simple, because we are dealing with a linear governing equation for which an exact solution exists. For the case of a pressure-dependent permeability, the governing equations become non-linear, and we no longer have exact solutions. Therefore, a numerical method is used to examine the integral solutions found in this work. The numerical code has been modified by implementing the permeability functions (34) and (35) to take into account the effects of pressure on formation permeability. The parameters of fluids and rock used for this numerical simulation are given in Table 1 and are also provided on the figures.

A comparison between the injection pressures at the inlet computed from the integral solution and the numerical model are shown in Figs. 3 and 4, for the permeability functions of Eqs. (34) and (35), respectively. It is obvious that the integral solution matches the numerical results very well for the entire injection period, while the constant-permeability (const. k) calculations lead to larger errors as injection pressure increases. The integral solutions are always expected to introduce some error; however, comparisons in Figs. 3 and 4 show that the integral solution for one-dimensional linear flow is excellent for applications.

5.2. Check on radial flow integral solution

5.2.1. Comparison with exact solution

For an infinite-acting radial system with a constant permeability, the exact (Theis) solution for slightly compressible fluid flow is [5]:

$$P(r,\, t) = P_{\rm i} + \left[\frac{q_{\rm inj}\mu}{4\pi kh}\right]\left[-Ei\left(-\frac{1}{4t_{\rm D}}\right)\right] \tag{38}$$

where $q_{\rm inj}$ is the volumetric injection rate, a constant, and $t_{\rm D}$ is the dimensionless time, defined as

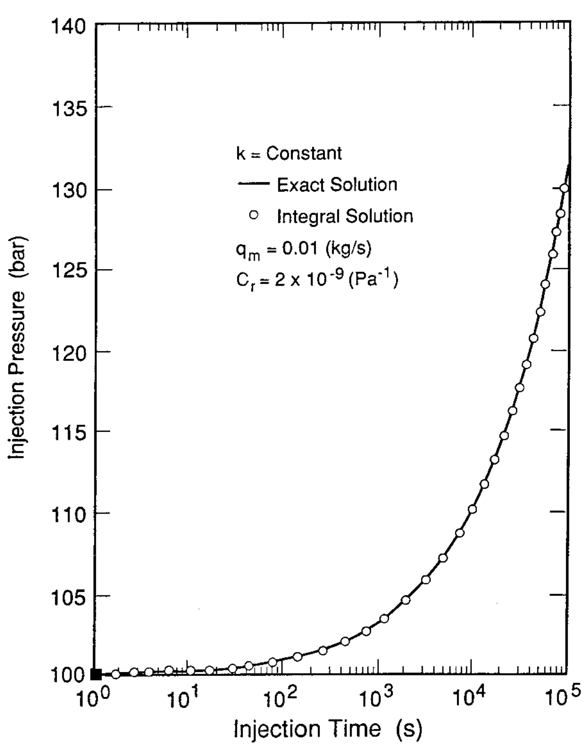

Fig. 2. Comparison of injection pressures calculated from integral and exact solutions for linear flow in a constant permeability medium.

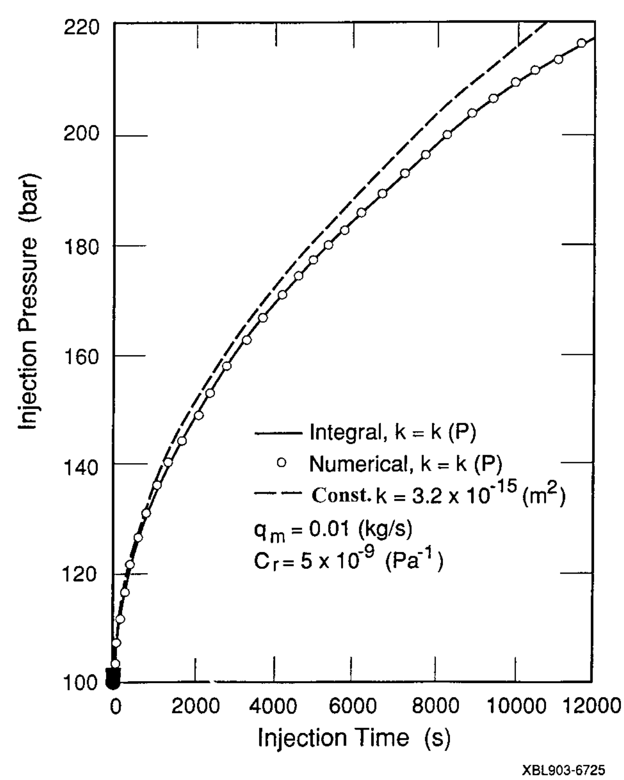

Fig. 3. Comparison of injection pressures calculated from integral, exact (constant permeability) and numerical solutions for linear flow in a permeability-dependent medium with permeability function (34).

$$t_D = \frac{kt}{\phi \mu C_t r^2} \quad (39)$$

If Eq. (24) is used to represent the pressure profile for radial fluid flow in a constant-permeability porous medium, the solution is given by Eq. (25). It has been found [29,30] that the integral solution in this form overestimates the pressure buildup due to injection by about 5–10%. On the other hand, the integral solution using a pressure profile of Eq. (27), with $P_n(r)$ being a second-degree polynomial, is given by Eq. (28). A comparison study [29] indicates that, essentially, no differences can be observed in the wellbore injection pressure calculations from the Theis solution and the integral solution (28).

5.2.2. Comparison with numerical solution

Brace's [2] permeability of model (34) is used to examine the integral solution for the radial flow case. The input parameters are given in Table 1. The calculations of the numerical, Theis and two integral solutions, are shown in Fig. 5. The integral solution, Eq. (25), with a pressure profile like (24), gives the best approximation to the problem, while both Theis and the other integral solution (28) result in larger errors. It is interesting to note that the integral solution (25) is poor for the constant permeability calculations. Obviously, the pressure profile for flow in a permeability-dependent medium deviates from the logarithmic distribution due to changes in permeability, and a pressure profile such as Eq. (24) best represents the physics.

A comparison of the different solutions for an order-of-magnitude smaller compressibility, $C_r = 5 \times 10^{-10}$ Pa^{-1}, is given in Fig. 6. In this case, the integral solution (28) is better than Eq. (25), even though certain errors with (28) are apparent. As discussed by Wu et al. [30], the pressure profile of Eq. (24) or solution (25) should be used in the integral solution in order to include effects of significant changes in permeability due to pressure variation. The integral solution (28) gives better accuracy if the medium is closed to rigid. Another alternative form of solutions was derived in Wu et al. [30] for the radial flow problem, which gives a better accuracy for intermediate ranges of compressibility. We have performed many tests by comparing integral solutions with numerical simulation results and have found that the accuracy of the integral solutions depends mainly on rock compressibility, among others. For applications of those solutions, one may substitute different forms of integral solutions into the original governing Eq. (9) or (10) to find a certain minimum or norm, in order to determine which solution should be used for a given problem when no numerical code is available.

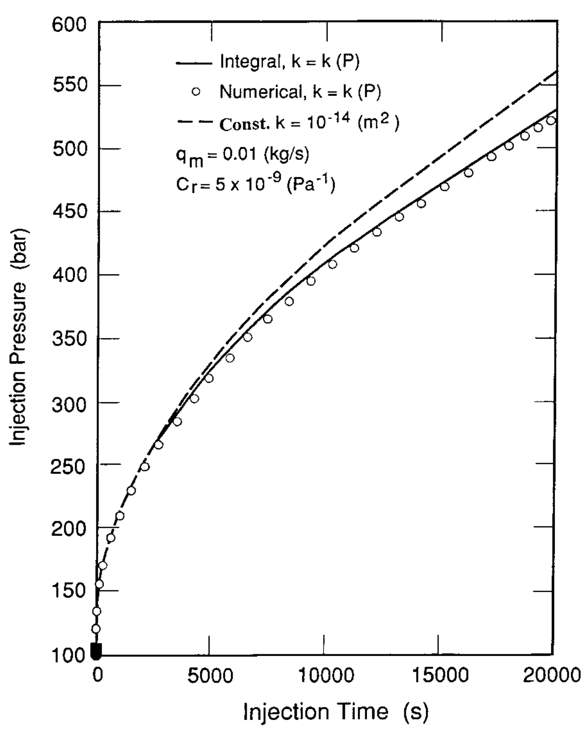

Fig. 4. Comparison of injection pressures calculated from integral, exact (constant permeability) and numerical solutions for linear flow in a permeability-dependent medium with permeability function (35).

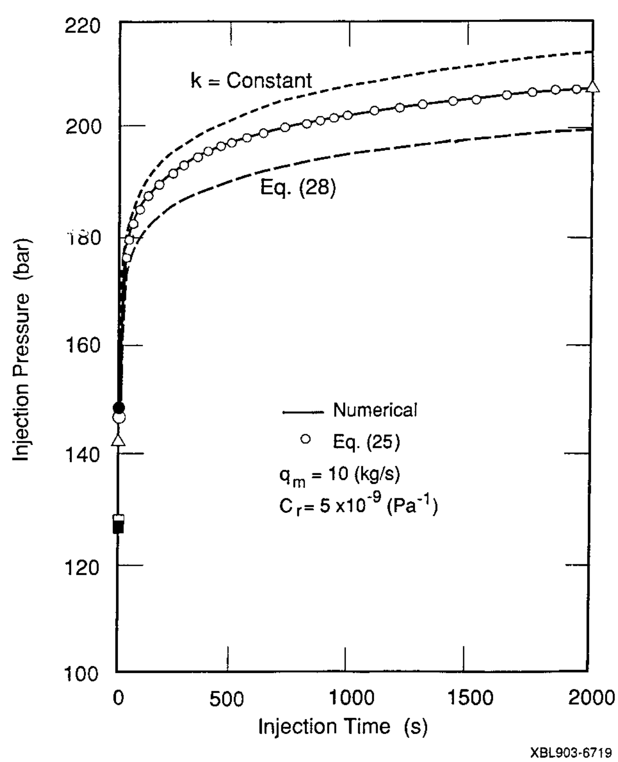

Fig. 5. Comparison of injection pressures calculated from integral and numerical solutions for radial flow in a strongly permeability-dependent medium with permeability function (34).

6. Flow through a horizontal fracture

Fluid flow through fractured media is of fundamental importance in many problems relating to energy recovery from the subsurface reservoirs and to nuclear waste disposal in geologic media. A number of physical models for fractures have been proposed to study transport phenomena in fractured media, and considerable progress has been made in understanding flow behavior of fluids through fractures since the 1950s. The simplest model is a set of parallel horizontal fractures, with constant spacing and initial aperture (see Fig. 7). This fracture model and the integral solutions obtained in this paper are used to examine the effects of coupled stress and fluid flow through a horizontal fracture system. The parameters of fluid and rock for this study are shown in Table 2. The formation is assumed to be subject to vertical uniaxial stress. Then, the aperture b is given by

$$b = b_{\mathrm{i}} + \frac{2D}{E}(P - P_{\mathrm{i}}) \tag{40}$$

where b_{i} is the initial aperture, D is the half-spacing between fractures, and E is Young's modulus of the intact rock. Fracture permeability is described by the cubic law [28] as

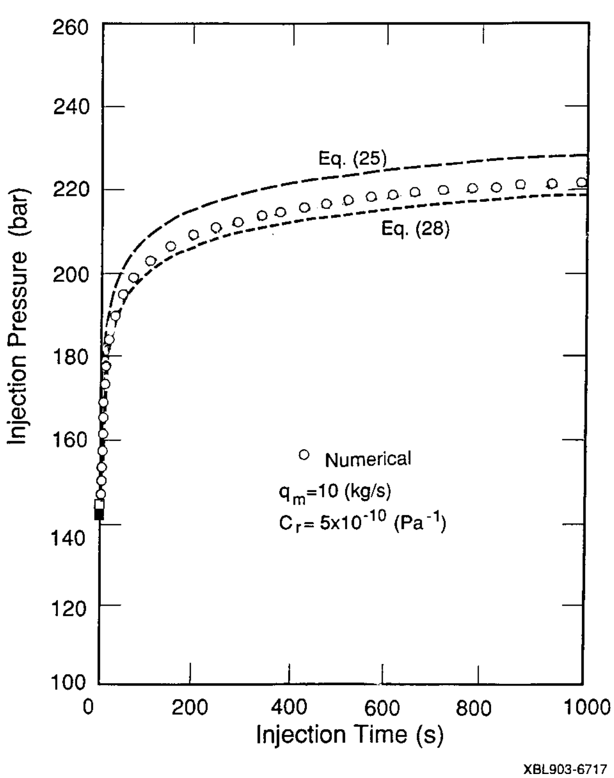

Fig. 6. Comparison of injection pressures calculated from integral and numerical solutions for radial flow in a weakly permeability-dependent medium with permeability function (34).

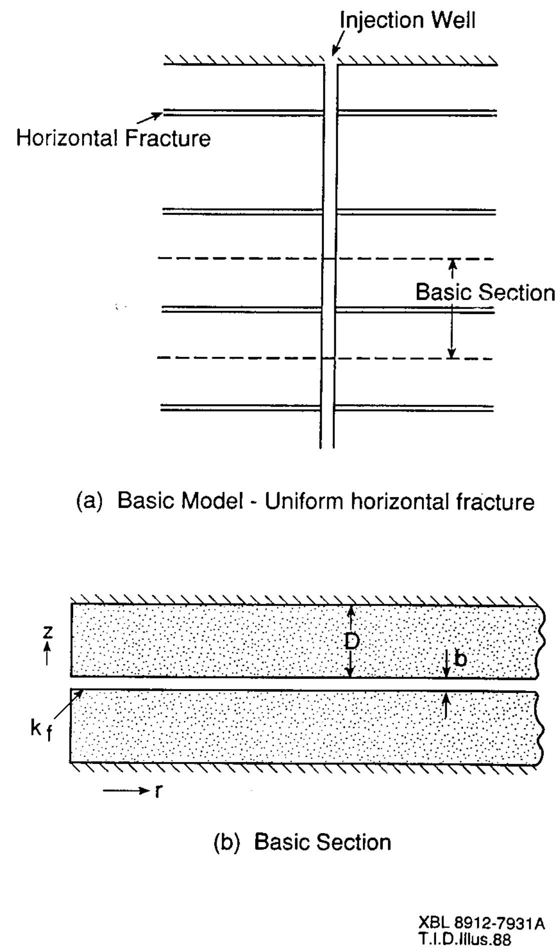

Fig. 7. Schematic of a horizontal fracture system.

$$k = \frac{b^2}{12} = \frac{b_{\mathrm{i}}^2}{12}\left[1 + \frac{2D}{Eb_{\mathrm{i}}}(P - P_{\mathrm{i}})\right]^2 \tag{41}$$

The effective permeability of fractures in the continuum sense, as used in Darcy's law, is then

$$k_{\mathrm{eff}} = k_{\mathrm{f}}\frac{b}{2D} = \frac{1}{3}\phi^3 \tag{42}$$

where ϕ is the porosity of the fracture system, given

Table 2
Parameters for flow through a horizontal fracture

Parameter	Value	Unit
Initial pressure	$P_{\mathrm{i}} = 10^7$	Pa
Initial aperture	$b_{\mathrm{i}} = 10^{-3}$, 10^{-4}	m
Initial fluid density	$\rho_{\mathrm{w}} = 975.9$	kg/m^3
Half-spacing	D = 0.25, 0.1	m
Fluid viscosity	$\mu = 0.35132 \times 10^{-3}$	Pa s
Fluid compressibility	$C_{\mathrm{f}} = 4.556 \times 10^{-10}$	Pa^{-1}
Young's modulus	$E = 5 \times 10^{11}$	Pa
Water injection rate	$q_{\mathrm{m}} = 1.0$	kg/s
Wellbore radius	$r_{\mathrm{w}} = 0.1$	m
Initial fracture permeability	$k_i = 1.66667 \times 10^{-10}$	m^2

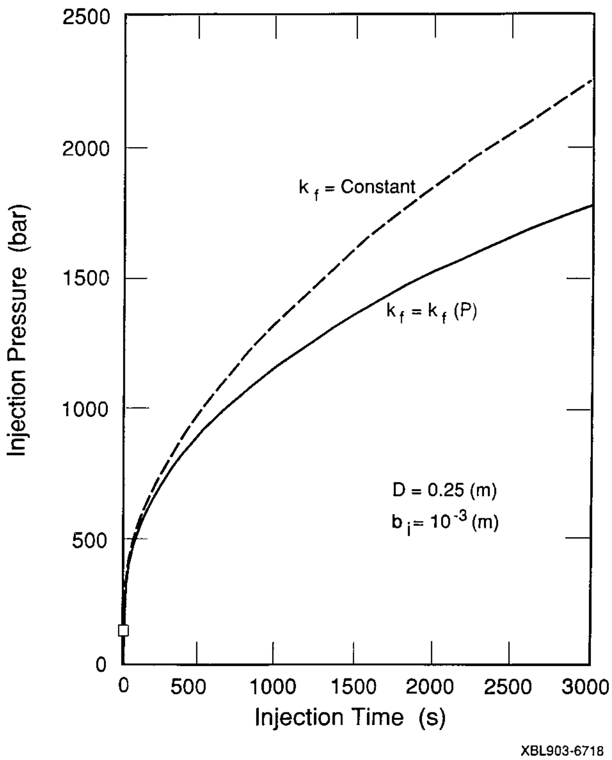

Fig. 8. Comparison of injection pressures in a linear fracture system with and without including effects of permeability changes.

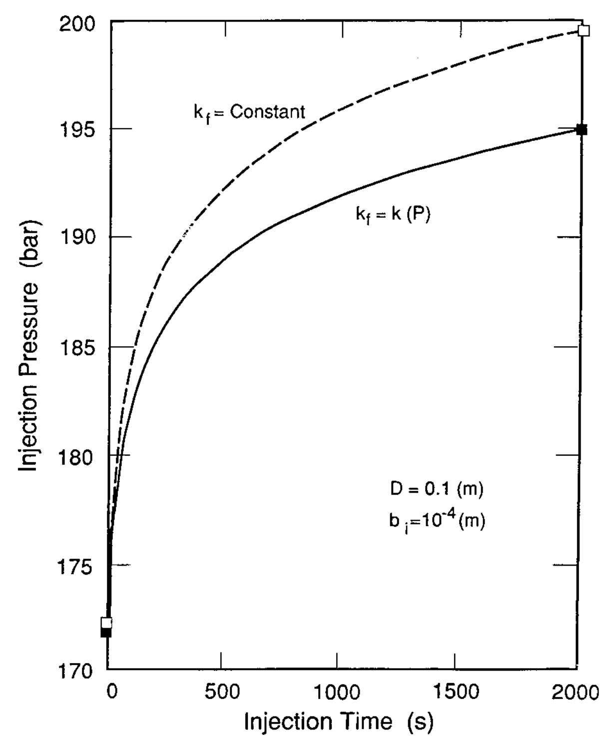

Fig. 9. Comparison of injection pressures in a radial fracture system with and without including effects of permeability changes.

by

$$\phi = \frac{b}{2D} = \frac{b_i}{2D}\left[1 + \frac{2D}{Eb_i}(P - P_i)\right] \tag{43}$$

Note that the cubic dependence of permeability on porosity in (42) is identical to Brace's model (34) for $\beta = 2$. Since the system is uniform and symmetric, only one basic section needs to be considered, as shown in Fig. 7b.

A comparison of the injection pressures in a linear horizontal fracture system, for a constant and a pressure-dependent permeability, is given in Fig. 8, for a constant mass injection rate condition at the inlet $x = 0$. Here, the integral solution (25) is used. Fig. 8 shows that, as the injection pressure gets higher, the injection pressure would be overestimated if fracture permeability were taken as a constant.

Fig. 9 shows the differences in injection pressures from constant and pressure-dependent fracture permeability solutions at the wellbore for a radial flow problem with different fracture characteristics. The constant-permeability solution always overestimates the pressure response at the wellbore as injection pressure reaches a very high value. Since the parameters used in these calculations are reasonable for actual fluid flow through fractures, we conclude that neglect of permeability dependence on pressure will lead to large errors in calculated fluid flow through fractured media for a high injection pressure operation.

7. Conclusions

1. The integral method, as commonly used for heat conduction analysis, has been applied to study fluid flow through permeability-dependent porous media. The approximate analytical solutions for one-dimensional linear and radial flow in a semi-infinite system at a specified injection rate are obtained by the integral technique. The solutions provide a good approximate solution to a general non-linear governing flow equation with arbitrary constitutive correlations of permeability, porosity and fluid density as functions of pore pressure.
2. More suitable pressure profiles are discussed for obtaining integral solutions to radial flow. Two published permeability models are used to examine the accuracy of the integral solutions by comparison with the exact solution and the numerical simulations for fluid flow through permeability-dependent porous media. Excellent agreement has been obtained for both linear and radial flow solutions. The integral solutions obtained from this paper are confirmed to give accurate results for engineering applications.

3. The effects of pressure on permeability are discussed by integral solutions for one-dimensional single-phase, slightly compressible linear and radial flow through a horizontal fracture. The results show that for flow in fractured media, ignoring pressure dependence of permeability may lead to large errors in flow behavior prediction under high-pressure operations.
4. The analytical solutions provided in this paper for the coupled fluid flow and rock permeability variation problem will find their applications in the following three fields: (i) to obtain some physical insight into hydraulic–mechanic coupling phenomena of porous medium flow; (ii) to determine some fluid and formation properties by well test analysis or laboratory test methods; and (iii) to verify numerical codes that include effects of pressure-dependent fluid and rock properties.

Acknowledgements

For a critical review of the manuscript the authors are grateful to J. Hinds. The authors would also like to thank N. G. W. Cook for his encouragement and valuable suggestions for this work. This work was supported by the Director, Office of Energy Research, the Office of Basic Sciences, Engineering and Geosciences Division of the US Department of Energy under Contract No. DE-AC03-76SF00098.

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