

Physics-guided deep learning using Fourier neural operators for solving the acoustic VTI wave equation

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ABSTRACT

Many real-world seismic modeling and imaging applications require computing frequency-domain numerical solutions of acoustic wave equation (AWE). However, obtaining such solutions in media characterized by strong parameter contrasts and anisotropy poses significant practical challenges to existing numerical solvers, especially for 3D scenarios. Physics-informed neural networks (PINN) provide a computationally efficient alternative approach for AWE solutions. However, PINNs solve only a single instance of AWE and need to be re-trained for each different subsurface models and frequencies. Fourier neural operators, on the other hand, can solve AWE for a wide range of models and frequencies with a single set of network configuration and parameters. This method, though, requires a tremendous amount of data, which can be difficult and expensive to obtain. Here, we propose a methodology that combines PINNs with Fourier neural operators to learn AWE solution operators that are valid for a wide range of frequencies without requiring any training data. We present two numerical examples that demonstrate the capabilities of the proposed method in modeling the acoustic wavefield accurately and efficiently in the frequency domain.

1 INTRODUCTION

Frequency-domain numerical solutions of the acoustic wave equation (AWE) in anisotropic media form the key computational kernel of a variety of full-waveform modeling, imaging, inversion applications that are of interest to scientists from various disciplines ranging from exploration and planetary seismology to medical imaging. However, obtaining such solutions with available numerical methods requires inverting a large impedance matrix, which is both challenging and expensive, especially for 3D scenarios Song et al. (2021).

Physics-informed neural networks (PINN) provide an alternative approach to partial differential equation (PDE) solutions by imposing physical constraints to the loss function during training (Raissi et al., 2017b,a). PINNs can compute wavefield partial derivatives with respect to spatial and temporal coordinates with an accuracy up to the machine precision by using the automatic differentiation procedure. Recently, many successful PINN-based wave-equation solvers have been demonstrated. Moseley et al. (2020) and Voytan and Sen (2020) obtain time-domain PINN-based solutions for acoustic wave equation. Song et al. (2021) solve the frequency-domain acoustic wave equation for scattered wavefields. Rao et al. (2020) present time-domain solutions of elastodynamics on Cartesian coordinates without training data.

Despite their computational advantages over conventional numerical methods, PINN-based AWE solvers still suffer from theoretical and practical limitations. Conventional PINN-based applications aim to find a mapping between inputs (e.g., spatiotemporal coordinates, subsurface parameters) and the desired AWE solution (e.g., scalar pressure and/or particle velocity vector) in finite-dimensional Euclidean spaces. Therefore, they are limited to the specific spatiotemporal discretization (i.e., solution grid and frequency) and subsurface parameters that are used during the training. These networks need to be re-trained if one needs to obtain AWE solutions with a new set of subsurface parameters or at another frequency. Practitioners mentioned above utilize artificial neural networks (ANN) to remove the dependency of the wave equation solution to a specific discretization, however ANNs are computationally expensive and require training an enormous number of parameters especially for 3D applications.

Recently, scientists propose neural operators for obtaining resolution-independent PDE solutions (Li et al., 2020b,a). These

methods aim to learn infinite-dimensional mappings (operators) from any input parameters to PDE solutions. Therefore, they are not limited to a single instance of PDEs and can provide solutions for a wide variety of parameters and grid discretizations using a single set of network configuration. Li et al. (2020a) parameterized neural operators in Fourier space (i.e., Fourier neural operators) to solve a family of PDEs for a range of parameters. The Fourier space allows the utilization of global operators that are able to capture long-distance spatial dependencies and provides a natural framework for AWE solutions in the frequency domain. However, Fourier neural operators rely on a supervised network training procedure and require a tremendous amount of data generated using conventional numerical (e.g., finite differences, finite elements) solvers for a wide range of parameter variations.

In this work, we combine the physics-informed deep learning methodology with neural operators in the Fourier space to obtain the AWE solutions in VTI media without using training data. To facilitate the neural network training, we split the acoustic wavefield into background and scattered terms. The background wavefield satisfies the isotropic AWE, which can be solved analytically, and we predict the monochromatic scattered wavefield via a neural network. Because we train neural operators in a high-dimensional Fourier space, our methodology allows us to use multiple frequencies simultaneously during training and generalize well to scenarios where one wants to obtain AWE solutions on different frequencies than the ones used in the training. We provide two numerical examples to demonstrate the quality of the results obtained with the proposed method as well as its advantages over conventional PINN-based workflows.

2 METHOD

2.1 The pure P-wave equation in VTI media

The constant-density acoustic wave equation can be written in the frequency domain as

$$-\omega^2 u(\mathbf{x}, \omega) - L^2 u(\mathbf{x}, \omega) = f(\mathbf{x}, \omega), \quad (1)$$

where $\mathbf{x} = [x, y, z]$ are spatial coordinates; ω is the angular frequency; $u(\mathbf{x}, \omega)$ is the scalar pressure wavefield; $f(\mathbf{x}, \omega)$ is the source function; L^2 operator is obtained using the exact phase velocity expression in VTI media (Harlan, 1995) as

$$-L^2 = v_z^2 k_z^2 + v_h^2 k_r^2 + (v_n^2 - v_h^2) k_r^2 k_z^2 / k_\rho^2, \quad (2)$$

where v_z is the P-wave velocity along the symmetry axis (i.e., vertical); $v_n = v_z \sqrt{1 + 2\delta}$ is the NMO velocity; $v_h = v_z \sqrt{1 + 2\epsilon}$ is the horizontal velocity; ϵ and δ are dimensionless anisotropy parameters; k_x, k_y , and k_z are wavenumbers in the x, y, and z directions, respectively; $k_r = k_x^2 + k_y^2$ and $k_\rho = k_x^2 + k_y^2 + k_z^2$.

Because point-source injection poses practical challenges for PINN-based solvers (Song et al., 2021; Voytan and Sen, 2020), we first split the acoustic wavefield and the vertical velocity into background and scattered terms ($u = u_0 + \delta u, v_z = v_{z_0} + \delta v_z$), where v_{z_0} and u_0 are the background (i.e., reference) P-wave velocity, and wavefield, respectively, and δu is the perturbations from the reference wavefield. The background pressure wavefield satisfies the isotropic acoustic wave equation

$$-\omega^2 u_0(\mathbf{x}, \omega) - L_0^2 u_0(\mathbf{x}, \omega) = f(\mathbf{x}, \omega), \quad (3)$$

where $-L_0^2 = v_{z_0}^2 k_\rho^2$. We obtain the wave equation for scattered pressure in VTI media by subtracting equation 3 from 1 and using $u = u_0 + \delta u$ as

$$-(\omega^2 + L^2) \delta u(\mathbf{x}, \omega) = (\omega^2 + L^2 + L_0^2) u_0(\mathbf{x}, \omega). \quad (4)$$

Note that equation 4 does not contain a point source term. Assuming an isotropic and homogeneous background allows us to obtain the reference wavefield $u_0(\mathbf{x}, \omega)$ from analytical AWE solutions.

2.2 Physics-constrained deep learning

To establish a deep learning framework to solve equation 4, we use a neural network system, where the inputs are real and imaginary components of the background wavefield at different frequencies and the outputs are complex monochromatic scattered pressure wavefields at multiple frequencies. We train our network by imposing equation 4 as physical constraints on the learning process through minimizing the mean-squared loss function

$$f = \frac{1}{N} \sum_{i=1}^N \|(\omega^2 + L^2 + L_0^2) u_0(\mathbf{x}, \omega) + (\omega^2 + L^2) \delta u(\mathbf{x}, \omega)\|_2^2. \quad (5)$$

To evaluate the L^2 and L_0^2 operators, we first transform the background and scattered wavefields to wavenumber domain using the Fourier Transform, which is available in most modern deep-learning packages (e.g., PyTorch, Tensorflow), and multiply with

wavenumber vectors according to equation 2. We then transform the results back to spatial coordinates and scale with appropriate velocities.

2.3 Fourier neural operators

To obtain AWE solutions independent of the grid discretization and resolution as well as the frequency, we first lift network inputs (i.e., background wavefields) to a high-dimensional “channel” space using a neural network that contains 1×1 convolutional layers. We then use a series of Fourier layers to learn a solution operator that maps background wavefields to scattered wavefields at multiple frequencies. In each Fourier layer, we first perform a 2D Fourier transform and apply a linear transformation on the lower Fourier modes and filter out the higher modes to establish regularization during training. Finally, we perform an inverse Fourier transform. After Fourier layers, we project features back to the image (i.e., spatial) domain using 1×1 convolutional layers.

3 NUMERICAL EXAMPLES

3.1 Two-layer medium

Our first example presents a simple two-layer medium, with $x = 2.5$ km and $z = 2.5$ km, where the top layer is isotropic $v_z = 2.5$ km/s and the bottom layer is characterized by vertical transverse isotropy with $v_z = 4.0$ km/s and anisotropy parameters $\epsilon = 0.24$ and $\delta = 0.1$. The source is located at $x = 1.25$ km and $z = 1$ km.

The network contains two 1×1 convolutional layers after the input layer to lift the background wavefields obtained from the analytical AWE solution to a high-dimensional space, and two 1×1 convolutional layers before the output layer to transform features back to the spatial domain. In between convolutional layers, we use five Fourier layers. We train our network with 64 frequencies ranging from 5 Hz to 15 Hz, and we use 16 frequencies for validation. During the training, we perform 10000 epochs with a learning rate of 0.0005. Figure 3.2 presents the real component of monochromatic wavefields obtained using a numerical solver for equation 4 and using the proposed methodology at four different frequencies. Note that network predictions match to wavefields obtained using the numerical solver in terms of both shape and amplitude, as all wavefields are plotted within the same amplitude range.

3.2 Layered VTI medium

For our second example, we use a portion of the Marmousi model 3.2 with constant anisotropy parameters $\epsilon = 0.2$ and $\delta = 0.1$. The models have same dimensions as the two-layer model. We place the source at $x = 1.25$ km and $z = 0$ km and use the same network architecture and training parameters as the previous example. Figure 3.2 shows the real component of four monochromatic wavefields generated using a numerical solver and predicted by the neural network. Similar to the previous example, the network predicts accurate wavefields.

4 CONCLUSIONS

We propose a physics-based deep learning methodology to solve the AWE in the frequency domain. The proposed method utilizes neural operators that can find mappings between monochromatic background and scattered pressure wavefields at multiple frequencies. These operators are defined in the Fourier space to learn AWE solution operators that are independent from spatial discretizations and are able to predict solutions at frequencies that are not included during the training process. Two numerical examples demonstrate the prediction and generalization capabilities of our proposed methodology in generating accurate AWE solutions.

5 ACKNOWLEDGEMENTS

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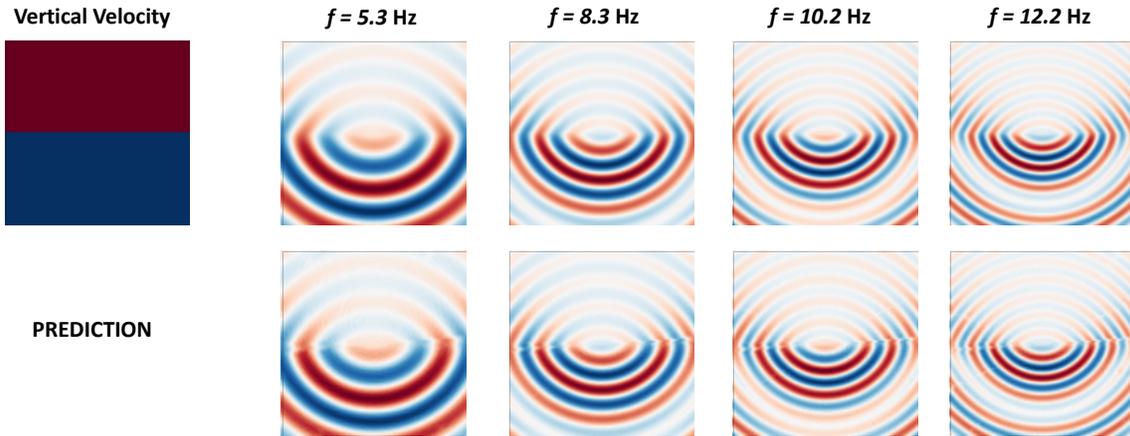


Figure 1. The real component of monochromatic scattered wavefields for the two-layer model obtained using a numerical solver (top row) and predicted by the proposed neural network (bottom row).

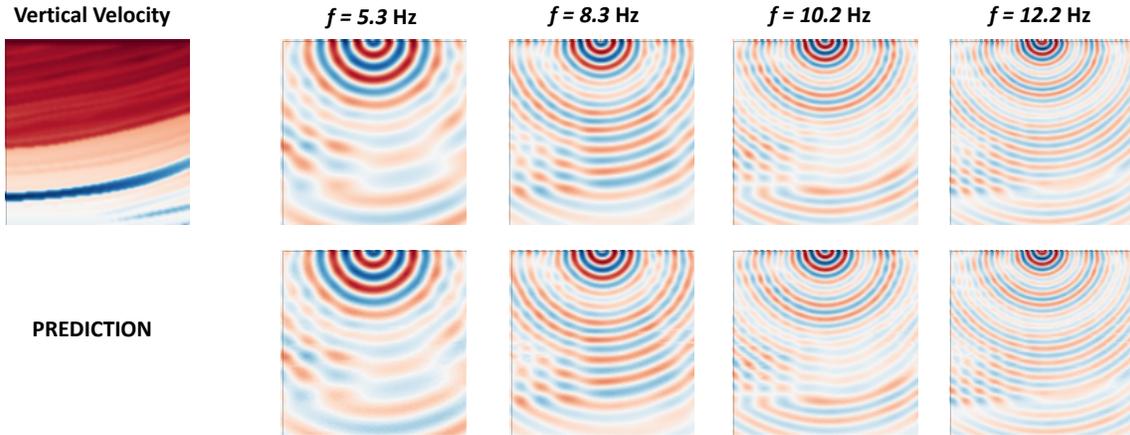


Figure 2. The real component of monochromatic scattered wavefields for the layered VTI model obtained using a numerical solver (top row) and predicted by the proposed neural network (bottom row).

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