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Ray Theoretical Modeling for Seismic Surveys Along a Common Trend (Strike) in Layered Acoustic Media

by

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ABSTRACT

Pseudo zero-offset raypaths, determined using common shot experiments along a dip line, are used to determine corresponding raypaths for a source receiver configuration along the common trend in a 2.5-D layered acoustic media.

Ray theory is used to derive an expression for the amplitude of the wave propagating away from the source. Then we derive the geometrical spreading, reflection and transmission effects for the given source-receiver geometry.

We finally show that the difference between the expression for the in-plane and out-of-plane wave propagations in a 2.5-D earth is that we have an extra term in the out-of-plane spreading in the strike rays when compared to the dip ray amplitude factor.

Given the pseudo zero-offset data and the slowness in the strike direction, we show that all the parameters for the experiment are determined.

Except for the slowness in the strike direction being constant, this experiment is three dimensional.

1. INTRODUCTION

Areas of interest for hydrocarbon exploration (sedimentary basins) are very often 2.5-D in nature. That is, the geology along the strike is invariant.

The mathematical definition of a 2.5-D earth is that the structural variations are essentially 2-D cylindrical in three dimensions while the wave propagation is 3-D (Bleistein, 1986).

Often, reflection seismic surveys are conducted in a grid of dips and strikes. The ray theoretical modeling and inversion for a 2.5-D earth, with the source and receiver configuration along the dip line was developed by Bleistein (1986) and Docherty (1987). In this paper the forward problem for a source and a receiver configuration along the common trend in a 2.5-D earth is solved.

This work was motivated by Marathon Oil Company, who is a sponsor of the Consortium Project on Seismic Inverse Methods for Complex Structures at the Center for Wave Phenomena. The University of Houston built a 2.5-D model at their Seismic Acoustics Lab and shot along the dip and strike of the model. This data was donated to the Center for Wave Phenomena. As the Center for Wave Phenomena did not have modeling and inversion codes for strike lines, it was necessary that they be created.

We begin, in Section 2, with the discussion of the kinematics of rays along the common trend (strike) of a layered acoustic media. That is, we show how the raypath in the strike direction can be determined from its corresponding pseudo zero-offset raypath along the dip direction. Then, in Section 3, we review some results from asymptotic solutions of the reduced wave equation (Keller and Lewis, 1964) and ray theory (Bleistein, 1984) and (Bleistein, 1988), and develop an expression for the amplitude of the wave propagating in the strike direction. The amplitude factor will also be used in our common strike inversion code that will be developed in the near future.

Finally, we finish with some examples generated by the pseudo zero-offset and common strike shot code. These codes are ready to be used.

2. KINEMATICS OF COMMON STRIKE RAYS

A mathematical model in which there is no variation of speed in a given direction (e.g, x_2) is called 2.5-D. In our case, this means that the acoustic velocity is represented as $c(x_1,x_3)$ from which it follows that p_2, the second component of slowness, is constant on rays. The direction of constant p_2 is known as the strike in geology, as shown in Figure 1.

Given a source-receiver configuration parallel to the common trend of a geological structure, specular reflection points directly under the profile line miss the profile except in the least interesting case of horizontal flat beds. Hence, any non 3-D migration or inversion technique that tries to image these points fails because data associated with these points are not recorded. On the other hand, a 2-D migration will erroneously place out-of-plane events in-plane.

Consider a ray crossing $n+1$ interfaces between source and receiver. In layer i with constant velocity c_i, located between interfaces $i-1$ and $i(i>0)$, the traveltime τ of the wave must satisfy the eikonal equation,

$$(\nabla\tau)_i^2 = \frac{1}{c_i^2} \quad . \tag{2.1}$$

By using the method of characteristics to solve this equation one can show that the gradient of the traveltime or slowness vector is constant in each layer and can be expressed as,

$$(\nabla\tau)_i = (p_{1i},p_{2i},p_{3i}) \quad . \tag{2.2}$$

The continuity condition for the tangential component of slowness at interfaces, otherwise known as Snell's law, has an interesting consequence in our case. Since p_{2i} is one of the tangential components of slowness, due to the cylindrical nature of the model, we have

$$p_{2i} = p_{2(i+1)} = p_2 \quad , \tag{2.3}$$

which is a constant along the ray.

This remarkable property gives us an opportunity to solve the eikonal equation in a much simpler way than the 3-D ray tracing. That is, we fix p_2 and rewrite (1) as

$$p_{1i}^2+p_{3i}^2 = \frac{1}{v_i^2} \quad , \tag{2.4}$$

where

$$v_i = \frac{c_i}{\sqrt{1-c_i^2 p_2^2}} \quad , \tag{2.5}$$

is a velocity that is always higher than the true velocity in the layer. This means that by solving the 2-D pseudo zero-offset problem defined by equation (2.4) and velocities from equation (2.5), we are able to completely determine the offset ray for the corresponding value of p_2.

For this we define Z_{i+1} as a plane parallel to the x_2 axis and containing the segment of the pseudo zero-offset ray between the interface i and $i+1$ (Figure 1). It follows from (2.4) that Z_{i+1} (i=0,...,n) are the planes of wave propagation in the strike direction.

The incremental distance traveled by the ray in the strike direction through layer i is given by

$$\Delta x_{2i} = p_2 v_i \sqrt{(\Delta x_{1i})^2 + (\Delta x_{3i})^2} \quad , \tag{2.6}$$

where Δx_{1i} and Δx_{3i} are the incremental distances through layer i for the pseudo zero-offset raypath. We can see that if the root in (2.5) is complex, there is no propagating wave (post critical angle). This happens whenever p_2 is greater than the total slowness in the layer under investigation.

The incremental traveltime through layer i is also given exclusively in terms of the kinematics of the pseudo zero-offset section by

$$\Delta \tau_i = \left| \frac{1+p_2^2 v_i^2}{c_i^2} \left[(\Delta x_{1i})^2 + (\Delta x_{3i})^2 \right] \right|^{1/2}$$

$$= \frac{v_i}{c_i^2} \sqrt{(\Delta x_{1i})^2 + (\Delta x_{3i})^2} \quad . \tag{2.7}$$

Given any value of p_2 and the pseudo zero-offset raypath determined using (2.5), we are sure that the reflected specular ray from the $i+1$ interface, will lie in the Z_{i+1} planes and pass somewhere through the profile line. But we still do not know what value of p_2 gives the specular ray that comes to our specific receiver that is located at an offset $= x_2^{i+1}$.

From (2.5), we had seen that p_2 ranges between 0 and $p_{2max} = \min(1/c_i)$, $0 \le i \le n+1$. This helps us to control our value of p_2. $p_2 = 0$ corresponds to propagation in the (x_1,x_3) plane and $p_2 = p_{2max}$ corresponds to propagation in the x_2 direction along the interface i. Hence, for p_2 greater than zero and less than p_{2max}, we use (2.6) to calculate the travel distance x_2^{n+1} of the ray in the strike direction. Then we check if this value is greater or less than the offset. When the percentage error falls within a prescribed bound we keep this ray.

In this manner we are able to calculate the total distance traveled by the ray from the source to the receiver. To find the total traveltime of the specular ray, we divide the ray segment within each layer by the velocity of the wave in the layer and then add the results. We will derive these results in greater detail in the next section.

3. 2.5-D COMMON-STRIKE AMPLITUDE

In this section we derive an expression for the amplitude of the wave propagating from a point source in a layered acoustic medium to a receiver along the trend of the medium.

To do this, we review some results from asymptotic solutions of the reduced wave equation (Keller and Lewis, 1964) and the ray theory (Bleistein, 1984) and then solve our specific problem. We start with the time-reduced wave (Helmholtz) equation.

$$\nabla^2 U(\boldsymbol{x},\omega)+\frac{\omega^2}{c^2(\boldsymbol{x})}U(\boldsymbol{x},\omega) = 0 \quad . \tag{3.1}$$

Here $\boldsymbol{x}=(x_1,x_2,x_3)$ and $U(\boldsymbol{x},\omega)$ is the scalar potential.

We introduce the WKBJ solutions for $U(\boldsymbol{x},\omega)$ defined by

$$U(\boldsymbol{x},\omega) \sim e^{i\omega\tau(\boldsymbol{x})}\sum_{n=0}^{\infty}\frac{A_n(\boldsymbol{x})}{(i\omega)^n} \quad . \tag{3.2}$$

We will discuss only the leading order term and thus set

$$U(\boldsymbol{x},\omega) \sim A(\boldsymbol{x})e^{i\omega\tau(\boldsymbol{x})} \quad . \tag{3.3}$$

Below we will refer to $A(\boldsymbol{x})$ and $\tau(\boldsymbol{x})$ as the amplitude and traveltime of the wave from the source to the position $\boldsymbol{x}$.

Substituting $U(\boldsymbol{x},\omega)$ into equation (3.1) and then equating the coefficients of ω^2 and ω in equation (3.2) to zero, we obtain the eikonal equation,

$$(\nabla\tau)^2 = \frac{1}{c^2(\boldsymbol{x})} \quad , \tag{3.4}$$

and the transport equation,

$$2\nabla A\cdot\nabla\tau+A\nabla^2\tau = 0 \quad . \tag{3.5}$$

Introducing a running parameter σ along the ray (Bleistein, 1984), we rewrite the equation of the ray as

$$\boldsymbol{x} = (x_1, x_2, x_3) = \boldsymbol{x}(\sigma) \quad . \tag{3.6}$$

The method of characteristics for the eikonal equation yields the following system of ordinary differential equations for $\boldsymbol{x}$

$$\frac{d\boldsymbol{x}(\sigma)}{d\sigma} = \nabla\tau \quad . \tag{3.7}$$

From this equation, it follows that

$$\frac{d}{d\sigma}\left(\frac{d\boldsymbol{x}}{d\sigma}\right) = \frac{d}{d\sigma}(\nabla\tau)$$

$$= \frac{1}{2}\nabla\left(\frac{1}{c^2(\boldsymbol{x})}\right) \quad . \tag{3.8}$$

Introducing the vector $\boldsymbol{p} = (p_1, p_2, p_3)$ the total slowness is

$$\boldsymbol{p} = \nabla\tau \quad . \tag{3.9}$$

Therefore, the ray equations can be rewritten as follows:

$$\frac{d\boldsymbol{x}}{d\sigma} = \boldsymbol{p} \quad , \tag{3.10}$$

$$\frac{d\boldsymbol{p}}{d\sigma} = \frac{1}{2}\nabla\left(\frac{1}{c^2(\boldsymbol{x})}\right) \quad , \tag{3.11}$$

$$\frac{d\tau}{d\sigma} = \nabla\tau \cdot \frac{d\boldsymbol{x}}{d\sigma} = \frac{1}{c^2(\boldsymbol{x})} \quad . \tag{3.12}$$

In particular, from the second equation in (3.11),

$$\frac{dp_2}{d\sigma} = \frac{1}{2}\frac{\partial}{\partial x_2}\left[\frac{1}{c^2(\boldsymbol{x})}\right] = 0 \quad , \tag{3.13}$$

and p_2 is a constant along a ray, given by its initial value.

Using equations (3.10), (3.11) and (3.12), and some initial conditions, that is, values of $\boldsymbol{x}$, τ and $\boldsymbol{p}$ at some specified value of σ, constitutes a well-posed problem. These equations can be used to find the ray path.

Next we investigate the transport equation to develop an expression for the amplitude of the wave.

Following Lewis and Keller (1964) and Bleistein (1984), rewrite the transport equation as

$$\nabla \cdot (A^2 \nabla \tau) = 0 \quad . \tag{3.14}$$

In regions where the velocity is constant, we use the divergence theorem to obtain

$$\int_V \nabla \cdot (A^2 \nabla \tau) dV = \int_S A^2 \nabla \tau \cdot \hat{\boldsymbol{n}} dS = 0 \quad , \tag{3.15}$$

where V is the volume bounded by a tube of rays sealed at both ends by a surface of constant σ. Since no rays are dispersing on the side of the ray tube, the surface integral then reduces to an integral over the ends of the tube.

Following Lewis and Keller (1964) and Docherty (1987), we shrink the tube of rays to the central ray in the ray tube and obtain

$$A^2(\sigma) p(\sigma) dS = A^2(\sigma_0) p(\sigma_0) dS_0 \quad , \tag{3.16}$$

where σ is an arbitrary point of the ray and σ_0 is the value of σ on some initial surface. For our specific problem, that is, a 2.5-D layered acoustic media with source and receiver along a line of common trend, we choose a pair of parameters $(\beta,\ p_2)$ that characterize a ray within the ray tube. Here p_2 is the constant value of the second component of the slowness vector along a ray, and β is the angle the ray makes with the z direction as was used by Docherty (1987). The ray equations now take the form

$$\boldsymbol{x} = \boldsymbol{x}(\sigma, \beta, p_2) \quad , \tag{3.17}$$

with σ, β and p_2 as the ray coordinates; β, and p_2 label the ray and σ varies along the ray.

To model a strike line ray, introduce a point source at $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ (Bleistein, 1986). The initial data for our problem is then

$$\boldsymbol{x}(0,\beta,p_2) = \boldsymbol{\xi} \quad , \tag{3.18}$$

$$\tau(0,\beta,p_2) = 0 \quad , \tag{3.19}$$

$$\boldsymbol{p}(0,\beta,p_2) = \left[\sin\beta\sqrt{c(\boldsymbol{\xi})^{-2}-p_2^2},\ p_2,\ \cos\beta\sqrt{c(\boldsymbol{\xi})^{-2}-p_2^2}\right] \quad . \tag{3.20}$$

In the neighborhood of the point source, we consider the velocity to be constant. Hence, the wavefronts close to the source are spherical surfaces, centered at the source, and the solution to the reduced wave equation is the 3-D Green's function,

$$U(\boldsymbol{x},\omega) = \frac{1}{4\pi\,|\boldsymbol{x}-\boldsymbol{\xi}|}e^{i\omega|\boldsymbol{x}-\boldsymbol{\xi}|/c} \quad . \tag{3.21}$$

We consider, now, the solution of (3.16) for $A(\sigma)$. Note, first, that the cross-sectional area of our ray tube is given by

$$dS = \left|\hat{\boldsymbol{n}}\cdot\frac{\partial \boldsymbol{x}}{\partial \beta}\times\frac{\partial \boldsymbol{x}}{\partial p_2}\right| d\beta dp_2$$

$$= J d\beta dp_2 \quad . \tag{3.22}$$

Using equation (3.8) we can see that

$$\hat{\boldsymbol{n}} = \frac{1}{p(\sigma)}\frac{d\boldsymbol{x}}{d\sigma} \quad ; \tag{3.23}$$

therefore, the ray Jacobian J can be rewritten as

$$J = \frac{1}{p(\sigma)}\left|\frac{d\boldsymbol{x}}{d\sigma}\cdot\frac{\partial \boldsymbol{x}}{\partial \beta}\times\frac{\partial \boldsymbol{x}}{\partial p_2}\right| \quad . \tag{3.24}$$

Hence, the amplitude of the wave at any σ is given as

$$A(\sigma) = A(\sigma_0)\left[\frac{p(\sigma_0)}{p(\sigma)}\frac{J(\sigma_0)}{J(\sigma)}\right]^{1/2} \quad . \tag{3.25}$$

From equation (3.25), note that the amplitude of the wave away from our source is proportional to the square root of the ratio of the ray Jacobians at σ_0 and σ. This is a measure of the expansion or contraction of the ray tube known as geometric expansion or geometrical spreading.

In this equation A(σ_0), $p(\sigma_0)$ and $p(\sigma)$ are known. We need to find expressions for $J(\sigma_0)$ and $J(\sigma)$.

We begin by looking for an expression for $J(\sigma_0)$. For $\sigma_0 = 0$ and $J(\sigma_0) = 0$ the solution method breaks down. Hence, we have to appeal to a canonical problem that produces $A(\sigma_0)$. From our ray equation

$$\boldsymbol{x} = (x_1, x_2, x_3) = \boldsymbol{x}(\sigma) \quad . \tag{3.26}$$

Therefore, around the source

$$x_1 - \xi_1 = \sigma \sin\beta \frac{\sqrt{1-p_2^2 c_1^2}}{c_1} \quad ,$$

$$x_2 - \xi_2 = p_2 \sigma \quad ,$$

$$x_3 - \xi_3 = \sigma \cos\beta \frac{\sqrt{1-p_2^2 c_1^2}}{c_1} \quad . \tag{3.27}$$

Using equation (3.27) we obtain

$$\frac{\partial x_1}{\partial \beta} = \sigma \cos\beta \frac{\sqrt{1-p_2^2 c_1^2}}{c_1} \quad ,$$

$$\frac{\partial x_2}{\partial \beta} = 0 \quad ,$$

$$\frac{\partial x_3}{\partial \beta} = -\sigma \sin\beta \frac{\sqrt{1-p_2^2 c_1^2}}{c_1} \quad , \tag{3.28}$$

$$\frac{\partial x_1}{\partial p_2} = \frac{-p_2 c_1 \sigma \sin\beta}{\sqrt{1-p_2^2 c_1^2}} \quad ,$$

$$\frac{\partial x_2}{\partial p_2} = \sigma \quad ,$$

$$\frac{\partial x_3}{\partial p_2} = \frac{-p_2 c_1 \sigma \cos\beta}{\sqrt{1-p_2^2 c_1^2}} \quad , \tag{3.29}$$

$$\frac{\partial x_1}{\partial \sigma} = \sin\beta \frac{\sqrt{1-p_2^2 c_1^2}}{c_1} \quad ,$$

$$\frac{\partial x_2}{\partial \sigma} = p_2 \quad ,$$

$$\frac{\partial x_3}{\partial \sigma} = \cos\beta \frac{\sqrt{1-p_2^2 c_1^2}}{c_1} \quad . \tag{3.30}$$

Using equations (3.28), (3.29) and (3.30) with equation (3.24) we have a very simple expression for $J(\sigma_0)$, namely,

$$J(\sigma_0) = \frac{\sigma_0^2}{c_1} \quad . \tag{3.31}$$

For our point source,

$$A(\sigma_0) = \frac{1}{4\pi(x-\xi)} \quad ,$$

$$= \frac{c_1}{4\pi\sigma_0} \quad . \tag{3.32}$$

Thus, $A(\sigma_0) \sqrt{J(\sigma_0) p(\sigma_0)} = 1/4\pi$ and equation (3.25) can be rewritten as

$$A(\sigma) = \frac{1}{4\pi} \frac{1}{\sqrt{p(\sigma) J(\sigma)}} \quad . \tag{3.33}$$

In a medium with a smooth velocity profile, equation (3.33), with knowledge of $J(\sigma)$, can be used to calculate the amplitude of the wave at any σ. However, the purpose of this paper is to determine the amplitude and traveltime of the wave in a layered acoustic media with jumps in velocity across the interface and an observation profile along the common trend. As a result of this equation, equation (3.15) is valid only within a layer. At the interfaces we need to include transmission and reflection effects.

On the i^{th} segment of the ray we have

$$\int A_i^2 \nabla \tau \cdot \hat{\boldsymbol{n}}_i dS_i = \int A_{(i-1)'}^2 \nabla \tau_{(i-1)'} \cdot \hat{\boldsymbol{n}}_{(i-1)} dS_{(i-1)} \quad . \tag{3.34}$$

From Docherty (1987), the unprimed subscripts refer to the incident ray and the primed subscripts refer to the transmitted ray. At the i^{th} interface the transmitted and incident amplitudes of the wave are related by

$$A_{i'} = K_i A_i \quad , \tag{3.35}$$

where K_i is the transmission or reflection coefficient at this interface, depending on whether we have transmission or reflection.

At the first interface with velocity c_1,

$$\int A_1^2 \nabla \tau_1 \cdot \hat{\boldsymbol{n}}_1 dS_1 = \frac{1}{(4\pi)^2} d\beta dp_2 \quad . \tag{3.36}$$

On the i^{th} ray segment with velocity c_i,

$$\frac{\cos\theta_i}{c_i} \int A_i^2 dS_i = \frac{\cos\theta_{(i-1)}}{c_{(i-1)'}} \int A_{(i-1)'}^2 dS_{(i-1)} \quad , \tag{3.37}$$

where θ_i is the angle of incidence at the interface i and $\theta_{(i-1)'}$ is the angle of emergence into the layer i.

Since $c_i = c_{(i-1)'}$,

$$\int A_i^2 dS_i = \frac{\cos\theta_{(i-1)'}}{\cos\theta_i} \int A_{(i-1)'}^2 dS_{(i-1)}$$

$$= \frac{\cos\theta_{(i-1)'}}{\cos\theta_i} K_{(i-1)}^2 \int A_{(i-1)}^2 dS_{(i-1)} \quad . \tag{3.38}$$

At the end of the ray, at interface $n+1$ that is the receiver location,

$$\frac{A_{n+1}^2}{c_{n+1}} dS_{n+1} = \frac{\cos\theta_{n'}}{c_{n+1}} K_n^2 \int A_n^2 dS_n \quad . \tag{3.39}$$

Using equations (3.36), (3.38) and (3.39), we are now able to find the final amplitude of the ray

$$A_{n+1}^2 = \frac{c_1 d\beta dp_2}{(4\pi)^2 dS_{n+1}} \prod_{n=0}^{\infty} K_i^2 \frac{\cos\theta_{i'}}{\cos\theta_i} \quad . \tag{3.40}$$

Hence, given a ray path, we are able to calculate A_{n+1}^2, but we still need to find the value of dS_{n+1} for our model.

Next we calculate dS_{n+1} for a survey along the common trend in a 2.5-D earth

$$dS_{n+1} = J d\sigma dp_2 \quad . \tag{3.41}$$

Our objective is to determine the Jacobian given by equation (3.24),

$$J(\sigma) = \frac{1}{p(\sigma)} \begin{vmatrix} \dfrac{\partial x_1}{\partial \sigma} & \dfrac{\partial x_1}{\partial \beta} & \dfrac{\partial x_1}{\partial p_2} \\ \dfrac{\partial x_2}{\partial \sigma} & \dfrac{\partial x_2}{\partial \beta} & \dfrac{\partial x_2}{\partial p_2} \\ \dfrac{\partial x_3}{\partial \sigma} & \dfrac{\partial x_3}{\partial \beta} & \dfrac{\partial x_3}{\partial p_2} \end{vmatrix} \quad . \tag{3.42}$$

We simplify $J(\sigma)$ using equations (3.28), (3.29) and (3.30) to obtain,

$$J(\sigma) = \frac{1}{p(\sigma)} \begin{vmatrix} \frac{\partial x_1}{\partial \sigma} & \frac{\partial x_1}{\partial \beta} & \frac{\partial x_1}{\partial p_2} \\ p_2 & 0 & \sigma \\ \frac{\partial x_3}{\partial \sigma} & \frac{\partial x_3}{\partial \beta} & \frac{\partial x_3}{\partial p_2} \end{vmatrix} . \tag{3.43}$$

The nonzero terms in the second row of the determinant are the out-of-plane spreading terms. Expanding about the second row in the above determinant we get

$$J(\sigma) = \frac{1}{p(\sigma)} \left[p_2 \left| \frac{\partial(x_1, x_3)}{\partial(\beta, p_2)} \right| + \sigma \left| \frac{\partial(x_1, x_3)}{\partial(\sigma, \beta)} \right| \right] . \tag{3.44}$$

In the above determinants, we know only $\partial x_1/\partial\sigma$ and $\partial x_3/\partial\sigma$. Hence we use the chain rule for Jacobi determinants to get a simplified version of equation (3.44).

$$\left| \frac{\partial(x_1, x_3)}{\partial(\beta, p_2)} \right| = \left| \frac{\partial(x_1, x_3)}{\partial(\tau, \beta)} \frac{\partial(\tau, \beta)}{\partial(p_2, \beta)} \right|$$

$$= \left| \frac{\partial \boldsymbol{x}^{(2)}}{\partial \tau} \times \frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta} \right| \begin{vmatrix} \frac{\partial \tau}{\partial p_2} & \frac{\partial \tau}{\partial \beta} \\ \frac{\partial \beta}{\partial p_2} & \frac{\partial \beta}{\partial \beta} \end{vmatrix}$$

$$= \left| \frac{\partial \boldsymbol{x}^{(2)}}{\partial \tau} \times \frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta} \right| \begin{vmatrix} \frac{\partial \tau}{\partial p_2} & \frac{\partial \tau}{\partial \beta} \\ 0 & 1 \end{vmatrix} , \tag{3.45}$$

where $\boldsymbol{x}^{(2)} = (x_1, x_3)$.

Since $\partial \boldsymbol{x}^{(2)}/\partial\beta$ is taken at constant τ, it is tangent to the wavefront. Moreover, $\partial \boldsymbol{x}^{(2)}/\partial\tau$ points in the direction of the ray that is orthogonal to the wavefront. Hence, equation (3.45) reduces to

$$\left|\frac{\partial(x_1, x_3)}{\partial(\beta, p_2)}\right| = \left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \tau}\right|\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right|\frac{\partial \tau}{\partial p_2}$$

$$= \frac{c^2}{v}\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right|\sum_{j=1}^{j=3}\frac{\partial \tau}{\partial x_j}\frac{\partial x_j}{\partial p_2}$$

$$= \frac{c^2}{v}\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right|\sum_{j=1}^{j=3} p_j \frac{\partial x_j}{\partial p_2}$$

$$= \frac{c^2 \sigma\, p_2}{v}\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right| \quad . \tag{3.46}$$

Note that we have used (2.7) in (3.45) to get to (3.46), and the two terms $p_j \partial x_j / \partial p_2$, $j=1$ and $j=3$ have been ignored. This is because test results for a complicated geological model have indicated that the variation of the in-plane component of the raypath is essentially independent of the value of p_2. Similarly,

$$\left|\frac{\partial(x_1, x_3)}{\partial(\sigma, \beta)}\right| = \frac{1}{v}\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right| \quad . \tag{3.47}$$

Combining these results in (3.44) we obtain,

$$J(\sigma) = \frac{c}{v}\left[\sigma\, p_2^2 c^2 + \sigma\right]\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right|$$

$$= \frac{\sigma c}{v}\left[p_2^2 c^2 + 1\right]\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right| \quad . \tag{3.48}$$

Hence, since $dS_{n+1} = J d\beta dp_2$,

$$dS_{n+1} = \frac{\sigma c}{v}\left[p_2^2 c^2 + 1\right]\left|\frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta}\right| d\beta dp_2 \quad . \tag{3.49}$$

Using equation (3.40), (3.41) and (3.49) we obtain,

$$A_{n+1}^2 = \frac{c_1}{(4\pi)^2} \left[\frac{1}{\frac{\sigma c}{v}(p_2^2 c^2 + 1) \left| \frac{\partial x^{(2)}}{\partial \beta} \right|} \right] \prod_{n=0}^{\infty} K_i^2 \frac{\cos \theta_{i'}}{\cos \theta_i} \quad . \tag{3.50}$$

As in Docherty (1987), we express $\partial x^{(2)}$ in terms of $\partial \xi$, a vector measured on the observation surface along the dip direction. That is, we make the final approximation,

$$\left| \frac{\partial x^{(2)}}{\partial \beta} \right| = \left| \frac{\partial \xi}{\partial \beta} \right| \cos \theta_{n+1}^v \quad , \tag{3.51}$$

where θ_{n+1}^v is the angle between the normal to the (x_1, x_3) plane and the zero-offset ray. Therefore, using equation (3.48) and (3.51) we obtain,

$$J = \frac{\sigma c}{v}(p_2^2 c^2 + 1) \left| \frac{\partial \xi}{\partial \beta} \right| \cos \theta_{n+1}^v \quad . \tag{3.52}$$

Depending on whether we are doing ray tracing in a smooth velocity profile or layered acoustic media, the expression of J in equation (3.52) can be used in equations (3.33) or (3.41) respectively. Since we are in a layered medium, the amplitude for our ray at the receiver location is given by

$$A_{n+1} = \frac{1}{(4\pi)} \left(\frac{c_1}{\frac{\sigma c}{v}(p_2^2 c^2 + 1) \left| \frac{\partial \xi}{\partial \beta} \right| \cos \theta_{n+1}^v} \right)^{1/2} \prod_{i=1}^{n} K_i \left[\frac{\cos \theta i'}{\cos \theta_i} \right]^{1/2} . \tag{3.53}$$

The value of σ can be calculated by solving equation (3.8). For our layered acoustic medium case,

$$\sigma = \sum_{i=1}^{n+1} c_i \Delta d_i \quad , \tag{3.54}$$

where Δd_i is the length of the 3-D ray segment in the i^{th} layer. The length of the ray segment is calculated using the ray path as was shown in chapter one. Using equation (3.44) in equation (3.43), we obtain our final result for the amplitude

$$A_{n+1} = \frac{1}{(4\pi)} \left[\left[\sum_{i=1}^{n+1} \frac{c_i^2 \Delta d_i}{c_1 v_i} \right] \left[p_2^2 c_{n+1}^2 + 1 \right] \right]^{-1/2} \left[\left| \frac{\partial \xi}{\partial \beta} \right| \cos \theta_{n+1}^v \right]^{-1/2}$$

$$\cdot \prod_{i=1}^{n} K_i \left[\frac{\cos \theta i'}{\cos \theta_i} \right]^{1/2} \quad . \tag{3.55}$$

In equation (3.55), the first square root term is the out-of-plane spreading factor. The second square root term is the in-plane spreading term. The last square root allows for the expansion or contraction of the ray tube at an interface. Comparing the final expression for the amplitude to that of Docherty's (1987) equation (2.49), it is apparent that the only difference between the two expressions for the amplitudes are the out of plane terms. If we set p_2=0, our equation reduces to that of Docherty. This should be the case since $p_2 = 0$ implies that the survey is conducted along the dip of the geological section under consideration.

Once we know p_2 from the ray path, we are able to calculate the amplitude of the wave at the receiver. The only unknown in equation (3.55) is $\partial\xi/\partial\beta$. This was also the case with Docherty. He approximated this by

$$\left| \frac{\partial \xi}{\partial \beta} \right| \sim \left| \frac{d\xi}{d\beta} \right| \sim \frac{\Delta \xi}{\Delta \beta} \quad , \tag{3.56}$$

where $\Delta\xi$ is twice the receiver spacing and $\Delta\beta$ is the increment in takeoff angle for shooting rays. In his experiment x_2 was always equal to zero.

Although there is one source and one receiver for each experiment, we use three receivers along the dip line. This is because we need to determine $\partial\xi/\partial\beta$. This is done with the source acting as one of the receivers and the other two equidistant from it on each side. Hence our $\partial\xi/\partial\beta$ approximation is the same as Docherty's.

Note that (3.51) can be rewritten as

$$\left| \frac{\partial \boldsymbol{x}^{(2)}}{\partial \beta} \right| = \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{p}}_v(\sigma) \sqrt{g} \left| \frac{\partial \xi}{\partial \beta} \right| \quad , \tag{3.57}$$

where $\hat{\boldsymbol{n}}$ is the unit normal to the observation surface at ξ, $\hat{\boldsymbol{p}}_v$ is the slowness in the (x_1, x_3) plane, and g is the differential surface element on the observation surface.

Hence, a useful representation of (3.55) for the corresponding inverse problem is

$$A_{n+1} = \frac{1}{(4\pi)} \left[\left[\sum_{i=1}^{n+1} \frac{c_i^2 \Delta d_i}{c_1 v_i} \right] \left[p_2^2 c_{n+1}^2 + 1 \right] \right]^{-1/2} \left[\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{p}}_v(\sigma) \sqrt{g} \left| \frac{\partial \xi}{\partial \beta} \right| \right]^{-1/2}$$

$$\cdot \prod_{i=1}^{n} K_i \left[\frac{\cos \theta i'}{\cos \theta_i} \right]^{1/2} . \qquad (3.58)$$

4. EXAMPLES OF SYNTHETIC ZERO OFFSET AND COMMON-STRIKE SHOT RECORDS

In this final section, we present the results of ray tracing with our equations for two models. Our ray tracer is a modified version of Docherty's (1987), which is based on homotopy or continuation procedure. The homotopy technique for ray tracing was developed by H.B. Keller and two of his graduate students, D.J. Perozzi (1980) and J.A. Fawcett (1983) at Caltech.

Figure 2 is a perspective rayplot for a shot profile along the strike of a cylindrical model where velocities increase from 1000 m/s in the first layer to 4000 m/s in the bottom layer. The minimum and maximum offsets are 0.0 m and 4531 m respectively, and the depth to the reflection point is about 1700 m. Despite a certain complexity in the geometry of the boundaries, the reflection points are practically aligned with the strike. From Table 1 and Table 2 we can see that the zero-offset and pseudo zero-offset raypaths are essentially the same.

This interesting result motivated us to do further investigations; hence, we increased the velocities in each layer by 4000 m/s and repeated the experiment. As can be seen from Table 3 and Table 4, the difference between the zero-offset and pseudo zero-offset raypaths is again negligible. At last we again increased the velocities by 4000 m/s. From Table 5 and Table 6 we can see that the zero-offset pseudo zero-offset raypaths are again essentially the same. This suggests that the propagation planes do not change with offset, at least when the reflectors are smooth.

Figure 3 is a cross-section of a 2.5-D physical model supplied to the Center for Wave Phenomena by Marathon Oil to test modeling and inversion codes.

Figure 4 is a zero-offset ray plot for the 2.5-D model in Figure 3. Despite the large number of shot points (100), there are still some shadow zones on the last layer towards the middle of the model. That figure also shows five caustics generated by the five synclines of the 4^{th} layer. Figure 5 is a common offset rayplot for the same model, for 100 source-receiver pairs aligned with the strike. First, we notice that the reflection points from all layers lie almost exactly in the vertical plane containing the midpoints of the common offset experiment. Second, it is apparent that both caustics and shadow zones observed on the zero-offset section are still present in the strike section.

Figures 6 and 7 are the zero-offset and common-strike shot records for the rays in Figures 4 and 5 respectively. As could be expected, the traveltimes on the offset section are larger than on the zero-offset section while the amplitudes are smaller, due to the increase of the travel path.

5. CONCLUSIONS

Given the pseudo zero-offset raypath, calculated with velocities $1/\sqrt{c_{n+1}^{-2} - p_2^2}$, we have shown that the out-of-plane raypath can be determined. We have derived an expression for the true amplitude of the wave propagating away from a point source along the strike (out-of-plane) direction in a 2.5-D acoustic medium. In the case of a layered medium, we have seen that the amplitude consists of out-of-plane, and in-plane spreading factors, reflection and transmission coefficients at each interface and spreading or contraction due to jump of discontinuities. All these components of the amplitude are easily calculated once the raypath is determined.

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APPENDIX:

Reflection and Transmission Coefficients

In this appendix we derive an expression for the reflection and transmission coefficients for our model. The assumptions made in the derivation of the reflection and transmission coefficients is that the $U(\boldsymbol{x},\omega)$ and its normal derivatives are continuous across the interface. For a layered acoustic medium of constant density Bleistein (1984) gave expressions for the reflection and transmission coefficients.

Here we specialize Bleistein's expression for the reflection and transmission coefficients to the special geometry of consideration in the paper.

At the i^{th} interface, the reflection coefficient is

$$K_i = \frac{\cos\theta_i - \left[\dfrac{c_i^2}{c_{i+1}^2} - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[\dfrac{c_i^2}{c_{i+1}^2} - \sin^2\theta_i\right]^{1/2}} \quad . \tag{A-1}$$

However, for our experiment at the i^{th} interface

$$\cos\theta_i = \frac{\boldsymbol{p}\cdot\hat{\boldsymbol{n}}}{|\boldsymbol{p}|} \quad , \tag{A-2}$$

but

$$\boldsymbol{p}\cdot\hat{n} = \boldsymbol{p}_v\cdot\hat{\boldsymbol{n}} + p_2(\hat{x}_2\cdot\hat{\boldsymbol{n}}) \quad , \tag{A-3}$$

where $\boldsymbol{p}_v$ is the total slowness in the (x_1,x_3) plane. Since

$$\hat{x}_2\cdot\hat{\boldsymbol{n}} = 0 \quad , \tag{A-4}$$

it follows that

$$\boldsymbol{p}\cdot\hat{\boldsymbol{n}} = \boldsymbol{p}_v\cdot\hat{n} \quad . \tag{A-5}$$

Hence,

$$\cos\theta_i = \frac{|p_v|\cos\theta_i^v}{|p|}$$

$$= \cos\alpha_i \cos\theta_i^v \quad , \tag{A-6}$$

where α_i is the angle which the ray makes with the (x_1, x_3) plane at the i^{th} interface and θ_i^v is the angle which the zero-offset ray makes with the normal to the surface at the i^{th} interface. In our experiment,

$$\cos\alpha_i = \sqrt{1 - p_2^2 c_i^2} \quad . \tag{A-7}$$

We can see that, once we know the ray path, we can easily determine $\cos\alpha_i$ only from the knowledge of p_2 and the velocities of the layers.

The transmission coefficient at the i^{th} interface is given by

$$K_i = \frac{2\cos\theta_i}{\cos\theta_i + \left[\dfrac{c_i^2}{c_{i+1}^2} - \sin^2\theta_i\right]^{1/2}} \quad . \tag{A-8}$$

From equation (A-1), we can see that the critical reflection, K_i=1, occurs when $c_i = c_{i+1}\sin^2\theta_i$. And beyond this we get postcritical reflections. That is, the square root in both the numerator and denominator in equation (A-1) becomes imaginary. Thus, the reflection coefficient becomes complex. Since the numerator is the complex conjugate of the denominator, the magnitude of the reflection coefficient of the postcritical reflection remains unity. The postcritical reflection coefficient is

$$K_i = e^{i\phi_i}, \tag{A-9}$$

where

$$\phi_i = -2\,\mathrm{sgn}(\omega)\left[\frac{\left(\sin^2\theta_i - \dfrac{c_i^2}{c_{i+1}^2}\right)^{1/2}}{\cos\theta_i}\right] \tag{A-10}$$

is the phase of the postcritical reflection coefficient. The most important thing in the angle of incidence in our experiment is that its cosine is the product of the cosines of the angles that the ray segment makes with respect to the (x_1, x_3) plane and the normal to the x_2 axis at the interface, refer to (A-6).

Velocities in m/s 1000, 2000, 3000, 4000

Source Location
$x_1 = 6000.00$, $x_2 = 0.0$, $x_3 = 0.0$

Receiver Location
$x_1 = 6000.00$, $x_2 = 0.0$, $x_3 = 0.0$

x_1	x_2	x_3
6000.00	0.00	0.00
5996.95	0.00	1059.69
5644.10	0.00	2543.81
4710.04	0.00	4619.52
5644.10	0.00	2543.81
5996.95	0.00	1059.69
6000.00	0.00	0.00

Table 1: Zero offset ray path.

Receiver Location
$x_1 = 6000.00$, $x_2 = 4531.05$, $x_3 = 0.0$

x_1	x_2	x_3
6000.00	0.00	0.00
6027.31	192.75	1053.01
5735.08	774.85	2532.95
4781.63	2265.52	4651.73
5735.08	3756.20	2532.95
6027.31	4338.29	1053.01
6000.00	4531.05	0.00

Table 2: Strike ray path.

Velocities in m/s 5000, 6000, 7000, 8000

Source Location
$x_1 = 6000.00$, $x_2 = 0.0$, $x_3 = 0.0$

Receiver Location
$x_1 = 6000.00$, $x_2 = 0.0$, $x_3 = 0.0$

x_1	x_2	x_3
6000.00	0.00	0.00
5714.09	0.00	1119.92
5179.96	0.00	2585.90
4339.81	0.00	4452.91
5179.96	0.00	2585.90
5714.09	0.00	1119.92
6000.00	0.00	0.00

Table 3: Zero offset ray path.

Receiver Location
$x_1 = 6000.00$, $x_2 = 4555.43$, $x_3 = 0.0$

x_1	x_2	x_3
6000.00	0.00	0.00
5723.76	439.63	1117.94
5197.06	1159.46	2584.86
4353.63	2277.71	4459.13
5197.06	3395.97	2584.86
5723.76	4115.79	1117.94
6000.00	4555.43	0.00

Table 4: Strike ray path.

Velocities in m/s 9000, 10000, 11000, 12000

Source Location
$x_1 = 6000.00$, $x_2 = 0.0$, $x_3 = 0.0$

Receiver Location
$x_1 = 6000.00$, $x_2 = 0.0$, $x_3 = 0.0$

x_1	x_2	x_3
6000.00	0.00	0.00
5629.46	0.00	1136.87
5051.16	0.00	2592.19
4235.05	0.00	4405.77
5051.16	0.00	2592.19
5629.46	0.00	1136.87
6000.00	0.00	0.00

Table 5: Zero offset ray path.

Receiver Location
$x_1 = 6000.00$, $x_2 = 4469.23$, $x_3 = 0.0$

x_1	x_2	x_3
6000.00	0.00	0.00
5636.47	493.08	1135.49
5062.36	1216.03	2591.76
4244.20	2234.62	4409.89
5062.36	1216.03	2591.76
5636.47	3976.15	1135.49
6000.00	4469.23	0.00

Table 6: Strike ray path.

Kinematics of Strike Rays

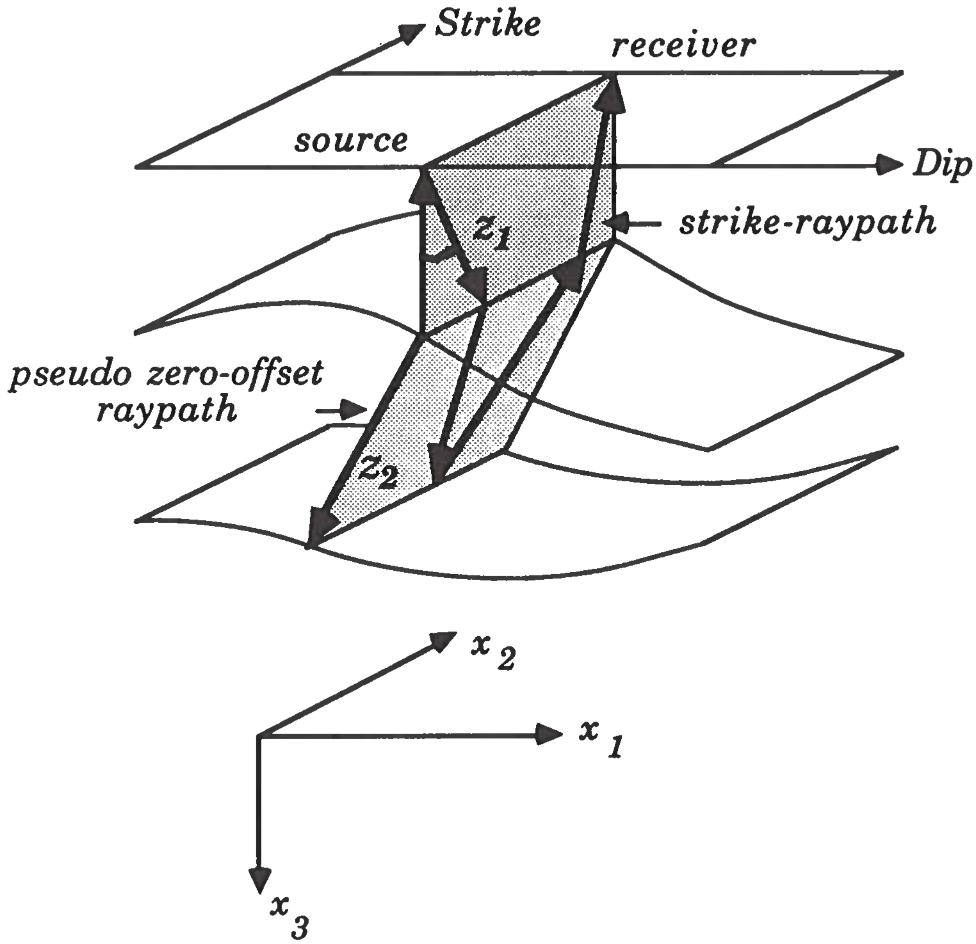

Figure 1: Strike rays are confined to propagation (Z_{n+1}) planes.

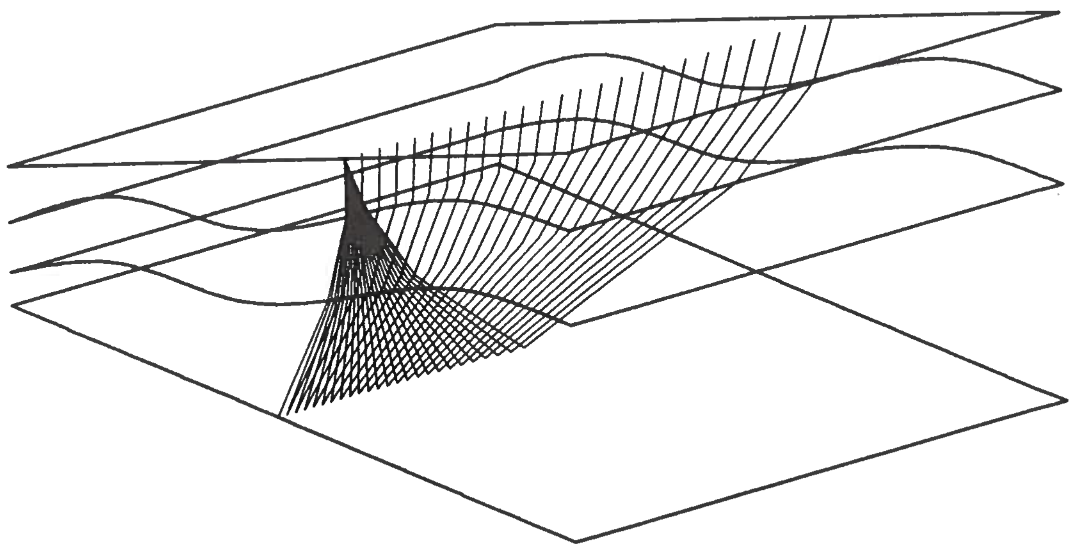

Figure 2: Perspective rayplot for a shot profile along strike.

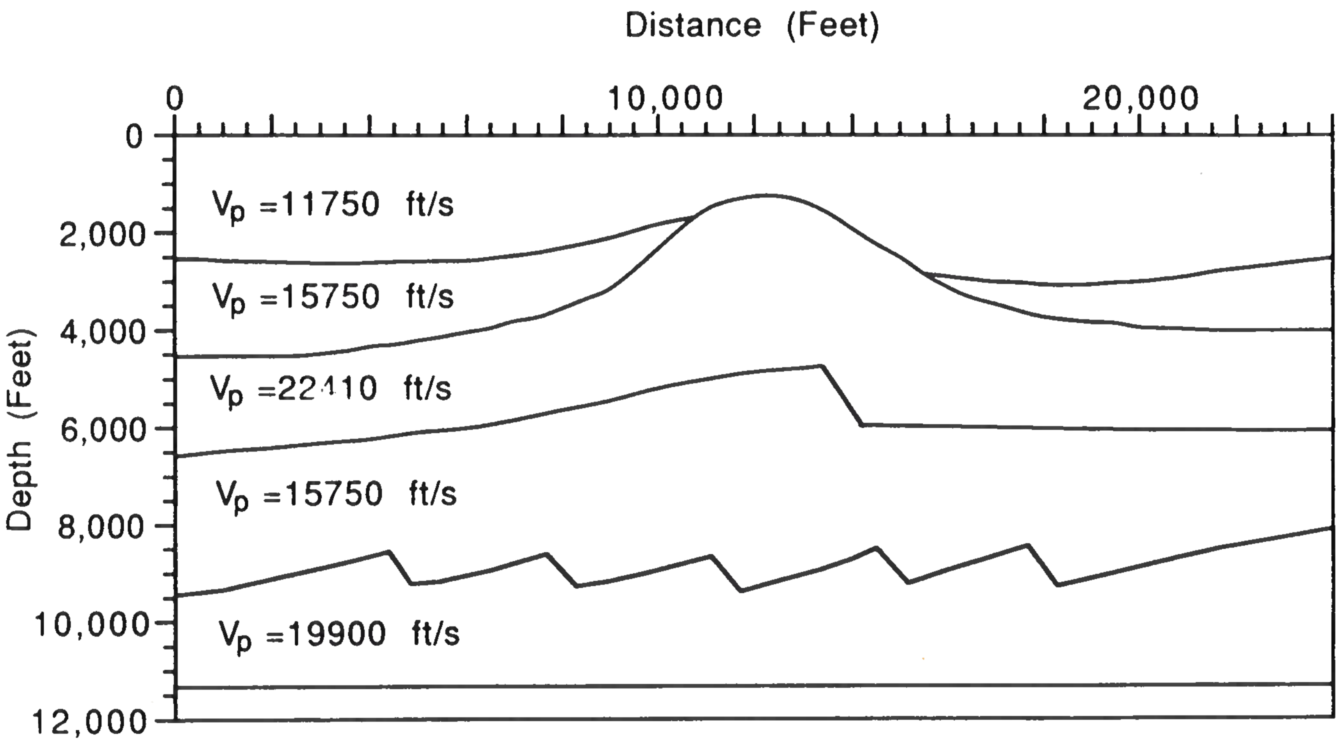

Figure 3: 2.5-D physical model (courtesy of Marathon Oil Company)

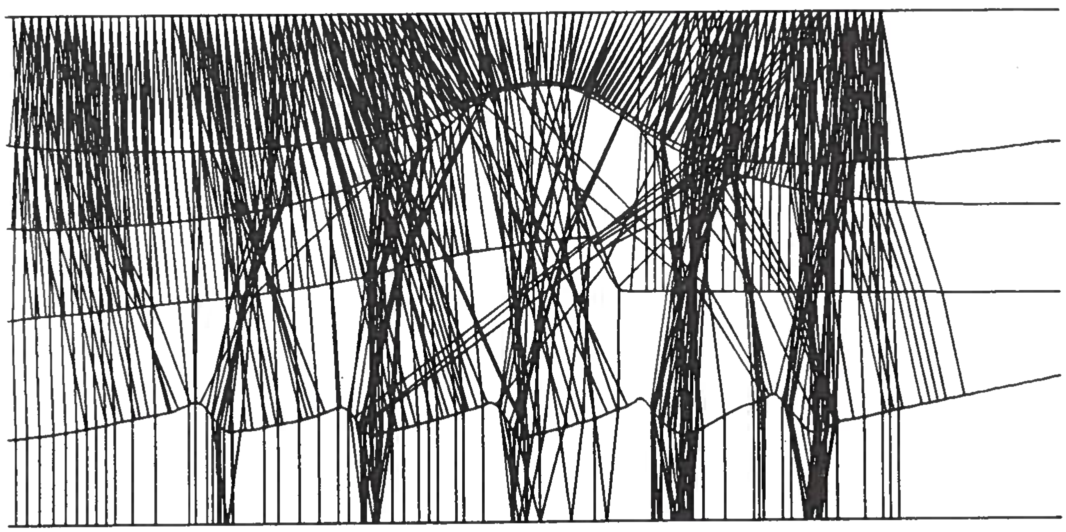

Figure 4: Zero-offset rayplot for a 2.5-D model.

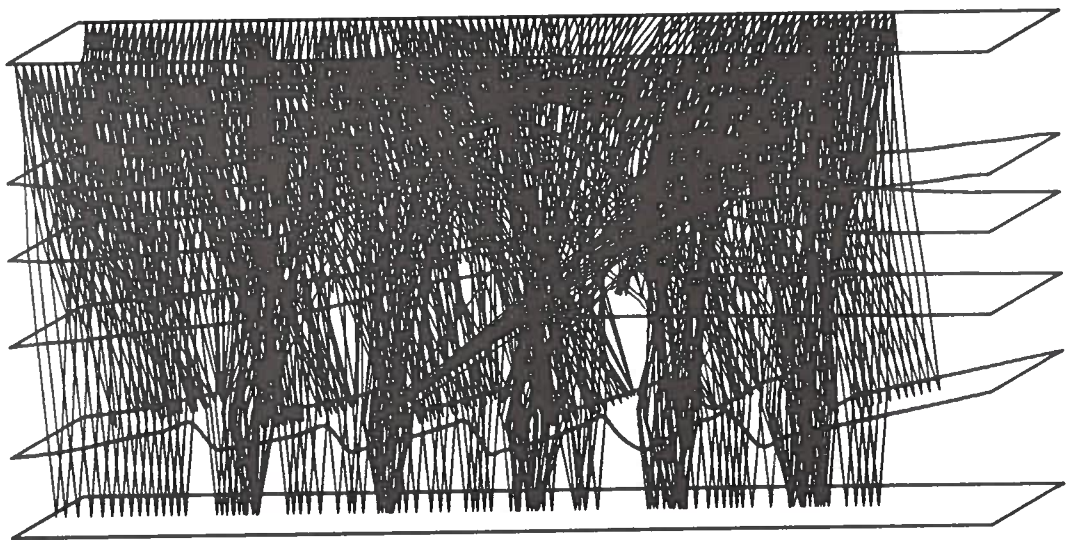

Figure 5: Common offset rayplot source-receiver along strike.

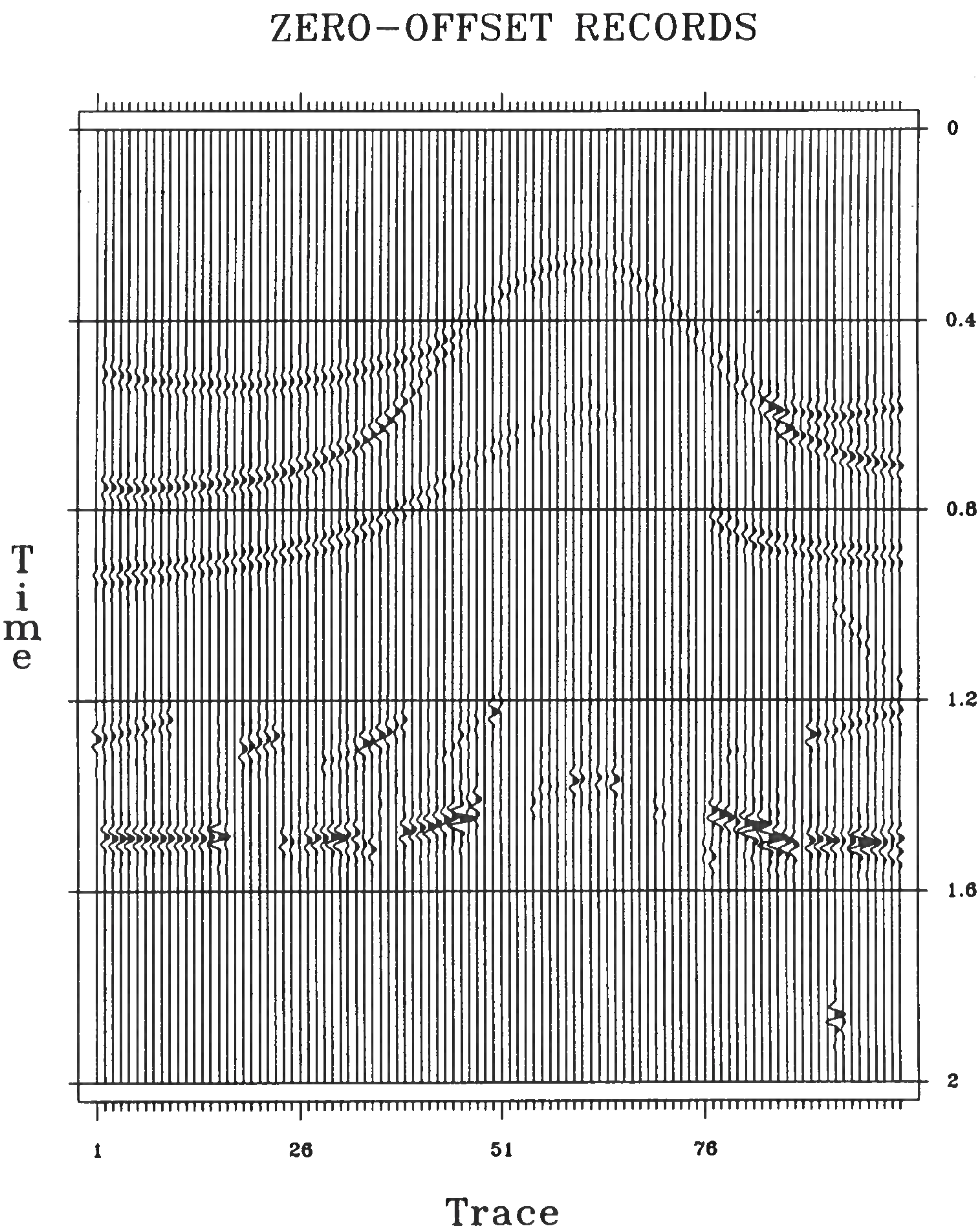

Figure 6: Pseudo zero-offset shot records for the 2.5-D model in Figure 3.

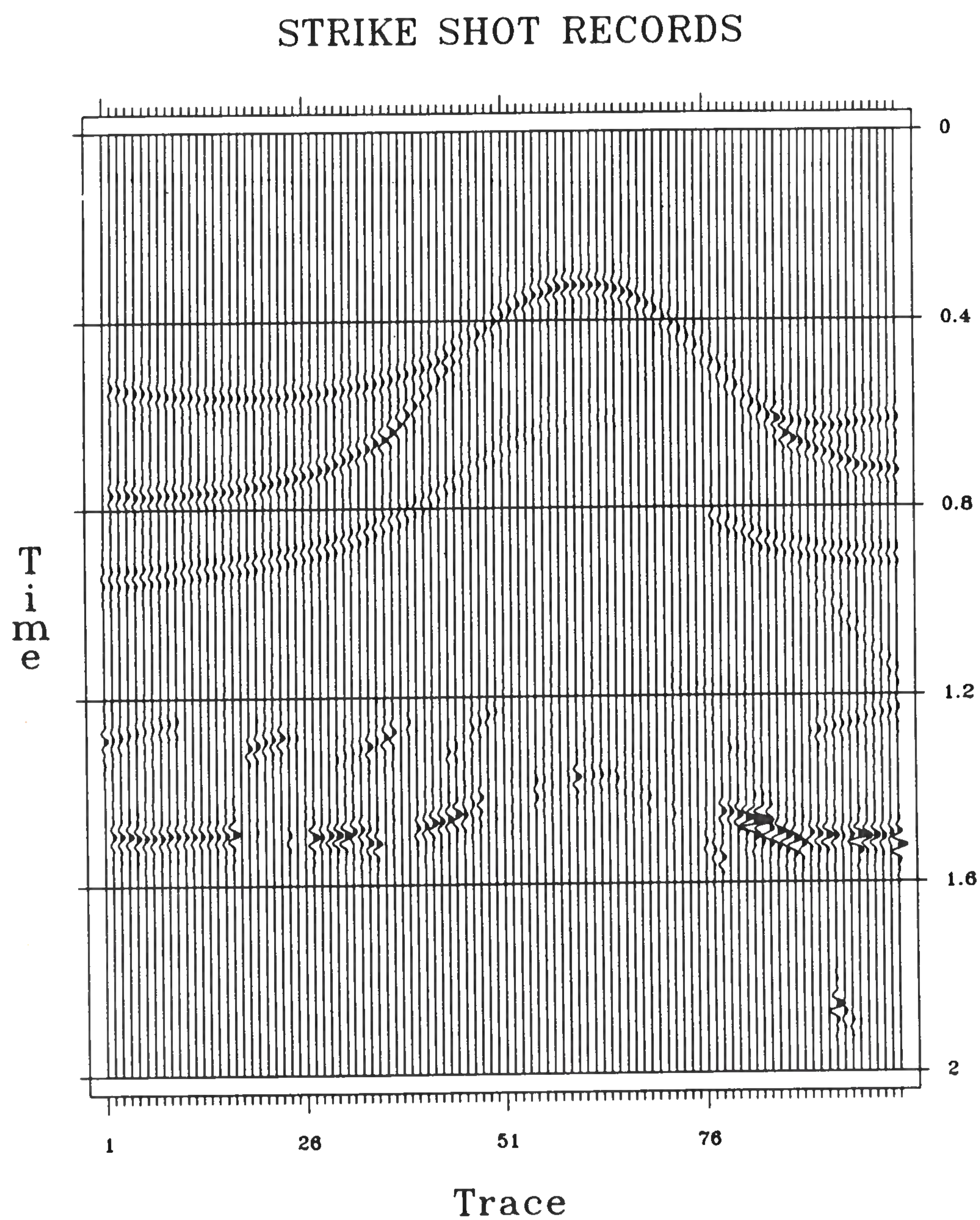

Figure 7: Common offset shot records for the 2.5-D model in Figure 3.