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ABSTRACT
Coda wave interferometry constitutes a technique where multiple scattered waves recorded at a limited number of receivers can be used to infer temporal changes, much smaller than a wavelength, in a medium. This technique can be used in warning mode, where one aims at detecting the presence of temporal changes in the medium only, or in diagnostic mode, where the temporal change is quantified. Because of the limited hardware requirements of coda wave interferometry, this technique can have important applications that include volcano monitoring, monitoring of hydrocarbon reservoirs and the detection of flaws in non-destructive testing.

Key words: detection of change, scattering

Most deterministic imaging techniques are based on the use of single scattered waves only. Presently it is not clear to what extent multiple scattered waves can be used for deterministic imaging (Scales & Snieder, 1997). However, in many applications, such as non-destructive testing, volcano monitoring or time-lapse oil reservoir diagnostics, one is primarily interested in detecting temporal changes in the medium, rather than in imaging them. Here we introduce coda wave interferometry whereby multiple scattered waves are used to detect changes much smaller than a wavelength by using the multiple scattering medium as an interferometer. For quasi-random perturbations of the positions of point scatterers, or for a change in the wave-velocity, one can retrieve estimates of this perturbation from multiple scattered waves by a simple cross-correlation in the time domain.

A model of a strongly scattering medium is shown in the left part of Figure 1, where the black dots denote isotropic point scatterers. For a source indicated by the asterisk, the wave-field at the triangle is shown by the black line in the right panel. In the numerical example shown here, the wave-field is computed using a deterministic variant (Groenenboom & Snieder, 1995; Snieder & Scales, 1998) of Foldy's method (Foldy, 1945). Given the mean free path ($l = 20.1$ m) and the wave velocity ($v = 1,500$ m) one can infer that after $t = 5.4 \times 10^{-2}$ s the waves are on average scattered more than three times. The later part of the signal is called the *coda*; this term originates from musical notation where the coda denotes the closing part of a musical piece.

Suppose that one repeats this multiple scattering experiment after the medium is perturbed. The perturbed scatterers are denoted by the open dots in Figure 1, and the corresponding modeled wavetrain is shown by a dotted line The perturbation in the scatterer location is only 1/30 of the dominant wavelength. For the sake of clarity this perturbation is greatly exaggerated in Figure 1; without this exaggeration the perturbation cannot be discerned. Throughout the remainder we refer to the signals from the unperturbed medium as unperturbed signals, and to the signals from the perturbed medium as perturbed signals. For the early part of the signal the waves have not scattered often, rendering the path lengths of these waves insensitive to the small perturbations of the scatterers (small compared to the dominant wavelength of the probing wave), which causes the unperturbed and perturbed signals to be almost identical. However, the multiple scattered waves are increasingly sensitive with time for the perturbations of the scatterer locations because the waves bounce more often among scatterers as time increases. The correlation between the unperturbed and perturbed signals therefore decreases with time.

The time-windowed correlation coefficient of these

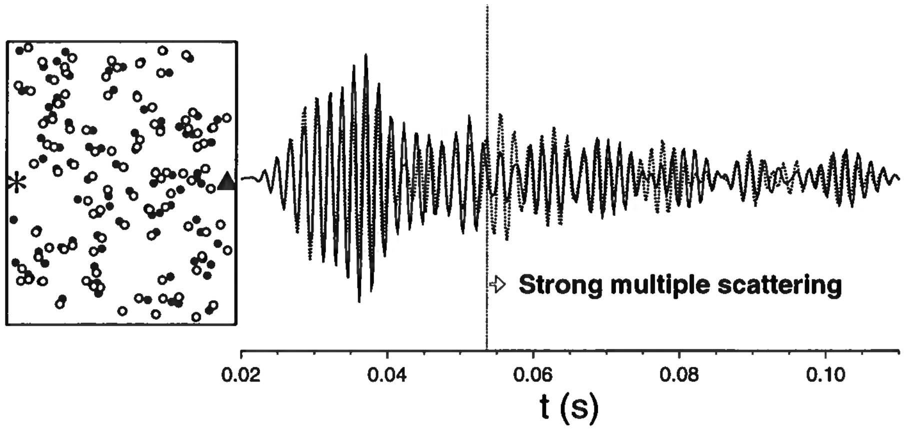

Figure 1. Location of 100 scatterers before and after the perturbation (filled dots and open dots respectively) with the source (asterisk) and receiver location (triangle). Thee scatterers are placed in an area of 40 × 80 m. The waveforms recorded before and after the perturbation at the receiver are shown on the right in solid and dotted lines respectively.

signals can readily be computed and used as a sensitive diagnostic for changes in the medium. Given the fact that one only needs a repeatable source, one or more receivers, and a simple computer to carry out the cross-correlation, one can build a simple and sensitive system for detecting temporal changes in media.

Coda wave interferometry is similar to speckle pattern interferometry (Maret, 1995; Fomin, 1998) and diffusing wave spectroscopy (Pine *et al.*, 1988; Sheng, 1995). However, the latter techniques rely on an ensemble average of the intensity of interfering multiple scattered waves, whereas coda wave interferometry does not. In speckle pattern interferometry and diffusing wave spectroscopy, this ensemble average is in practice approximated by a spatial average, which requires recording the wavefield over an extended region in space. Coda wave interferometry, however, employs the interference of multiple scattered waves in time rather than in space by using the waveforms of transient signals rather than the intensity of a stationary signal. For this reason, only a limited number of sensors of the wavefield are required. For many applications, including those in geophysics and non-destructive testing, this is an important advantage of coda wave interferometry. Another difference between coda wave interferometry and diffuse wave spectroscopy is that the latter technique is based on the diffusion equation of the intensity (Pine *et al.*, 1988; Sheng, 1995) rather than a wave equation.

For point scatterers whose positions are perturbed independently in the different directions with an r.m.s. perturbation δ, one can retrieve this perturbation from the correlation of the coda waves. The unperturbed wave-field can be written as a Feynman path summation (Snieder, 1999) over all possible paths P:

$$u_{unp}(t) = \sum_P A_P S(t - t_P) \ , \tag{1}$$

where $u_{unp}(t)$ denotes the unperturbed signal, t_P is the travel time along path P, A_P is the corresponding amplitude and $S(t)$ is the source signal. In the following analysis we assume that when the scatterers are perturbed, the dominant effect on the wave-form arises from the change τ_P in the arrival time associated with path P:

$$u_{per}(t) = \sum_P A_P S(t - t_P - \tau_P) \ . \tag{2}$$

A path is defined here as a sequence of scatterers that are encountered. (Note that in the numerical experiment the perturbed waveform is computed numerically (Snieder and Scales, 1998).) The auto-correlation of the source signal is defined as $C(t) \equiv \int_{-\infty}^{\infty} S(t'+t)S(t')dt'$.

Given measurements of the unperturbed and the perturbed signals, one can readily compute the time-windowed correlation coefficient

$$R(t,T) \equiv \frac{\int_{t-T}^{t+T} u_{unp}(t')u_{per}(t')dt'}{\left(\int_{t-T}^{t+T} u_{unp}^2(t')dt' \int_{t-T}^{t+T} u_{per}^2(t')dt'\right)^{1/2}} \ , \tag{3}$$

where the time window is centered at time t with duration $2T$. When the expressions (1) and (2) are inserted, double sums $\sum_{PP'}$ over all paths appear. In these double sums, the cross-terms with different paths ($P \neq P'$) are incoherent and average out to zero when the DC-component of the source signal is equal to zero. This means that in this approximation:

$$R(t,T) \approx \frac{\sum_{P(t,T)} A_P^2 C(\tau_P)}{\sum_{P(t,T)} A_P^2 C(0)} \ , \tag{4}$$

where $\sum_{P(t,T)}$ denotes a sum over the paths with arrival

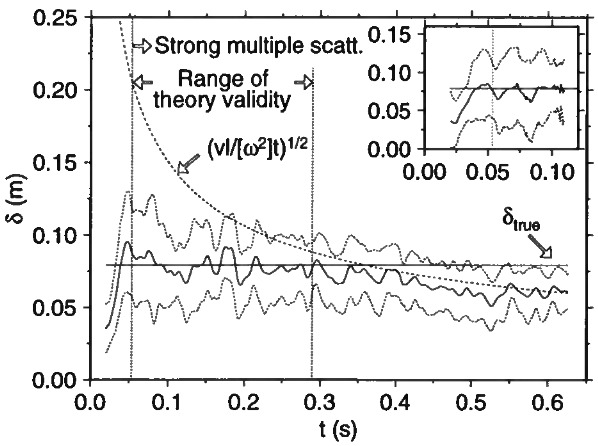

Figure 2. The value of δ obtained from the time-windowed cross correlation as a function of the center time t of the time window (solid line) plus or minus one standard deviation (dotted lines). The true rms displacement value δ_{true} is shown by the horizontal solid line. For the case of noise-contaminated data the obtained value of δ and δ_{true} are shown in the panel in the upper right corner. The dotted vertical line denotes the same demarcation between the single scattering and multiple scattering regime as in the main panel.

times within the time window of the cross-correlation. For small perturbations τ_P, a second-order Taylor expansion of $C(\tau_P)$ can be used to derive that $C(\tau_P) = C(0)\left(1 - \frac{1}{2}\bar{\omega}^2\tau_P^2\right)$, with $\bar{\omega}^2$ the mean-squared frequency. Using this gives

$$R(t,T) = 1 - \frac{1}{2}\bar{\omega}^2\left\langle\tau^2\right\rangle , \tag{5}$$

with $\left\langle\tau^2\right\rangle$ the variance of the arrival time perturbation for the paths associated with arrivals in the time window. When the scatterers are perturbed independently with r.m.s. displacement δ, this variance is given by

$$\left\langle\tau^2\right\rangle = \frac{2\delta^2 t}{vl_*} , \tag{6}$$

where l_* is related to the mean free path by $l_* = l/\left(1 - \langle\cos\psi\rangle\right)$ and is usually referred to as the transport mean free path(Maret, 1995; Ishimaru, 1997); ψ denotes the scattering angle (Snieder and Scales, 1998). For isotropic scattering, as in our numerical experiment, the transport mean free path l_* is equal to the mean free path l. In deriving (6) we assume that the number of scatterers encountered is on average given by $n = vt/l$.

Using (5) and (6), one retrieves the average scatterer perturbation from the time-windowed correlation coefficient:

$$\delta^2 = \left(1 - R(t,T)\right)\frac{vl_*}{\bar{\omega}^2 t} . \tag{7}$$

In this expression, δ^2 does not depend on time. Estimating δ from the cross-correlation for time windows centered at different times t therefore provides a consistency check of the method. The mean value of δ obtained from the red and black waveforms in Figure 1 and of 20 other receivers is shown in Figure 2 as a function of the center t of the time window, for $T = 2\times10^{-2}$ s. Its variance is indicated by the dashed lines. The true value of r.m.s. displacement is given by $\delta_{true} = 8\times10^{-2}$ m, as indicated by the horizontal solid line; this displacement equals only 1/30 of the dominant wavelength. Between $t = 5.4\times10^{-2}$ s and $t = 2.9\times10^{-1}$ s, there is an excellent agreement between the value of δ obtained from coda wave interferometry and the true value of δ. For early times ($t < 5.4\times10^{-2}$ s) the waves are not scattered often, and the relation $n = vt/l$ is not valid. For late times ($t > 2.9\times10^{-1}$ s), the second-order Taylor expansion of the auto-correlation of the source signal is inaccurate (by $\approx 15\%$), and the theory breaks down. However, given the source wavelet $S(t)$, one can easily verify for which times the second-order Taylor expansion is accurate. For these later times the cross-correlation $R(t,T)$ is close to zero and according to (7) the inferred value of δ is given by $\sqrt{vl_*/\bar{\omega}^2 t}$, which is no longer a good estimate of δ_{true}.

Because the method is based on a cross-correlation, it is relatively robust in presence of noise. Figure 3 shows the noise level that is added in the form of two independent realizations to the unperturbed and perturbed signals in a numerical experiment. The scatterer displacement computed from the noise-corrupted waveforms using a modification of expression (7) that accounts for the average noise level is shown in the panel in the upper right of Figure 1; it agrees well with the true scatterer displacement. However, when noise is present the technique can be used only for times that are sufficiently short that the coda waves are not dominated by the noise.

In the previous example it was assumed that the perturbation of the medium consisted of independent perturbation of the location of point scatterers. However, the medium can be perturbed in different ways, and coda wave interferometry can be used to differentiate between different perturbations. Suppose for example that the velocity v changes with a constant value δv. In that case the arrival time perturbation along path P is to leading order given by $\tau_P = -(\delta v/v)t_P$. Equation (5) still holds, but for this perturbation expression (6) must be replaced by $\left\langle\tau^2\right\rangle = (\delta v/v)^2 t^2$. For this perturbation the time-windowed correlation function is thus given by

$$R(t,T) = 1 - \frac{1}{2}\bar{\omega}^2\left(\frac{\delta v}{v}\right)^2 t^2 , \tag{8}$$

hence the time-windowed cross correlation depends quadratically on time. It follows from equation (7) that for the perturbation in the scatterer locations the time-windowed cross correlation depends linearly on time: $1 - R(t,T) \sim t$. Therefore, the difference in time-dependence of the correlation function reveals the dif-

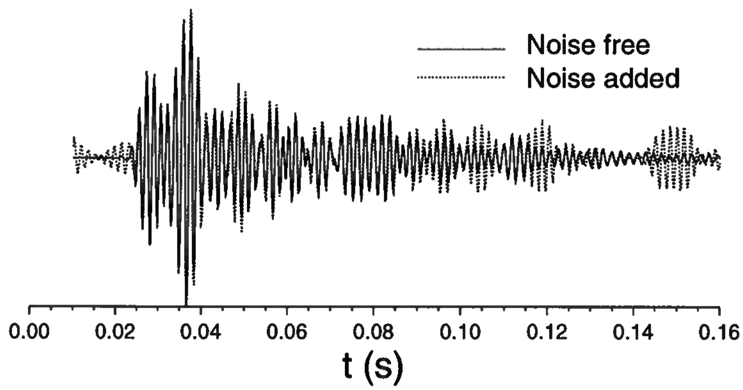

Figure 3. A waveform without noise (solid line) and with noise (dotted line).

ference in character of the changes in the medium. Also, when the perturbation consists of a displacement of the source location over a distance δ, only the wavepath to the first scatterer is perturbed. In that case $\langle \tau^2 \rangle = (\delta/v)^2$ and the time-windowed correlation function deduced from expression (5) is independent of time: $1 - R(t,T) \sim constant$. These examples show that the time-dependence of the correlation function provides information about the mode of change of the medium as well.

Coda wave interferometry requires only a limited number of sensors of the wavefield and the hardware requirements for this technique are modest. This opens up many new applications. The simplest way to use this technique is to employ coda wave interferometry in warning mode where one uses changes in the cross-correlation of the signals recorded with different shots to determine whether or not the medium has changed within a certain (small) tolerance. After a change has been detected one can perform more elaborate diagnostics of the medium or take other actions that are a consequence of the knowledge that the medium has changed. This approach can be used to monitor the changes in the internal structure of dams, for volcano monitoring, for detecting changes in oil reservoirs or aquifers, and for non-destructive testing applications where one wants to know only whether a sample has incurred a flaw or not. In a more advanced approach, coda wave interferometry can be used to quantify the change in the scattering medium as well, as shown in this report. For independent perturbations of the scatterer locations the theory presented here can be used to determine the r.m.s. displacement of scatterers, but the theory can also be used to infer a velocity change in the medium or a change in the source location. For the special case that the scatterers are embedded in a continuous medium, we have derived the imprint of an arbitrary strain tensor of this medium on the time-windowed cross-correlation, as well as the extension of the theory to mode-converted waves in elastic media. The derived relation between scatterer locations and an arbitrary strain tensor can be used to diagnose the strain within the scattering medium. Measuring a minute strain with coda wave interferometry can potentially be used as a diagnostic for impending fracture processes.

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