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Spectral footprint of intrabed multiples

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ABSTRACT
Understanding the "coloring effect" intrabed multiples have on seismic trace spectra is of great importance for intrinsic absorption estimation and model-based deconvolution. If earth reflectivity was white, then its impulse response, either transmitted or reflected, would be white too. However, earth reflectivity is known to be low-frequency deficient, i.e., "blue". An impulse, passed through such a "blue" sequence, is tailed by short-period multiples with a "red" spectrum. The high-frequency loss and dispersion experienced by the transmitted pulse are similar to those caused by propagation through inelastic media. That is why intrabed multiples are said to cause apparent attenuation.

In practice, however, we never observe isolated pulses on seismic traces. Rather, we see a *superposition* of arrivals and the spectrum of the trace is determined by the relative importance of counteracting reflection- and transmission-type contributions.

In this paper, the normal-incidence impulse responses of representative reflectivity series are modeled. The analysis of their spectral behavior over time and depth shows that: (i) VSP spectral ratio methods for absorption estimation will fail in "cyclic" layering with strong reflectivity unless the apparent attenuation is taken into account; (ii) surface-related multiple suppression will not aid in absorption estimation from surface data in a spatially-invariant-Q medium; (iii) stratigraphic filtering can be included in a wavelet model through the operator R_m/R, where R and R_m are the spectra of the reflectivity series and its impulse response respectively; both can be estimated from a nearby well.

Key words: apparent attenuation, intrabed multiples

Introduction

Thirty years ago, O'Doherty and Anstey suggested that an impulse transmitted through a "cyclic" layered sequence would experience apparent attenuation due to transmission losses and intrabed multiples. Shortly after, Schoenberger and Levin (1974) studied the spectral changes occurring as multiples are added into synthetic seismograms. They confirmed O'Doherty and Anstey's idea using real logs. They also acknowledged that in practice we do not observe individual reflections but a *multitude of overlapping pulses* and therefore the spectrum of that superposition deserves special attention. Further experiments in that direction were undertaken, for example, by Spencer *et al.* (1982) and a theoretical treatment given by Banik *et al.* (1985). Yet, the coloring effect short-period multiples have on the *superposition* of pulses, comprising the seismic trace, remains poorly understood. What "color" is the total impulse response of the elastic earth? How does it change along the trace? How does it vary with the depth of the receiver? How does it relate to the statistical properties of reflectivity? All these questions have a direct bearing on absorption-estimation and on data processing steps in which the frequency-selective action of the elastic earth is a factor (e.g., model-based deconvolution). The answers are sought in this paper. I analyze the impulse response of a horizontally layered elastic earth at normal incidence for two representative reflectivity series.

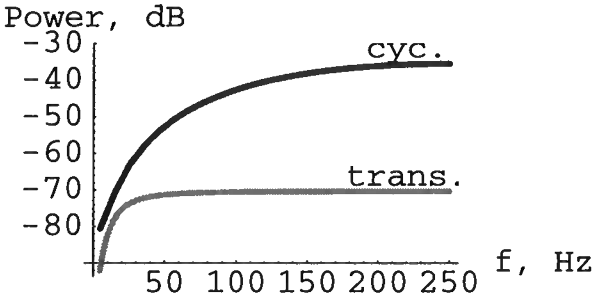

Figure 1. Example reflectivity spectra of "cyclic" and "transitional" geological sequences.

	Reflectivity 1 (cyclic)	Reflectivity 2 (transitional)
Process	ARMA(1,1)	ARMA(1,1)
θ	0.9	0.98
ϕ	0.1	0.8
mean	-0.0005	-0.0005
std	0.13	0.02
p	1.0	0.23
λ_1	0.09	0.007
λ_2	—	0.017

Table 1. Statistical properties of the two reflectivity series used throughout this paper: Both are modeled as ARMA(1,1) processes with autoregressive parameter θ and moving average parameter ϕ. These control the shape of the spectrum, i.e., the correlation between consecutive reflection coefficients (Walden & Hosken, 1985). The amplitudes of the reflection coefficients are drawn from a mixture of two Laplace distributions with a mixing proportion parameter p and scaling parameters λ_1 and λ_2 respectively (Walden & Hosken, 1986). These control the magnitude of the reflectivity.

Reflectivity Model

Two characteristics of a reflectivity series govern its impact on the trace spectrum. The first is the magnitude of the reflection coefficients which determines how energetic the multiples will be as compared with the primaries. The second is the frequency content of the reflectivity series. Reflection sequences are pseudo-white only above a corner frequency, below which their power spectrum falls as f^β, where $\beta \in [0.5; 1.5]$. Such a spectrum can be adequately modeled by ARMA(1,1) ((Walden & Hosken, 1985); for other stochastic models of reflectivity see (Saggaf & Robinson, 2000)). The stronger the deviation of reflectivity from whiteness, the stronger the coloring in its impulse response.

Reflectivity's magnitude and frequency content are not completely independent characteristics since the subsurface acoustic impedance can vary only in certain limits. Strong reflectivity series have markedly "blue" spectra, i.e., spectra whose power increases with frequency over most of the seismic frequency band (Figure 1, top curve). In such "blue" sequences, there is a negative correlation between consecutive samples, i.e., consecutive reflection coefficients tend to have the opposite sign. Thus, the corresponding acoustic impedance can sustain significant changes (giving rise to strong reflections) while staying within certain geological bounds. These are the "cyclic" sequences described by O'Doherty and Anstey (1971). They can cause strong apparent attenuation during transmission. At the other extreme, O'Doherty and Anstey considered "transitional" sequences in which consecutive reflection coefficients tend to have the same sign due to the steady increase of the acoustic impedance with depth. As the authors pointed out, in such sequences reflection coefficients should be small for the acoustic impedance to remain geologically feasible.

Later it was found out that "transitional" reflectivities are not high-frequency deficient, as first suggested by O'Doherty and Anstey. In fact, their spectra have a steep low-cut (after removing the mean) but become virtually white above a corner frequency, significantly lower than that for "cyclic" sequences (Figure 1). The argument, associating transitional geology with small reflection coefficients is still valid, however. Therefore, strong multiples are not likely to occur.

To investigate the spectral properties of the *superposition* of pulses, constituting a seismic trace, I shall consider two synthetic, but realistic, reflectivity series – one with a markedly "cyclic" character, and another with a markedly "transitional" character. The "transitional" series is similar to the reflectivity of Well 5 in the studies of Walden and Hosken ((1985), (1986)). The "cyclic" series is similar to that extracted from Well 8 in the same papers. It has large reflection coefficients which will give rise to strong multiples. To generate sequences with the desired statistical properties (Table 1; spectra depicted in Figure 1), I followed the procedure suggested by Walden (1993).

The emphasis in this study will fall on the case of "cyclic" reflectivity when multiples are expected to have the strongest effect.

Besides a realistic subsurface, the reflectivity model should include the earth surface. It is usually omitted in synthetic seismogram experiments but, as we will see, it makes a big difference to the trace spectrum behavior. Not modeling the earth surface is equivalent to assuming that any surface-related multiples in the data have been fully suppressed which, so far, is hardly achievable. Partial multiple suppression would only introduce unknown spectral distortions. That is why it is important to understand the spectral behavior of the *raw* trace (with all the multiples) and then compare it to the spectrum of a trace free of surface-related multiples (I shall call it *"filtered"* for short). Obviously, the latter can be generated by setting the surface reflection coefficient r_0 to zero. The raw trace can be obtained by setting $|r_0| = 1$.

The later assumption deserves some discussion. Certainly, $|r_0| = 1$ is appropriate for the physical ground-air interface (free surface). However, what really matters in our study is the portion of energy available for further reflection and detection after having hit the earth surface. Assuming horizontal layering and vertical propagation, that portion is 100%, i.e., $|r_0| = 1$ is appropriate. But how good is the 1D model in practice? Some reasons for concern are:

- near-surface scattering;
- deviations from horizontality (in the subsurface or topography);
- non-vertical propagation due to finite source-receiver offset.

The near-surface is often very heterogeneous, so scattering is likely. It would send some energy away from the vertical, making it unavailable for further reflections in a 1D model. If the loss is not too big, it may be roughly accounted for by diminishing the effective reflection coefficient of the surface ("roughly", because scattering is frequency dependent and the adequacy of such a compensation would depend on the statistical distribution of scatterers with different sizes). Slight deviations from horizontality or vertical propagation would be tolerable in practice because of the finite frequency range used. Thinking in terms of "fat rays" or Fresnel zones, a diminished surface reflection coefficient seems again appropriate to make the model more realistic.

It is not obvious how much to reduce the surface reflection coefficient. Analysis of real data would hardly help because there will be a trade-off with the geometrical spreading correction. Synthetic experiments show that the greatest changes on the trace occur during the first several percent reduction of r_0. That is why I chose to consider a model with $|r_0| = 0.9$ in addition to the other two (raw trace with $|r_0| = 1$ and filtered trace with $r_0 = 0$) in all cases when the difference between the raw trace and the filtered one is substantial.

For all experiments in this paper, the seismic source is an unit spike at the earth surface. Reflection coefficients are defined as seen from above by the displacement field. The next two sections (Surface Seismogram and VSP) are devoted to the "cyclic" geological setting, expected to cause problems with absorption estimation and deconvolution. After that, the same experiments are briefly repeated for "transitional" layering.

Surface Seismogram

A reflectivity series expected to produce strong apparent attenuation is shown in Figure 2. Also shown are its impulse responses for different models of the earth surface (different reflection coefficients r_0). Surface-related multiples make a striking contribution – not only is the energy level of the raw trace ($|r_0| = 1$) much higher than that of the filtered trace ($r_0 = 0$), but also the energy decay with time is very different. It is quite fast for $r_0 = 0$ and virtually absent for $|r_0| = 1$ over the whole 4-second duration of the record. This suggests that the bulk of energy, especially at late times, comes from a different part of the medium in each case (near-surface for $|r_0| = 1$ and depth for $r_0 = 0$; the latter needs investigation in the presence of strong scattering – see the Appendix).

To explore spectral changes with time, each trace was divided into time segments, 256 ms (128 samples) in length. Figure 3 shows the estimated spectra of the first 8 segments for each trace. The presence of a free surface at the top of the model makes a large difference. The spectrum of the raw surface trace (Fig. 3a) has the same character as the reflectivity spectrum, i.e, it is "blue". This is remarkable since the trace is comprised of "red" pulses, i.e., pulses which have been transmitted to a certain depth and back up, experiencing apparent attenuation along the way (O'Doherty & Anstey, 1971). This is an excellent illustration of the fact that the trace spectrum is governed by the *superposition* of the arrivals (which is determined by the reflectivity series) rather than by their individual spectra alone.

Another interesting characteristic of the raw trace spectrum (Fig. 3a) is that it is constant with time (over the few second time interval, typically recorded in seismic surveys). In contrast, the spectrum of the filtered trace is not stationary (Fig. 3c). At early times, it is "blue", but less steep than that of the raw trace. Over time, it looses predominantly high frequencies, so the late portion of the filtered trace eventually becomes high-frequency deficient. Since the source has input a spike to the medium (flat spectrum), the late portion of the filtered trace may be said to exhibit "apparent attenuation". In general however, the spectral effects due to propagation through a layered elastic medium are not identical to those caused by intrinsic attenuation (inevitable loss of high frequencies, i.e., "red" spectrum at all times).

The above observations have a direct bearing on intrinsic absorption estimation. Absorption estimation from surface data is highly desirable, though not practiced yet (for a recent development see (Dasgupta & Clark, 1998)). One of the main obstacles is our inability to record zero-offset traces. For the sake of argument, however, suppose we had a truly zero-offset trace acquired over a horizontally layered constant-Q medium*.

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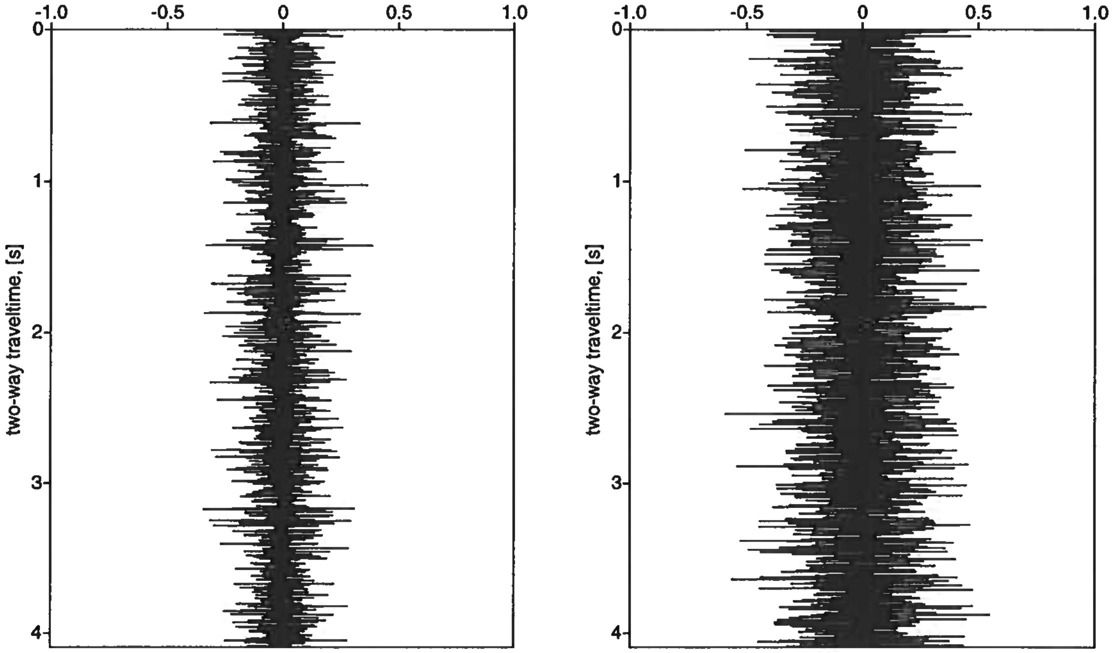
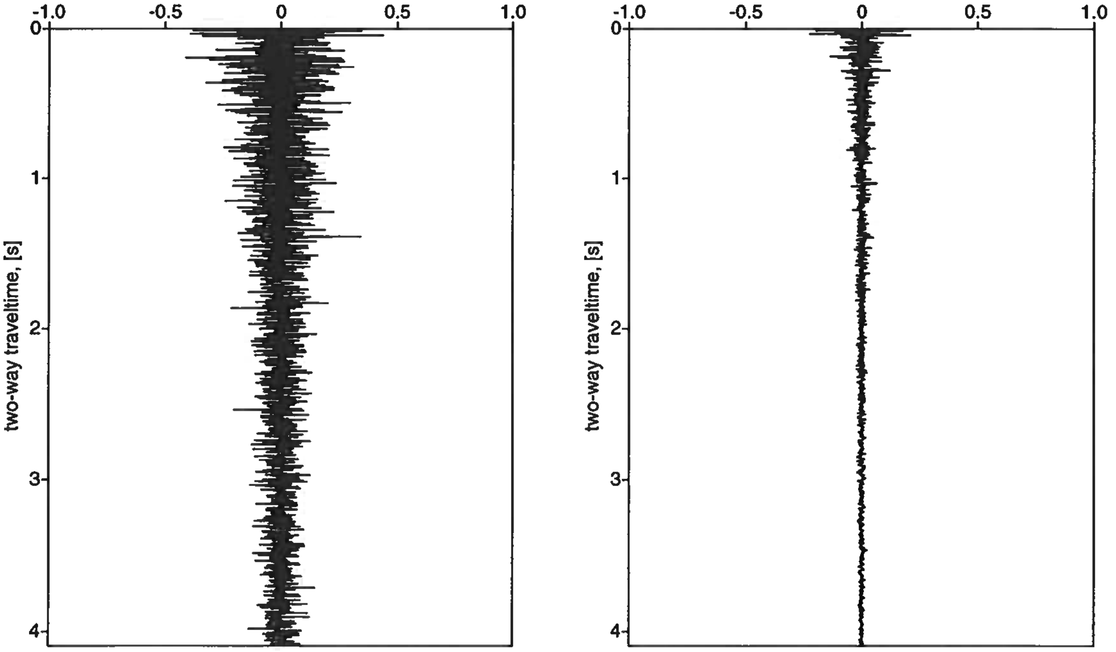
*In this paper, "constant-Q" means spatially-invariant Q. The term does not refer to the frequency dependence of the Q factor. Q is assumed to be frequency-independent over the seismic frequency band.



Reflectivity series

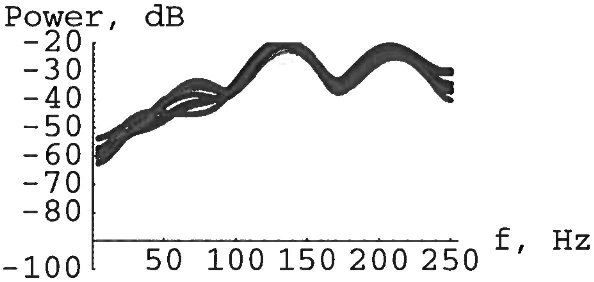
Impulse response r0 = - 1.0



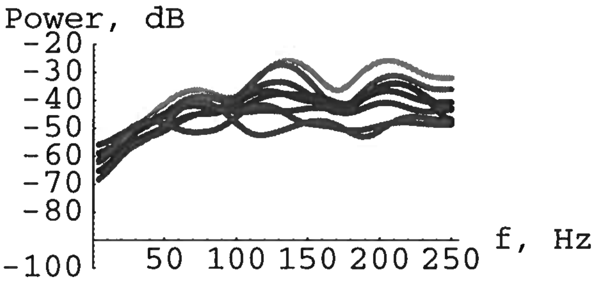
Impulse response r0 = - 0.9

Impulse response r0 = 0

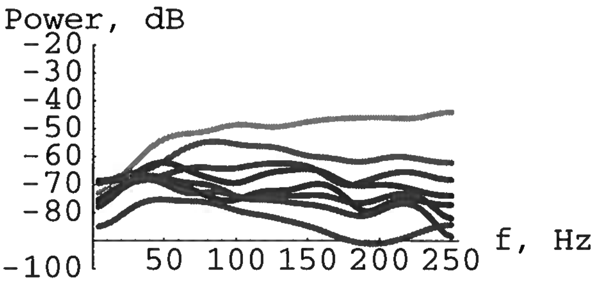
Figure 2. Reflectivity series and its surface-source/surface-receiver impulse response for different surface reflection coefficients.



(a) $|r_0| = 1.$



(b) $|r_0| = .9$



(c) $|r_0| = 0.$

Figure 3. Spectral change with time of the impulse response recorded at the earth surface. Each trace ($|r_0| = 1$, $|r_0| = .9$, and $|r_0| = 0$) has been divided into 256ms-long adjacent segments (128 samples per segment). Shown are the spectra of the first 8 segments of each trace. They practically coincide for $|r_0| = 1$. In the other two cases, lower curves correspond to later time windows.

Suppose also the source, detector and instrumentation signatures were known and could be removed from the trace to obtain the impulse response. If we attributed any spectral coloring of the raw impulse response to inelasticity, we would underestimate the absorption by about 0.12 dB/Hz (Fig. 3a). In other words, we would overestimate Q by 5% in a highly attenuating medium and by up to 20% in a low-loss medium (the higher the intrinsic Q, the larger the error) †.

Such a direct estimate of absorption from the trace spectrum is sensitive to uncertainties in the source and detector signatures, and to frequency-dependent coupling. For that reason, *spectral ratios* between portions of the trace are more likely to be used.

By looking at spectral ratios between portions of the *raw* trace, intrinsic absorption (of a constant-Q medium) can be estimated accurately since the spectrum of the elastic impulse response is stationary. For the more realistic case of $|r_0| = 0.9$, Q will be underestimated by up to 10% because some loss of high frequencies with time is evident in the elastic impulse response (Fig. 3b). For a trace without any surface-related multiples, an underestimate of Q by up to 20% can be expected.

Besides to intrinsic absorption estimation, the spectral shaping caused by intrabed multiples is important to model-based deconvolution. Denoting the elastic impulse response of the medium by r_m, the corresponding seismic trace can be written as $w_0 * r_m$, where the star means convolution and w_0 is some "basic" wavelet accounting for source and detector signatures, as well as constant-Q absorption (Connelly & Hart, 1985). On the other hand, the conventional convolutional model of the trace is $w * r$ where r is the reflectivity series and w is a wavelet. In order that both representations might be equivalent, the traditional wavelet w has to account for transmission losses and multiples (in addition to source and detector signatures and absorption). Its relation to the "basic" wavelet w_0 is easy to establish in frequency domain:

$$W_0 R_m = W_0 \frac{R_m}{R} R = WR \, , \tag{4}$$

where the capital letters stand for the Fourier images of the respective time series. This simple consideration

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†The basis for comparison between intrinsic and apparent attenuation throughout this paper is the following:

Amplitude loss due to intrinsic absorption in a constant-Q medium is described by

$$A = A_0 e^{-\alpha z} = A_0 e^{-\frac{\omega z}{2cQ}} = A_0 e^{-\frac{\omega t}{2Q}} \; . \tag{1}$$

Therefore, the power loss is

$$P = P_0 e^{-\frac{\omega t}{Q}} \; , \tag{2}$$

and the slope of the power spectrum in dB/Hz is

$$-20 \frac{2\pi t}{Q} \log_{10} e \approx -55 \frac{t}{Q} \; . \tag{3}$$

For the depth range of exploration seismology, the Q-factor is typically between 25 and 80. Thus, the power spectrum of a pulse which has travelled for 1 second through an absorbing homogeneous earth would have a slope of -2.2 dB/Hz in a "high-loss" medium with Q=25 and -0.7 dB/Hz in a "low-loss" medium with Q=80.

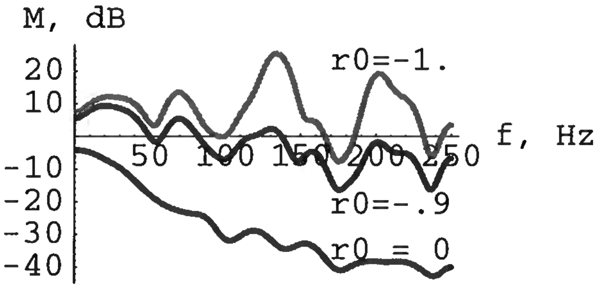
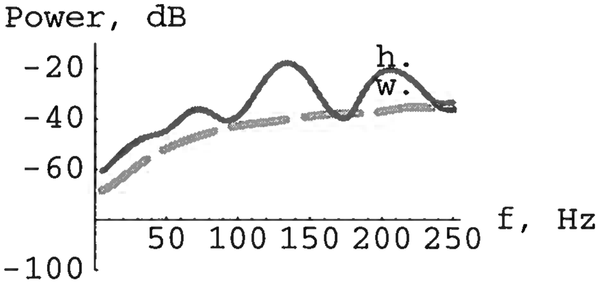


Figure 4. Power of the operator $M = \frac{R_m}{R}$ for Reflectivity 1 ("cyclic" layering).

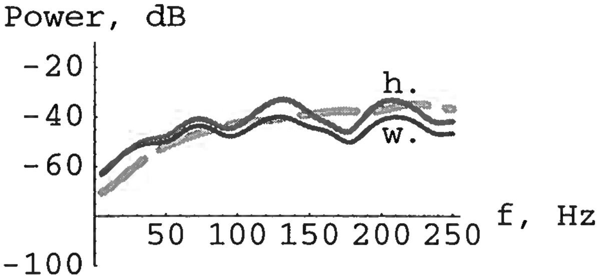
shows that transmission losses and multiples can be included in the wavelet model through the $\frac{R_m}{R}$ operator. If reflectivity was white, its impulse response would be white too (the elastic-earth filter would be frequency in-selective) and the operator $\frac{R_m}{R}$ would be simply a constant. When the primary reflectivity is not white, however, the operator $\frac{R_m}{R}$ is not trivial. Its power spectra for our experiments are shown in Figure 4. For the raw trace ($|r_0| = 1$), the slope of $\frac{R_m}{R}(f)$ is negligible and therefore, unaccounted multiples in the wavelet construction will not degrade model-based deconvolution. In the more realistic case of $|r_0| < 1.$, however, the slope of the $\frac{R_m}{R}$ operator is significant, i.e., multiples cause "apparent attenuation" of the basic wavelet.

The estimates of the operator $\frac{R_m}{R}$ shown in Figure 4 are based on the whole traces (4s long). Since trace spectra are not stationary for $|r_0| < 1$, a question may arise whether the last conclusions are not dictated by the choice of a time window for spectral estimation. Figure 5 shows two spectral estimates of the trace – one, based on the whole trace, and the other based on the first half of the trace; also depicted is the power spectrum of the reflectivity series. Despite the existence of a difference between the two estimates of the trace spectrum (especially for $r_0 = 0$), it is clear that, for a reasonably long time window, the trace spectrum rises with frequency less steeply (if at all) than the reflectivity spectrum. This, in turn, causes apparent attenuation of the basic wavelet (via $\frac{R_m}{R}$).

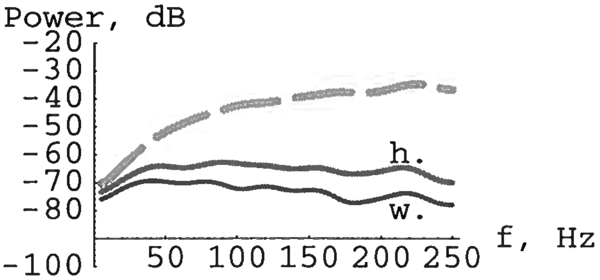
The fact that the impulse response recorded at the earth surface has a "weaker color" than the primary reflectivity seems to be universal and is easy to explain in the time domain. A steep reflectivity spectrum signifies a strong correlation between consecutive time samples of the reflectivity series. The surface impulse response differs from the primary reflectivity series due to the presence of transmission losses and multiples. Transmission losses cause a simple amplitude decay with time and are frequency independent (Schoenberger & Levin, 1974). Multiples, on the other hand, tend to destroy the correlation between consecutive primary reflections by over-imposing information about remote (weakly correlated) reflection coefficients.



(a) $|r_0| = 1.$



(b) $|r_0| = .9$

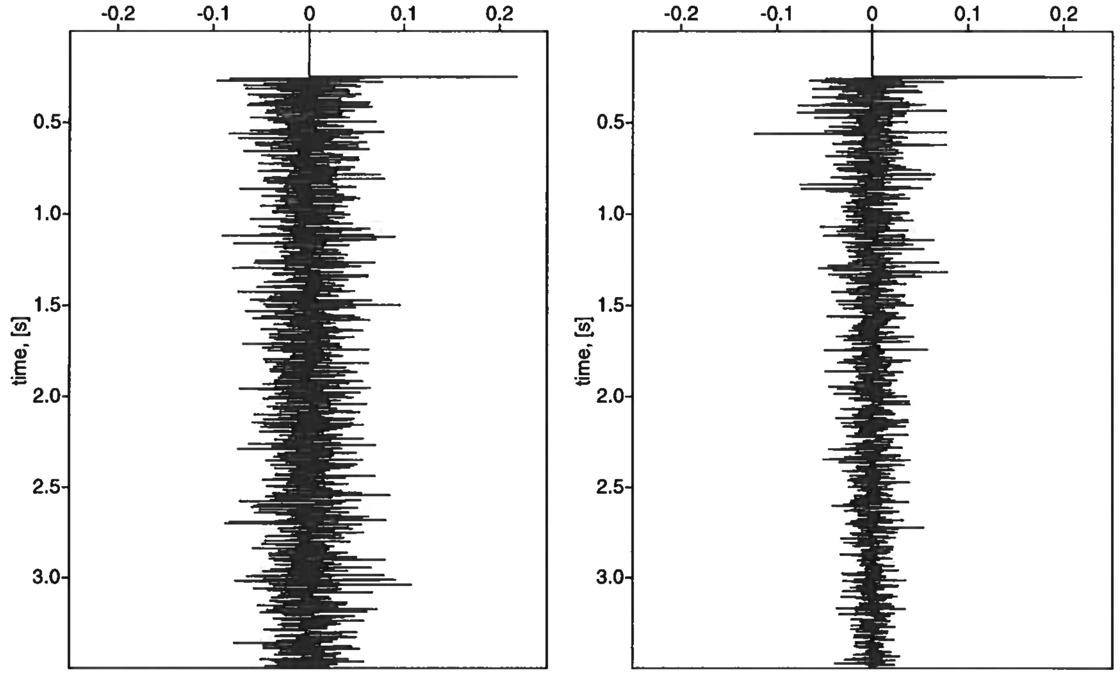


(c) $|r_0| = 0.$

Figure 5. Two estimates of the trace spectrum – one based on the whole trace (2048 time samples), and the other based on the first half of the trace (1024 samples). Shown also is the power spectrum of the reflectivity series (dashed).

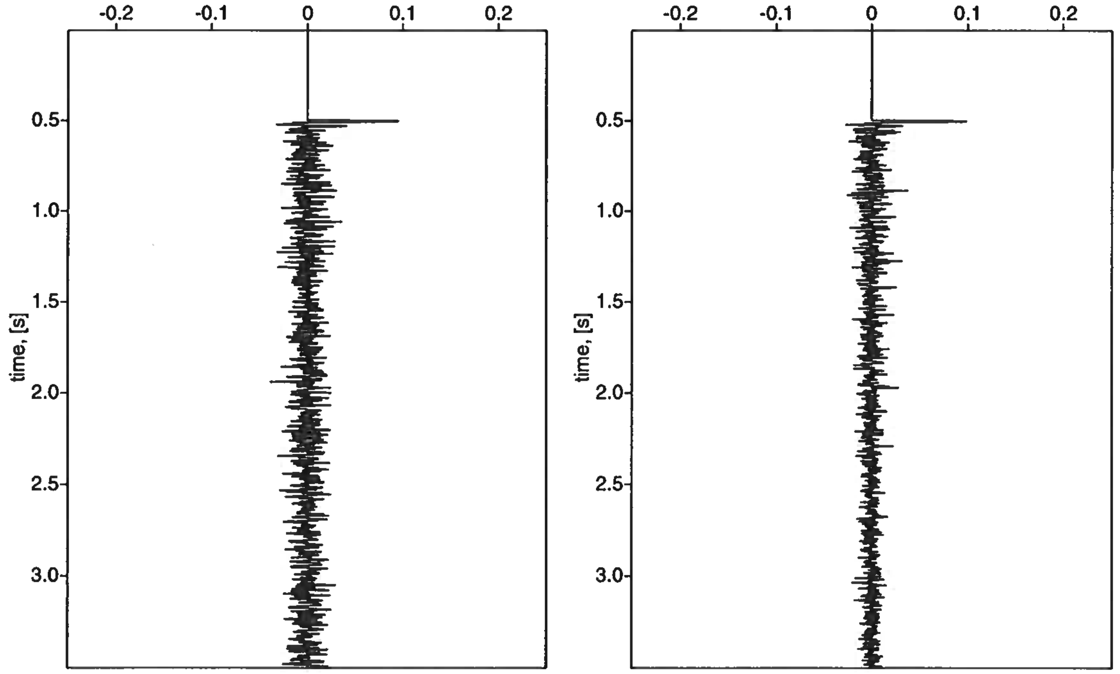
VSP

In order to investigate the spectral changes occurring with depth, two receivers were placed at 250 ms and 500 ms one-way traveltime from the surface. The model



Receiver 1, r0 = - 1.0

Receiver 1, r0 = 0



Receiver 2, r0 = - 1.0

Receiver 2, r0 = 0

Figure 6. Impulse response at two depths: Receiver 1 is at 250 ms one-way traveltime from the surface and Receiver 2 is at 500 ms from the surface.

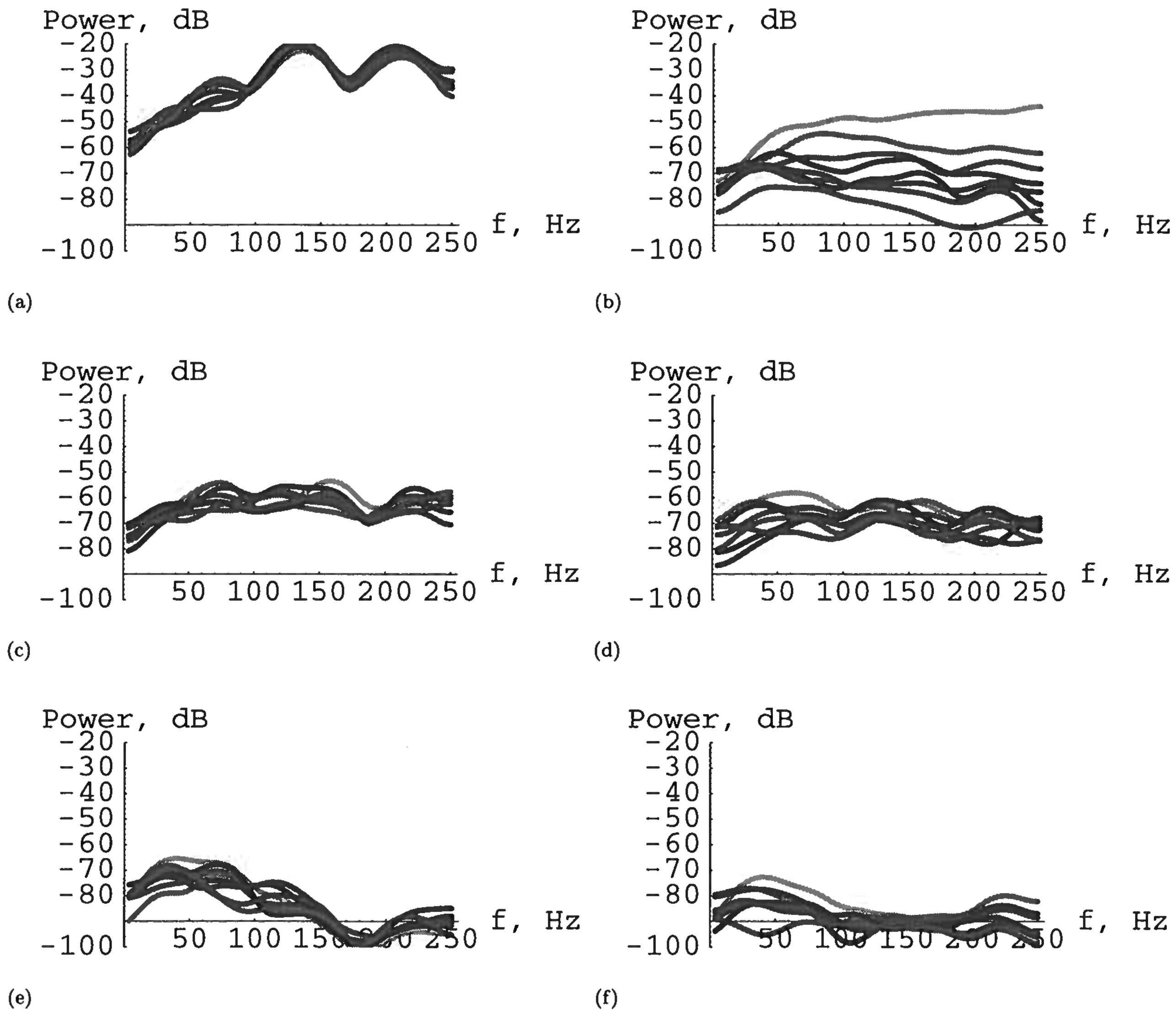


Figure 7. Spectral change with time for three receiver positions (top to bottom): surface (from Figure 3), 250 ms from the surface, and 500 ms from the surface. Left column [(a),(c),(e)] – with $|r_0| = 1$; right column [(b),(d),(f)] – with $r_0 = 0$. For each trace, spectra of adjacent time windows are shown. The length of the windows is 256ms (128 samples) and the first window on each trace starts at the first arrival.

with $|r_0| = 0.9$ is omitted during the VSP experiments because surface-related multiples have a much smaller impact on deep records than on surface data. The parallel consideration of models with $|r_0| = 1$ and $r_0 = 0$ should suffice.

The impulse response recorded by the two buried receivers is shown in Figure 6. The most significant change with depth is decrease in energy (compare Figure 6 to Figure 2 and note the difference in horizontal scale). Large transmission losses are typical for "cyclic" sequences with big reflection coefficients. The amplitude of the direct arrival has dropped by almost 80% from its initial value (a unit spike at the surface) at Receiver 1 (250 ms from the surface) and by an additional 80% traveling from Receiver 1 to Receiver 2 (500 ms from the surface), so the amplitude of the direct arrival at the lower station is only 5% of that of the initial spike. The spike of amplitude 0.1 appearing as a direct arrival at Receiver 2 is, in fact, on the third sample after the first break and is comprised almost entirely of multiples. It cannot be determined how many orders of multiples are sufficient to restore the pulse energy as done by O'Doherty and Anstey (1971) and Schoenberger and

Levin (1974) since the tail of the transmitted pulse interferes with reflections from below the receiver station.

Spectral change with time along the down-hole seismograms was investigated as in the previous section. The first time-window was shifted to begin at the first break on each trace. Spectra of adjacent time windows are shown in Figure 7 for all three receiver depths (surface and two down-hole stations). While the spectrum of the raw trace at the surface is essentially stationary, its estimates at the deep receivers show some, seemingly random, variability with time. For the filtered trace, temporal spectral changes are much smaller at the buried stations than at the surface. While the surface trace clearly looses high frequencies over time, no such trend is seen on the down-hole traces.

When the source-receiver distance increases, transmission effects become stronger. That is why deeper traces contain substantially fewer high frequencies than those recorded at shallower stations (Figure 7). High-frequency loss with depth must be taken into account when spectral-ratio methods are used to determine absorption from VSP experiments. Figure 8 shows the spectral ratios between the trace recorded at Receiver 2 and the surface trace. In the absence of surface-related multiples, the loss of high frequencies with depth seems mild [‡]. For the raw trace, the high-frequency loss with depth is huge: at 0.5 dB/Hz in 1 sec., it constitutes more than 70% of the loss rate in a high-Q medium (Q=80) and more than 20% in a low-Q medium (Q=25)!

Therefore, the apparent attenuation must be taken into account in "cyclic" geology. Being controlled by the gross shape of the reflectivity spectrum, it should be reasonably constant for a given field and can be assessed from nearby wells [§].

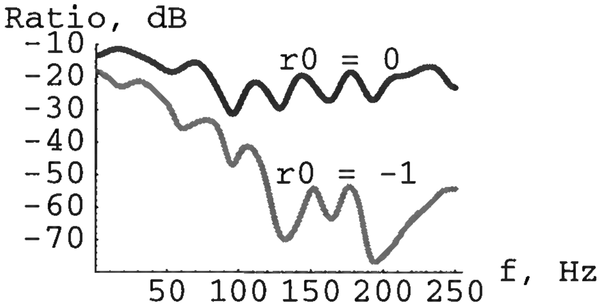


Figure 8. Spectrum change with depth – from the surface to Receiver 2: $\delta t = 500ms$ ("cyclic" layering).

Transitional Geology

After having analyzed the more unfavorable situation – large reflection coefficients and markedly non-white reflectivity – let us see what happens at the other end of the gamut when the reflection coefficients are small and have a pseudo-white spectrum above a low corner frequency. I repeated all the experiments described previously with the transitional-type sequence whose parameters are given in Table 1 and theoretical spectrum depicted in Figure 1. The reflectivity series and its impulse responses recorded at the earth surface are shown in Figure 9. Since the reflection coefficients are small, little energy is returned to the surface (note the horizontal scale and compare with Figure 2). Surface-related multiples make a strong contribution again. One important difference from the previous case (of "cyclic" layering) is that now the filtered trace ($r_0 = 0$) looks stationary and its early portion closely resembles the reflectivity series. This is because both multiples and transmission losses are weak.

The small transmission losses are also evident on the down-hole seismograms (Figure 10) where the direct arrival is by far the most energetic and its amplitude has dropped by less than 20% after 250 ms transmission (compared to a drop of more than 80% in the case of "cyclic" layering).

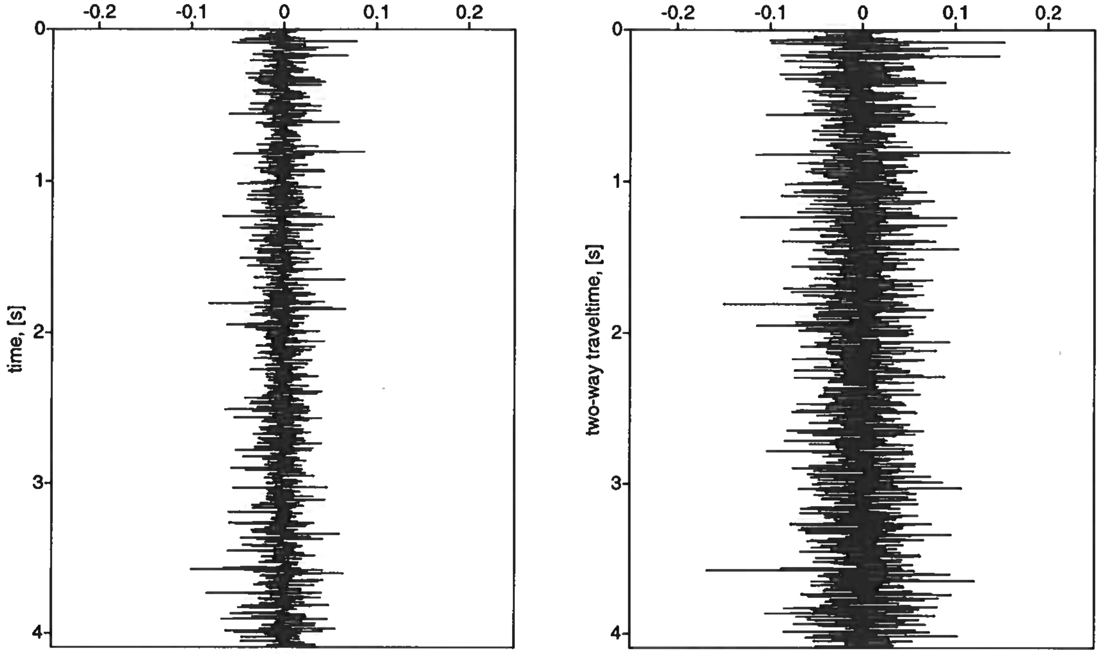
Weak multiples allow the spectra of surface seismograms to mimic the shape of the reflectivity spectrum (Figure 11). An important consequence is that the operator $\frac{R_m}{R}$ needed for proper model-based deconvolution reduces virtually to a scaling constant (Figure 13) like in the white-reflectivity case, despite the profound non-whiteness of the reflectivity at low frequencies.

Another important consequence of the weak multiples is that the trace spectrum does not change its shape as the receiver is lowered down a borehole (Figure 12). Therefore, VSP spectral-ratio methods are able to give an accurate estimate of the intrinsic absorption in a transitional-type geology.

Care should be taken, however, when absorption is

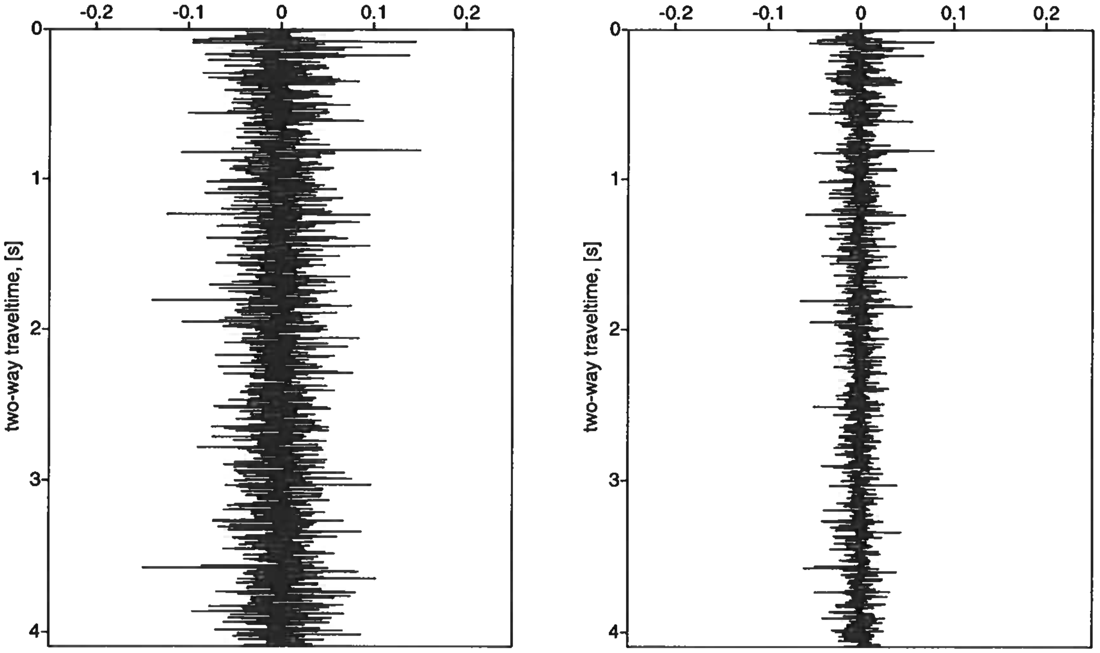
[‡] based on the whole 4s-long surface trace; if only an early portion of the surface trace is used for spectral estimation, the loss of high frequencies with depth will be more pronounced – compare the top curves in Figure 7(b) and Figure 7(f).

[§] Spencer *et al.* (1982) observed large variability of the stratigraphic filtering estimates at high frequencies caused by changes in the local stratigraphic detail. They concluded that filter estimation from nearby wells was unreliable. I believe their conclusion was dictated by the too coarse reflectivity model used. Nowadays earth reflectivity properties are much better known. Repeating the experiments of Spencer *et al.* with a more realistic reflectivity model, I did not find spectral variability to be too large even at high frequencies (up to the Nyquist). The main uncertainty comes from the limited number of time samples used for spectral estimation. For a reasonably long time window (e.g., 128 samples, at least), the "apparent attenuation" estimates for different reflectivity realizations were fairly consistent.



Reflectivity series

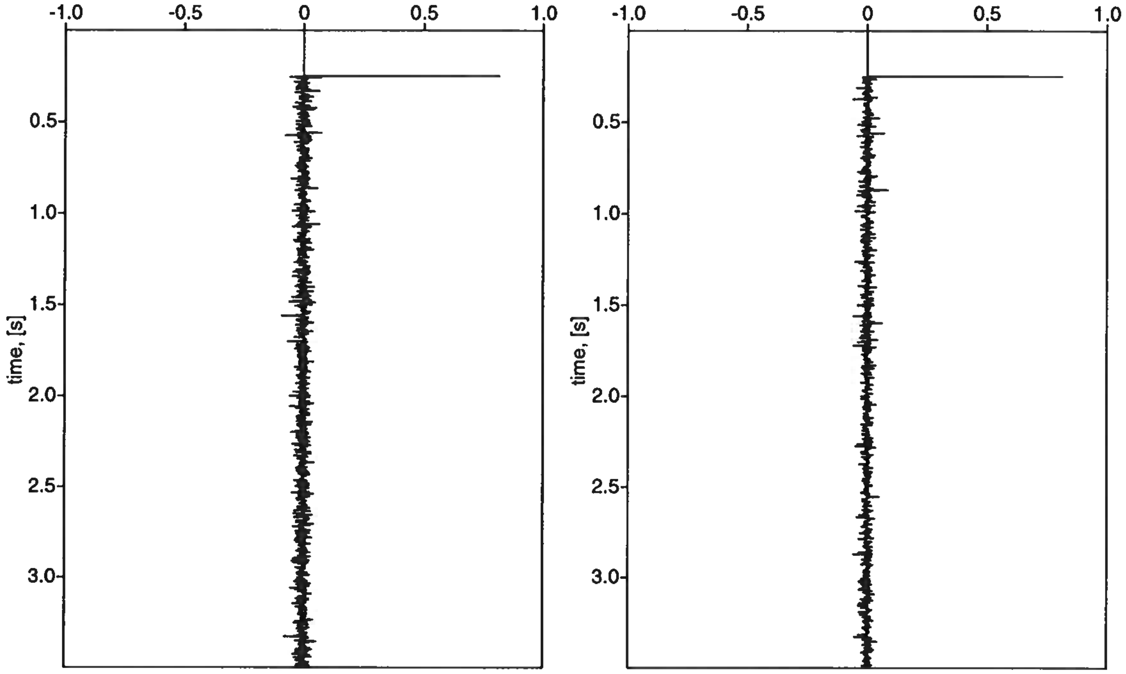
Impulse response r0 = - 1.0



Impulse response r0 = - 0.9

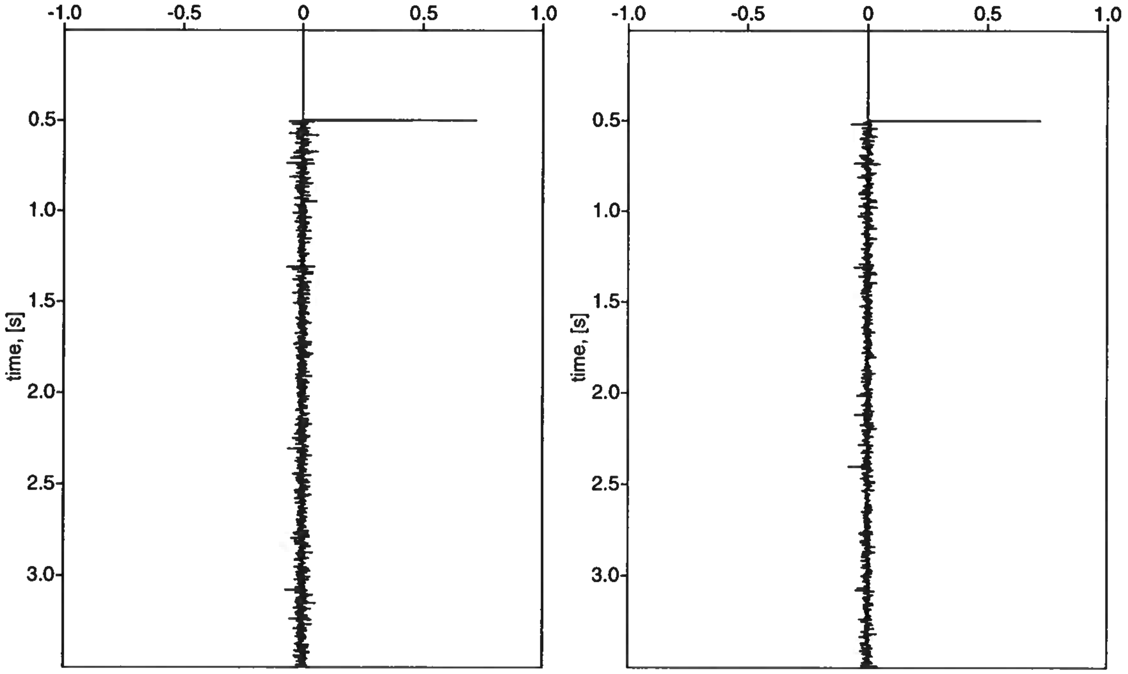
Impulse response r0 = 0

Figure 9. Reflectivity series and its surface-source/surface-receiver impulse response for different surface reflection coefficients.



Receiver 1, r0 = - 1.0

Receiver 1, r0 = 0



Receiver 2, r0 = - 1.0

Receiver 2, r0 = 0

Figure 10. Impulse response at two depths: Receiver 1 is at 250 ms one-way traveltime from the surface and Receiver 2 at 500 ms from the surface.

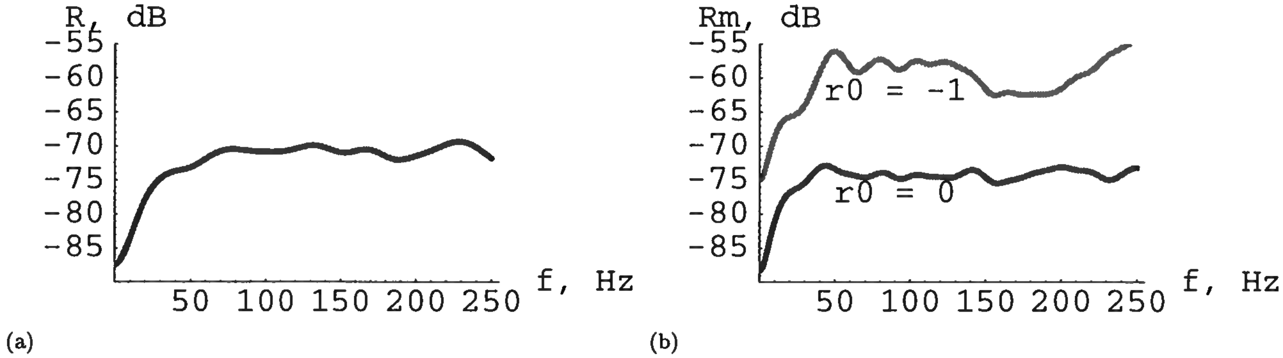


Figure 11. Power spectrum of Reflectivity 2 (a) and surface traces with and without surface-related multiples (b).

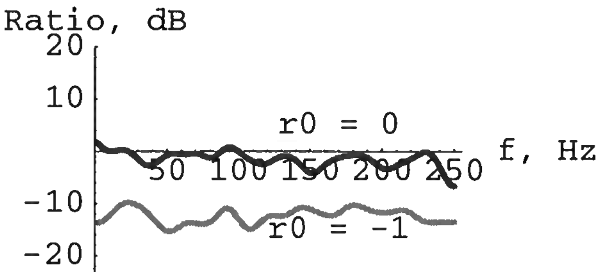


Figure 12. Spectrum change with depth – from the surface to Receiver 2: δt = 500ms (transitional layering).

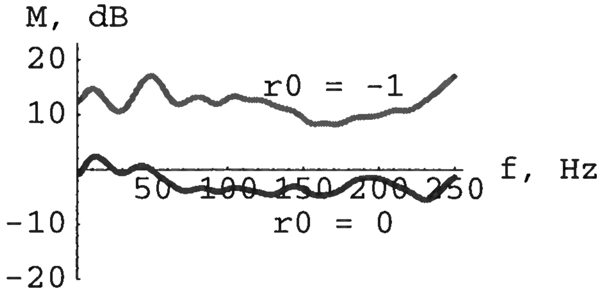


Figure 13. Power of the operator $M = R_m/R$ for Reflectivity 2 ("transitional" layering).

estimated directly from the trace (accounting for source, receiver and instrumentation signatures). Depending on noise characteristics and the fitting procedure used, only the low-frequency part of the trace spectrum may be employable for absorption assessment. That is exactly where the elastic impulse response is severely non-white and may lead to a substantial over-estimate of the medium Q-factor.

Conclusions

Based on the described experiments, the following conclusions can be made:

I. Absorption estimation through *VSP spectral-ratio* methods (currently conventional) should work well in a constant-Q "transitional" geology but *will* fail in "cyclic" media with large reflection coefficients unless the apparent attenuation is accounted for.

II. *Spectral-ratio* methods applied to a *raw surface trace* (zero offset) would give a good Q estimate. *Direct* absorption estimation from the raw surface trace spectrum would require information about the reflectivity spectrum (in addition to corrections for source and receiver signatures, etc.)

Suppressing surface-related multiples would not benefit absorption estimation from surface seismic data in a constant-Q medium. However, if absorption varies with depth, the raw surface trace would give a Q-estimate, heavily weighted by the Q-factor of the shallow subsurface. Removing the surface-related multiples would lessen the influence of the shallow zone at the expense of including apparent attenuation appraisal in the problem. The question of which part of the medium has contributed most to the effective Q estimate in the presence of strong scattering needs further investigation.

III. Stratigraphic filtering can be included in the wavelet model through the operator $\frac{R_m}{R}$, where R and R_m are the spectra of the reflectivity series and its impulse response respectively.

When *model-based deconvolution* is applied to a *raw* trace ($|r_0| = 1$), multiples need not be included explicitly in the wavelet model since the spectrum of the operator $\frac{R_m}{R}$ has a negligible slope.

In practice (non-zero offsets, non-horizontal layering, near-surface scattering), modeling the earth surface as a perfect reflector might be inappropriate. A reduced surface reflection coefficient above a "cyclic" sequence may cause a measurable decay of $|\frac{R_m}{R}|$ with frequency, e.g., apparent attenuation of the basic wavelet.

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REFERENCES

Banik, N. C., Lerche, I., & Shuey, R. T. 1985. Stratigraphic Filtering, Part I: Derivation of the O'Doherty-Anstey Formula. *Geophysics*, **50**, 2768–2774.

Connelly, D., & Hart, D. 1985. Model-based wavelet processing of deconvolved seismic data. *Pages 491–495 of: 55th Ann. Internat. Mtg., SEG, Expanded Abstracts.* SEG.

Dasgupta, R., & Clark, R. A. 1998. Estimation of Q from Surface Seismic Data. *Geophysics*, **63**, 2120–2128.

O'Doherty, R. F., & Anstey, N. A. 1971. Reflections on Amplitudes. *Geophysical Prospecting*, **19**, 430–458.

Saggaf, M. M., & Robinson, E. A. 2000. A United Framework for the Deconvolution of Traces of Nonwhite Reflectivity. *Geophysics*, **65**, 1660–1676.

Schoenberger, M., & Levin, F. K. 1974. Apparent Attenuation due to Intrabed Multiples. *Geophysics*, **39**, 278–291.

Spencer, T. W., Sonnad, J. R., & Butler, T. M. 1982. Seismic Q – Stratigraphy or Dissipation. *Geophysics*, **47**, 16–24.

Walden, A. T. 1993. Simulation of realistic synthetic reflection sequences. *Geophysical Prospecting*, **41**, 313–321.

Walden, A. T., & Hosken, J. W. J. 1985. An Investigation of the Spectral Properties of Primary Reflection Coefficients. *Geophysical Prospecting*, **33**, 400–435.

Walden, A. T., & Hosken, J. W. J. 1986. The Nature of Non-Gaussianity of Primary Reflection Coefficients and Its Significance for Deconvolution. *Geophysical Prospecting*, **34**, 1038–1066.

APPENDIX A: ENERGY DISTRIBUTION IN A STRONGLY SCATTERING MEDIUM

To understand the spectral properties of the impulse response of a strongly scattering medium (such as the first reflectivity sequence considered in this paper), it is useful to know from which part of the medium the energy recorded at a given time comes.

To check the energy distribution, I looked at the trace energy in the three equally spaced receivers – surface receiver, Receiver 1 and Receiver 2 – at times after the first break on Receiver 2.

In the presence of surface-related multiples (Figure A1a), the energy is concentrated near the surface and monotonically decreases with depth at all times ($t \in [0.5; 3.0]$ s).

In the absence of surface-related multiples (Figure A1b), the the energy is concentrated at intermediate depths (Receiver 1) for the monitored time interval, i.e., the scattered wave field exhibits "diffusion" rather than "ballistic propagation".

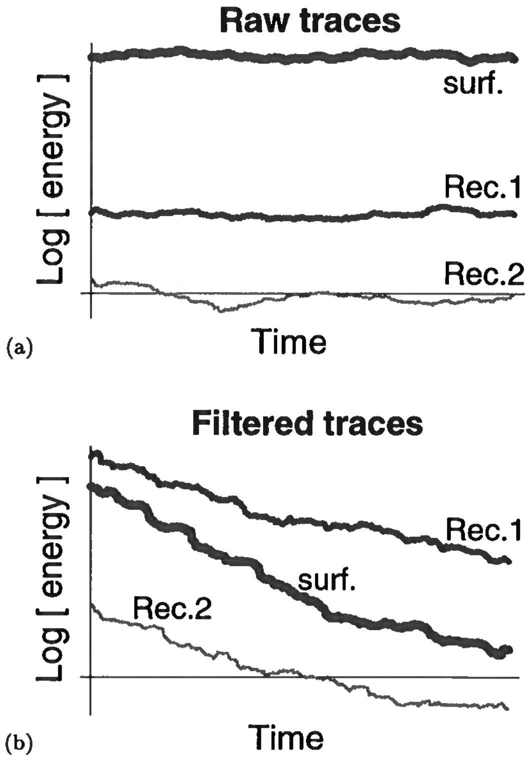

Figure A1. Energy distribution with depth after the ballistic arrival in the deepest receiver: (a) raw traces ($|r_0| = 1$); (b) filtered traces ($r_0 = 0$).