AVO analysis in finely layered azimuthally anisotropic media

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This paper appeared in the 1998 CWP Project Review, CWP-283.
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ABSTRACT

Over the years, amplitude-variation-with-offset (AVO) analysis has proved its usefulness in exploration of oil and gas reservoirs. However, the model conventionally used to interpret AVO anomalies – a single isolated interface between two isotropic half-spaces – is often too simplistic. Here, I examine what can be obtained from AVO responses for significantly more complicated reservoir model – a stack of plane azimuthally anisotropic layers, the model which can be used to simulate AVO signatures over finely layered fractured reservoirs.

I describe a processing technique which takes seismic data in frequency – slowness (ω – p) domain, properly corrects it for slant wave propagation in finely layered medium, and produces instantaneous AVO intercept and azimuthally varying AVO gradient as functions of vertical traveltime. Ideally, obtained AVO intercept and gradient are those which would be recorded in the case of isolated interfaces and in the absence of interference between closely spaced reflections. I use azimuthally dependent AVO gradient to obtain instantaneous AVO azimuth, which corresponds to the direction of the greatest AVO gradient. In fractured reservoirs, AVO azimuth is related to the orientation of vertical cracks, therefore, fracture characterization is one of potential applications of the described technique.

I perform numerical study to examine stability of obtained AVO azimuth with respect to errors in velocity model of the reservoir, inaccuracies in wavelet estimation, and random noise in the data. The results of numerical tests indicate that AVO azimuth is reasonable stable, and suggest the possibility of detecting principal directions of azimuthal anisotropy in layers which are thinner than half dominant seismic wavelength.

Key words: AVO analysis, azimuthally anisotropic media, fine layers

Introduction

Amplitude-variation-with-offset (AVO) analysis has been used for years to estimate reservoir lithology and fluid content. Since work of Ostrander and Gassaway (1983) and Ostrander (1984), who first demonstrated potential of AVO as an indicator of hydrocarbons, many studies have been undertaken to improve reliability of AVO analysis and overcome its numerous complications and pitfalls. An extensive list of inherent problems in measuring and interpreting offset-dependent reflectivity is given by Castagna (1993). Some of the factors causing amplitude variations such as spherical spreading, source-receiver directivity, and influence of free surface can be accounted for and approximately compensated during processing, the others require revision of conventional reflectivity model – an isolated interface between two sufficiently thick isotropic homogeneous layers. In this paper, I examine the combination of two factors which have the first-order influence on measured AVO signatures: fine layering and anisotropy within the reservoir. I study P-wave AVO response and construct corresponding processing algorithm for a model that consists of a stack of thin (as compare to the wavelength) azimuthally-anisotropic plane layers. The presence of fine layering in the model enables one to simulate and properly account for various tuning and interference phenomena. Permit-
ting azimuthal anisotropy in the layers allows to model AVO response for naturally-fractured reservoirs.

Inability of reflection coefficient from a single interface to describe real seismic reflections, which usually contain superposition of events from closely spaced interfaces, was recognized in the very beginning of applying AVO technology (e.g., Sherwood et al., 1983; Ostrander, 1984). Interference between reflections from adjacent interfaces causes the classical tuning phenomenon (Widess, 1973) and becomes increasingly complicated when considered as a function of offset or an incidence angle (Swan, 1988). As a result, an apparent AVO response is observed and the single-interface AVO signatures (i.e., AVO intercept and AVO gradient which would be recorded if interference does not exist) become obscured. Wapenaar et al. (1995) related this apparent AVO behavior to the fact that waves illuminating layered medium have different apparent vertical wavelengths \( \lambda_v \) at different incidence angles. Wapenaar et al. (1995) suggested considering the wave frequency \( \omega \) as a function of the incidence angle or the horizontal slowness \( p \) and change \( \omega \) in such a way to make \( \lambda_v \) independent on \( p \). The result was a technique which takes the data in \( t_0 - p \) (\( t_0 \) is the vertical time) domain, corrects it for apparent AVO phenomena due to fine layering, and reconstructs true reflectivity of acoustic media. The same idea was applied to laterally varying acoustic and elastic isotropic media by van Geloven and Wapenaar (1996, 1997). I express this idea in terms of slowness dependent plane-wave time delays \( t_0(p) \) and generalize the technique to azimuthally anisotropic media.

Anisotropy as well as fine layering is known to have a strong effect on seismic reflectivity. Numerical results of Wright (1984, 1987) indicate that transverse isotropy of overlying shales severely influences AVO gradient and must be accounted for in AVO analysis. Thomsen (1993), assuming weak anisotropy and small contrast in elastic properties across an interface, showed analytically that transverse isotropy with the vertical symmetry axis (VTI model) has first-order influence on angular dependence of P-wave reflection amplitude. Exact formulae for reflection coefficients in VTI media were derived by Graebner (1992) who used the horizontal slowness \( p \) rather than the incidence angle as independent parameter. I will follow the same idea here. Having slowness \( p \) as an argument is especially attractive because the slowness is preserved during wave propagation in plane layered media to be examined.

The influence of anisotropy on P-wave AVO signatures is even more pronounced for azimuthally anisotropic media. As was first numerically demonstrated by Mallick and Frazer (1992), AVO gradient becomes azimuthally dependent in this case. Since azimuthal anisotropy is often associated with aligned vertical cracks, the result of Mallick and Frazer (1991) suggests a possibility for fracture detection using measurements of P-wave AVO gradient in several azimuths. The relation between azimuthal variations in P-wave amplitudes and anisotropic coefficients, which can be derived from parameters of cracks, becomes apparent after studies done by Rüger and Tsvankin (1995) and Rüger (1996, 1997) who obtained analytical weak-anisotropy, small-contrast approximations for P-wave reflection coefficients in two important seismic models of vertical fractures — transversely isotropic media with a horizontal symmetry axis (HTI model) and orthorhombic media. They explicitly showed which combinations of anisotropic coefficients influence AVO signatures and, therefore, can be estimated from AVO inversion. The results of Rüger and Tsvankin (1995) and Rüger (1996, 1997) were extended to arbitrary anisotropy by Vavryčuk and Pšenčík (1997) and Zilmer et al. (1997). Sayers and Rickett (1997) numerically examined azimuthal variation in AVO response from the top and the base of a layer of fractured gas sand. Still, all the above mentioned studies concentrate on reflections from isolated interfaces and do not take into account fine layering.

From exploration standpoint, there is a great demand in characterizing fractured reservoirs, and multi-azimuth P-wave AVO studies have been performed in a number of exploration areas. Lefevre (1994) applied AVO-AVAX (amplitude versus offset and amplitude versus azimuth) analysis in the Paris basin, France and found that P waves, calibrated by shear-wave data, can detect fractures in the subsurface. Lynn et al. (1996) and Lynn et al. (1997) successfully used multi-azimuth P-wave AVO analysis in the Wind River Basin, Wyoming and at the Rulison Field, Piceance Basin, Colorado to determine fracture orientation within production intervals. Mallick et al. (1996) described a fracture detection method (the fractogram technique) applied to a 3-D P-wave data set over a gas field in central Wyoming. Although the method of Mallick et al. (1996) aims to produce principal directions of AVO gradient as functions of vertical time, influence of fine layering within reservoir was not removed from final fractogram displays.

Here, I develop a technique for azimuthal AVO analysis designed to reconstruct true AVO intercept and a quadratic form responsible for azimuthally varying AVO gradient as functions of the vertical time (or the depth, if the vertical velocity is known) in finely layered azimuthally anisotropic media. The word “true” in the previous sentence means that ideally I estimate series of spikes for AVO intercept and azimuthally dependent
AVO gradient which would be recorded if all interfaces are isolated from each other and interference of closely spaced reflections does not exist. In reality, the reconstructed AVO signatures are influenced by the shape and the frequency content of the seismic wavelet, thus, wavelet estimation becomes important for deconvolving and obtaining correct reflectivities. Then, the estimated AVO gradients can be interpreted in terms of differences of elastic parameters at each given vertical time using, for instance, linearized approximations derived by Vavryčuk and Pšeníčk (1997) or nonlinear inversion as suggested by Neves and de Hoop (1997).

I begin with introducing an exploding reflector-type model for a finely layered azimuthally anisotropic reservoir. This model contains plane-wave reflection coefficients of primaries. I demonstrate that the introduced model properly accounts for the influence of fine layering on AVO signatures and can be used to obtain the true AVO intercept and gradient. Then, I perform numerical study to show how errors in estimated wavelet, uncertainties in velocity model, and random noise distort the extracted AVO signatures.

Reflectivity model for layered reservoir
I model layered reservoir as a stack of plane azimuthally anisotropic layers between two homogeneous halfspaces $x_3 \leq 0$ and $x_3 \geq z$ (Figure 1) and concentrate on wavefield $U$ reflected from the reservoir and recorded at its top $x_3 = 0^*$. It is convenient to examine the wavefield $U$ in frequency-slowness domain, i.e., $U \equiv U(\omega, p_1, p_2) \equiv U(\omega, p)$ because both temporal frequency $\omega$ and the horizontal components $p_1$ and $p_2$ of the slowness vector are preserved during propagation of a plane wave within the reservoir.

Conventional convolutional model (e.g., Yilmaz, 1991)

$$U(\omega, p) = S(\omega) R(\omega, p)$$

(1)

in $\omega - p$ domain can be used to relate the wavefield $U(\omega, p)$ to the source spectrum $S(\omega)$ and the generalized $P$-wave reflection coefficient $R(\omega, p)$. Note that the source spectrum $S$ is independent of $p$ if the size of the source is smaller than the dominant wavelength (e.g., Buland et al., 1996), which is usually the case in practice. Assuming normal incidence (i.e., $p = 0$), one can deconvolve the wavefield $U(\omega, 0)$ and estimate the reflection coefficient $R(\omega, 0)$ and the source spectrum $S(\omega)$. Conventional assumptions under which deconvolution is being applied are discussed in Yilmaz (1991). The spectra $R(\omega, 0)$ and $S(\omega)$ can be Fourier-transformed into time domain to obtain the reflectivity series $r(t_0, 0)$ and the wavelet $s(t_0)$ as functions of the vertical time $t_0$.

After the source wavelet has been estimated using conventional processing, equation (1) explicitly gives the generalized reflection coefficient $R(\omega, p)$ for any slowness $p$. The initial objective is to express $R(\omega, p)$ in terms of individual $P$-wave reflection coefficients $r(t_0, p)$ from all interfaces within the reservoir. The obtained coefficients $r(t_0, p)$ then can be smoothed by fitting surfaces $G(p)$ at each vertical time $t_0$. The values $G(0) = r(t_0, 0)$ are AVO intercepts and the curvatures of $G(p)$ at $p = 0$ are AVO gradients in slowness domain.

A single-layer reservoir model
Let us begin with the simplest reservoir model that does not contain intermediate layers. Since, there is only one plane wave reflected from the reservoir base at depth $z$ (Figure 1), the reflectivity series in time domain contains a single spike $r(t_0, p)$ which can be related to the reflection response $R(\omega, p)$ as

$$R(\omega, p) = e^{2\omega t_p t_0} r(t_0, p).$$

(2)

The term $r(t_0, p)$ in this equation represents amplitude of the spike while traveltime $t_p \equiv t_p(t_0, p)$ describes slowness-dependent time delay; the factor $2\omega$ in the exponential stands to account for two-way propagation time. The relation between two time delays, $t_p$ and $t_0$, is given by:

$$t_p = \frac{z \cos \theta(p)}{V(p)} = t_0 \frac{v_0}{V(p)}.$$  

(3)

* This wavefield can be obtained as a result of true-amplitude migration applied to the wavefield recorded at the earth surface.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{The generalized reflection response of layered reservoir can be constructed from plane waves excited at all interfaces within the reservoir.}
\end{figure}
where $\theta(p)$ is the propagation angle (Figure 1), $V(p)$ is the phase velocity in direction $\theta(p)$, $v_0$ is the vertical group velocity, and $q(p)$ is the vertical component of the slowness vector.

**Multi-layer reservoir model**

Now, I generalize equation (2) and construct reflection response from a multi-layer reservoir. The simplest way of doing that is to sum up reflections from all interfaces at their appropriate time delays:

$$R(\omega, p) = \sum_k e^{2i\omega t_k} r(t_0, k, p),$$

(4)

where

$$t_0, k = \sum_{\ell \leq k} \Delta t_{0, \ell}$$

(5)

and

$$t_k(t_0, k, p) = \sum_{\ell \leq k} \Delta t_{0, \ell} v_0, \ell q_{\ell}(p).$$

(6)

In equations (5) and (6), $\Delta t_{0, \ell}$ is the interval one-way vertical traveltime in the $\ell$th layer, $v_0, \ell$ is the interval vertical group velocity, and $q_{\ell}(p)$ is the interval vertical slowness.

Equation (4) represents a linear "exploding reflector-type" reflectivity model which includes primaries only. This model accounts neither for internal multiples, nor for various converted waves which both present in true reflection response. Also, equation (4) does not compensate for propagation effects through interfaces at vertical times $t_{0, \ell}$ for $\ell < k$. Nevertheless, the simplistic reflectivity model (4) has one important feature: it is linear with respect to unknown reflection coefficients $r(t_0, k, p)$ which can be found from system of equations

$$E_j = E_{jk} r_k,$$

(7)

where

$$R_j = R(\omega_j, p), \quad E_{jk} = e^{2i\omega_j t_k} t_{0, k}(p),$$

(8)

and $r_k \equiv r(t_0, k, p)$.

Note that the time delays $t_0, k$ for the reflection coefficients $r_k$ are not unknowns; $t_0, k$ should be specified before solving equations (7). If there is no reflector at time $t_0, k$, corresponding reflection coefficient is supposed to be zero.

Ideally, equations (7) should be solved independently for each value of slowness to produce individual reflection coefficients $r_k$ as functions of $p$. It is possible if the number of frequencies $\omega_j$ in reflection response $R(\omega_j, p)$ is greater than the number of unknowns $r_k$, and system (7) is overdetermined. This constraint the maximum number of reflection coefficients that can be resolved. Another constraint is that any resolution or on the minimum value of $\Delta t_{0, \ell}$ is determined by spectral bandwidth of the response $R$. I will discuss these and other numerical issues of obtaining $r_k$ in the following sections; here, I examine some options for making the reflectivity model more accurate.

The reflectivity model (4) may be improved by including some of the factors - multiples, conversions, and propagation effects - which were initially ignored. There is not much freedom, though, because I would not like to increase the number of unknowns as those additional model parameters may introduce some trade-offs with the primary unknowns - the individual P-wave reflection coefficients $r_k$ - and degrade their resolution. Below, I discuss some possibilities and include the factors which can be expressed in terms of $r_k$ and, therefore, do not add new unknowns.

**Multiples**

Including multiples into the reflectivity model (4) is not difficult. However, it makes the model nonlinear with respect to $r_k$. For example, amplitude of the first-order multiple is proportional to $r_{k_1} r_{k_2} r_{k_3}$, where indices $k_i$ indicate the interfaces where the multiple has reflections. Nonlinearity of the model is an undesirable feature because it complicates calculating reflection coefficients and may lead to non-uniqueness of the solution. On the other hand, one may believe that the influence of multiples within the reservoir on reflection response $R(\omega, p)$ is not that significant. For example, reflection coefficient of about $r = 0.2$ indicates a fairly strong interface. There may be only a few such strong interfaces within the reservoir. Therefore, amplitude of the first-order multiple, reflected from these interfaces, can expected to be approximately 4% of amplitudes of corresponding primaries. Given the accuracy of measuring seismic amplitudes, an error of 4% can be ignored. Thus, I choose not to make reflectivity model (4) more complicated to improve its accuracy by just a few percent. Numerical examples presented below support this choice.

**Converted waves**

The problem with including various converted waves into the reflectivity model is that their reflection coefficients depend on parameter combinations of contacting layers which are different from those that control P-wave reflection coefficients $r_k$. This can be explicitly observed from linearized weak-anisotropy small-contrast approximations for reflection coefficients given by Rüger (1996). Exact reflection coefficients are expected to be even
more complicated. As a consequence, converted-mode reflection coefficients cannot be expressed in terms of \( r_k \). Therefore, to account for converted waves explicitly, some unknown parameters, which influence converted-mode AVO signatures but do not present in P-wave ones, should be added to the reflectivity model (4). This, again, complicates the reflectivity model and may reduce stability in estimating \( r_k \).

There is another consideration, which allows one to believe that conversions can be often approximately subtracted from composite reservoir response even without explicit estimating of corresponding converted-mode reflection coefficients. Let us concentrate on a common model of fractured reservoir which can be characterized by the presence of vertical cracks. All anisotropic media (HTI, orthorhombic, or monoclinic), which are used to simulate seismic response from a vertically fractured reservoir, have a horizontal symmetry plane. In those media, converted-wave reflection coefficients equal zero at normal incidence (at \( p = 0 \)) and depend linearly on slowness \( p \) for small \( p \) (or small offsets) as opposed to pure-mode reflection coefficients which are quadratic functions of \( p \) when \( p \) is small. This provides us with a simple tool for separating converted-mode reflections from pure-mode ones even if reflectivity model (4) is used.

Let us suppose that equations (7) have already being solved and \( r_k = r_k(p) \) are found. At small \( p \), reflection coefficients can be approximated by double a Taylor series

\[
r_k(p) = r_k(p_1, p_2) \approx \sum_{m+n \leq N} \frac{1}{(m+n)!} G_k^{(m,n)} p_1^m p_2^n \tag{9}
\]

where \( G_k^{(0,0)} \) is the normal-incidence reflection coefficient (AVO intercept), the coefficients \( G_k^{(m,n)} \) with \( m+n = 1 \) describe amplitudes of converted waves arriving at the vertical time \( 2t_{0,k} \), and \( G_k^{(m,n)} \) with \( m+n = 2 \) are the elements of a symmetric quadratic form which represents azimuthally dependent AVO gradient in the slowness domain. Criteria for choosing \( N \) will be discussed in the next section.

Thus, by fitting reflection coefficients \( r_k(p) \) with equation (9), I may hope to account for some influence of converted waves. I will verify this by performing numerical tests.

Propagation effects
Since I consider the reflection response \( R(\omega, p) \) for each plane wave (specified by slowness \( p \)) separately, the spherical spreading, which exists in time-offset domain, has already been removed by 3-D Fourier transformation to \( \omega - p \) domain. Another factor that can be accounted for is transmission effects.

At normal incidence (\( p = 0 \)), transmission coefficient through \( k \)th interface is \( 1 - r_k \). Since wave transmits through each interface twice on the way down and up and the reflection coefficients of down- and upgoing waves are opposite, amplitude reduction for two-way propagation is \( 1 - r_k^2 \). Therefore, due to transmission, reflection response \( R \) contains

\[
\tilde{r}_k = r_k \prod_{\ell < k} \left[ 1 - r_\ell^2 \right] \tag{10}
\]

instead of just \( r_k \). Thus, I can replace \( r_k \) by \( \tilde{r}_k \) in equation (7) and, after \( \tilde{r}_k \) are found, use equation (10) to obtain actual \( r_k \). Although this equation is nonlinear with respect to \( r_k \), it has a triangular-type structure which becomes apparent if equation (10) is presented in the form:

\[
\tilde{r}_0 = r_0, \\
\tilde{r}_1 = r_1 \left[ 1 - r_0^2 \right], \\
\tilde{r}_2 = r_2 \left[ 1 - r_1^2 \right] \left[ 1 - r_0^2 \right], \ldots, 
\tag{11}
\]

where \( r_0 \) is reflection coefficient from the top of reservoir.

Thus, equation (10) is easy to solve. Some difficulties appear if one wants to correct transmission effects for non-normal incidence, i.e., for \( p \neq 0 \). The problem is that reflection coefficient \( r_k(p) \) for downgoing wave approaching interface from the top is no longer opposite to that for upgoing wave which approaches the same interface from the bottom (e.g., Castagna, 1993). Therefore, strictly speaking, reflection coefficient is asymmetric, and equation (10) becomes incorrect. On the other hand, all known linearized weak-anisotropy small-contrast approximations show that reflection coefficients from two sides of the same interface are opposite to each other. This fact indicates that asymmetry in reflection coefficient is usually small and probably can be ignored for practical purposes. The numerical results support that and I will apply equation (10) to correct for transmission effects within reservoir for both normal and non-normal incidence.

Thus, the reflectivity model for layered reservoir, which will be used to extract individual reflection coefficients \( r_k(p) \), is given by

\[
R(\omega, p) = \sum_k c_{2i\omega} f_p(t_{0,k}, p) \tilde{r}(t_{0,k}, p), 
\tag{12}
\]

or, in matrix form,

\[
R_k = E_{jk} \tilde{r}_k, \tag{13}
\]

where \( \tilde{r}_k \equiv \tilde{r}_k(p) = \tilde{r}(t_{0,k}, p) \), and \( R_k \) and \( E_{jk} \) are defined by equations (8). Equation (10) relates reflection coefficients \( r_k \) to \( \tilde{r}_k \).
Extracting AVO intercept and gradient

Once equations (13) and (10) are solved, I obtain individual reflection coefficients \( r(p) \) as functions of the vertical time \( t_0 \). Then, using equation (9), I estimate AVO intercept

\[
A(t_0) = G(0,0)(t_0)
\]  
and symmetric matrix

\[
B(t_0) = \begin{pmatrix} B_{11}(t_0) & B_{12}(t_0) \\ B_{21}(t_0) & B_{22}(t_0) \end{pmatrix} = \begin{pmatrix} G^{(2,0)}(t_0) & G^{(1,1)}(t_0) \\ G^{(1,1)}(t_0) & G^{(0,0)}(t_0) \end{pmatrix}
\]

which describes azimuthally varying AVO gradient in the slowness domain. Quantities \( A \) and \( B \) represent AVO intercept and gradient at interface which corresponds to one-wave vertical traveltime \( t_0 \). They are an ideal input for AVO inversion schemes and can be interpreted in exactly the same way as AVO intercept and gradient at a single isolated reflector.

In reality, however, recorded reservoir response \( \hat{R}(\omega, p) \) may contain some portion of the source wavelet \( \hat{s}(t_0) \) which was not properly deconvolved. This portion of the wavelet can be put into equation (12) explicitly:

\[
\hat{S}(\omega) R(\omega, p) = \sum_k e^{i2\omega t_p(k, p)} [\hat{s}(t_0) \ast \hat{f}(t_0, p)]_k,
\]

where \( \hat{S}(\omega) \) is the portion the source spectrum which was not removed, and "\( \ast \)" denotes convolution. From equations (16), I can find the convolved reflection coefficients \( \hat{s}(t_0) \ast \hat{f}(t_0) \). Then, solving equations (10) with the left-hand sides \( \hat{s}(t_0) \ast \hat{f}(t_0) \), yields the result which is approximately [for small \( \hat{f}(t_0) \)] the convolution \( \hat{s}(t_0) \ast \hat{r}(t_0) \).

This approximation becomes exact if reflection coefficients \( \hat{r}(t_0) \) are so small that the influence of transmission effects can be ignored. Finally, I obtain \( A(t_0) \) and \( B(t_0) \) which are also approximately convolutions of true AVO intercept and gradient with wavelet \( s(t_0) \).

Even when the source wavelet is correctly estimated, obtained \( \hat{r}(t_0) \) may be still inaccurate because of errors in the anisotropic velocity model used to compute \( t_p(t_0, p) \). As equation (6) indicates, correct calculating \( t_p(t_0, p) \) requires knowing not only interval vertical velocities \( v_0 \) but also interval anisotropic coefficients which control the vertical slowness \( q(p) \). Such detailed information apparently cannot be extracted from seismic data. Therefore, in practice, velocity model used to compute \( t_p(t_0, p) \) in equations (16) will always be erroneous. This means that reflections from different interfaces within reservoir are going to be mispositioned which, in turn, will lead to errors in \( \hat{r}(t_0, p) \) and, consequently, to errors in AVO intercept \( A(t_0) \) and gradient \( B(t_0) \). Fortunately, for relatively thin reservoirs, errors in \( t_p(t_0, p) \) due to inaccuracy in anisotropic velocity model are small even if some average isotropic velocity model is used. As I demonstrate below, those errors do not lead to noticeable distortions in \( A(t_0) \) and \( B(t_0) \), thus, in fact, detailed velocity information within reservoir is not needed for AVO analysis.

Numerical examples

Here, I demonstrate capabilities of the developed technique and clarify various computational issues. I apply the technique to two reservoir models. For all examples presented, I generate reflection response \( R(\omega, p) \) using anisotropic reflectivity code. Thus, the response I examine contain all possible multiples and converted waves. I use equations (12) and (10) to find \( r_k(p) \); then, applying equations (9), (14), and (15), obtain AVO intercept \( A(t_0) \) and gradient \( B(t_0) \).

Model 1

The first model contains two orthorhombic layers between two isotropic half-spaces (Table 1). This model was intentionally designed in such a way that azimuthal variations of \( P \)-wave reflection coefficients from its interfaces have all theoretically possible shapes (for small slowness \( p \): a saddle, a maximum, and a minimum (Figure 2). Note that the range of \( p \) values is smaller in Figure 2a compared to that in Figure 2b and 2c. I reduced the range of \( p \) in Figure 2a to show more explicitly the shape of reflection-coefficient surface at small \( p \) or small offsets. Corresponding surface for conventional offsets (when incidence angle reaches 30°) is shown in Figure 3. Clearly, the order of the surface in Figure 3 is higher than quadratic. To accommodate those high-order variations in reflection coefficient, I use \( N = 4 \) in equation (9) but analyze only AVO intercept and gradient.

An important quantity which can be extracted from azimuthal AVO analysis is the azimuth \( \beta \) of greater principal value of AVO gradient (I will call \( \beta \) the AVO azimuth). It equals to the azimuth of eigenvector corresponding to the greater eigenvalue of matrix \( B \) [equation (15)]:

\[
\beta = \frac{1}{2} \tan^{-1} \left( \frac{2B_{12}}{B_{11} - B_{22}} \right).
\]

The principal values of AVO gradient (eigenvalues of matrix \( B \)) are given by

\[
b_{1,2} = \frac{1}{2} \left[ B_{11} + B_{22} \pm \sqrt{(B_{11} - B_{22})^2 + 4B_{12}^2} \right].
\]

I will be mostly interested in reconstructing \( \beta(t_0) \).

† This is similar to using stacking velocity obtained from nonhyperbolic velocity analysis of long-spread reflection moveout.
Table 1. Reservoir model 1: two orthorhombic layers between isotropic half-spaces. Density 2.5 g/cm³ is the same in all layers and half-spaces. Azimuths of [x₁, x₃] symmetry planes of the layers are measured counter-clockwise from the positive direction of axis p₂ = 0.

![Figure 2. P-wave reflection coefficients at three interfaces in model 1. Maximum incidence angle is 15° for the first interface (a) and 30° for the second (b) and the third (c) interfaces.](image1)

Figure 2. P-wave reflection coefficients at three interfaces in model 1. Maximum incidence angle is 15° for the first interface (a) and 30° for the second (b) and the third (c) interfaces.

since it relates to the symmetry directions of contacting anisotropic layers (assuming that the layers have orthorhombic or HTI symmetry) and, therefore, indicates the azimuth of vertical fractures, which is of great importance for seismic exploration. If only one of contacting layers is azimuthally anisotropic, fracture orientation in this layer is parallel or orthogonal to the azimuth θ. If both layers are azimuthally anisotropic, θ incorporates influence of anisotropy of both.

Figure 3. P-wave reflection coefficient at the first interface in model 1. Maximum incidence angle is 30°.

![Figure 3. P-wave reflection coefficient at the first interface in model 1. Maximum incidence angle is 30°.](image2)

Figure 3. P-wave reflection coefficient at the first interface in model 1. Maximum incidence angle is 30°.

Figure 4. Normalized AVO attributes as functions of vertical time estimated for model 1: (a) AVO intercept A, (b) principal values of AVO gradient [solid – b₁ and dashed – b₂] obtained from equation (18), and (c) AVO azimuth β (in degrees) given by equation (17).

![Figure 4. Normalized AVO attributes as functions of vertical time estimated for model 1: (a) AVO intercept A, (b) principal values of AVO gradient [solid – b₁ and dashed – b₂] obtained from equation (18), and (c) AVO azimuth β (in degrees) given by equation (17).](image3)

Figure 4. Normalized AVO attributes as functions of vertical time estimated for model 1: (a) AVO intercept A, (b) principal values of AVO gradient [solid – b₁ and dashed – b₂] obtained from equation (18), and (c) AVO azimuth β (in degrees) given by equation (17).

and used correct anisotropic velocity model given in Table 1 to calculate t_p(t₀, p) [see equations (12) and (6)]. The reflection response was convolved with a bell-shaped wavelet,

\[ s(t₀) = \exp(-\nu^2 t₀^2), \]

(19)
\[ \nu = 100 \text{ s}^{-1} \]; the side lobes of wavelets in Figure 4 appeared because of spline-interpolation between time samples \( \tau = 4 \text{ ms} \).

Figure 4 presents almost ideal result:

- Normal incidence amplitudes for all there reflections are equal (Figure 4a) as it is supposed to be (correct normal-incidence reflection coefficients, which can be calculated based on Table 1, are \( \pm 0.067 \)). This indicates that the correction for transmission effects [equation (10)] is sufficiently accurate, and ignoring multiples and converted waves for this model is justified.

- The signs of the principal values of AVO gradient shown in Figure 4b are in exact accordance with the shapes of reflection-coefficient surfaces (Figure 2). Reflection at \( t_0 = 0 \) has a saddle-type shape (Figure 2a); correspondingly, its \( b_1 \) and \( b_2 \) values [equation (18)] are of opposite sign. Reflection from the second interface at \( t_0 = 24 \text{ ms} \) has a maximum at normal incidence and both its \( b_1 \) and \( b_2 \) are negative. Similarly, for reflection from the third interface at time \( t_0 = 48 \text{ ms} \), which has the minimum at normal incidence, both principal values of AVO gradient are positive.

- The AVO azimuths \( \beta \) (Figure 4c), whose correct values are \( 60^\circ, -14^\circ, \) and \(-50^\circ \), were reconstructed with error less than \( 1^\circ \). This error can be attributed to the influence of ignored multiples and converted waves. The azimuths \( \beta \) for the first and third reflections correspond to the \( [x_2, z_3] \) and \( [x_1, z_3] \)-symmetry planes in the first and second orthorhombic layers (compare Figure 4c and Table 1).

Figure 5 demonstrates a more realistic example. It shows the results of applying the AVO procedure to the same reflection response as that in Figure 4 with the only difference: an incorrect isotropic model with constant

\[ t_p(t_0, p) = t_0 \sqrt{1 - v_0^2 (p_1^2 + p_2^2)} . \] (20)

Since intentionally incorrect velocity model was used, the matrix \( e^{2\pi i t_p(t_0, p)} \) in equations (12) is erroneous, which leads to incorrect values of \( \hat{R}(t_0, p) \). Moreover, due to incorrect \( t_p \), reflections at \( t_0 = 24 \text{ ms} \) and \( t_0 = 48 \text{ ms} \) are mispositioned which produces additional errors in \( A(t_0) \) and \( B(t_0) \). However, Figure 5 demonstrates that obtained AVO signatures are very close to the correct ones shown in Figure 4. Some differences can be observed between \( b_2 \) values for reflection at \( t_0 = 48 \). Also, there are errors up to \( 5^\circ \) in estimated AVO azimuths. To explain such good results, one should take
Table 2. Reservoir model 2: Barinas field, Venezuela. The model contains isotropic layers and half-spaces and one HTI layer. HTI anisotropy can be viewed as a special case of orthorhombic anisotropy with the following constraints on elastic constants: $c_{12} = c_{13}, c_{22} = c_{33}, c_{23} = c_{32} = 2c_{44}$, and $c_{44} = c_{66}$. Azimuth $\alpha$ (in degrees) is given for the symmetry-axis plane of the HTI layer with respect to the positive direction of axis $p_2 = 0$.

<table>
<thead>
<tr>
<th>Layer or half-space</th>
<th>Name</th>
<th>Thickness (m)</th>
<th>Density (g/cm$^3$)</th>
<th>Density normalized $c_{ij}$ (km$^2$/s$^2$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Paguey</td>
<td>2.3</td>
<td>9.97</td>
<td>5.17</td>
<td>9.97</td>
</tr>
<tr>
<td>2</td>
<td>Gobernador</td>
<td>300</td>
<td>2.4</td>
<td>12.88</td>
<td>5.46</td>
</tr>
<tr>
<td>3</td>
<td>La Morita</td>
<td>30</td>
<td>2.4</td>
<td>11.19</td>
<td>6.19</td>
</tr>
<tr>
<td>4</td>
<td>O member</td>
<td>21</td>
<td>2.5</td>
<td>12.34</td>
<td>5.22</td>
</tr>
<tr>
<td>5</td>
<td>P member</td>
<td>81</td>
<td>2.7</td>
<td>24.94</td>
<td>16.44</td>
</tr>
<tr>
<td>6</td>
<td>S shale</td>
<td>12</td>
<td>2.7</td>
<td>22.56</td>
<td>0.18</td>
</tr>
<tr>
<td>7</td>
<td>Aquitard1</td>
<td>154</td>
<td>2.4</td>
<td>23.04</td>
<td>12.80</td>
</tr>
<tr>
<td>8</td>
<td>Basement</td>
<td>2.6</td>
<td>30.70</td>
<td>16.98</td>
<td>30.70</td>
</tr>
</tbody>
</table>

into account that, since the whole reservoir is relatively thin (time thickness is 48 ms), errors in traveltimes $t_p$ due to incorrect velocity are small and cannot significantly impair obtained estimations of AVO signatures. This conclusion is very important because it indicates that a precise reservoir model is, in fact, not needed for azimuthal AVO analysis.

The previous two examples were somewhat artificial because frequency of the bell-shaped wavelet was high enough so there was no interference between reflections from different interfaces. A more realistic example is shown in Figure 6. I have chosen the same wavelet (15) but this time with $\nu = 7$ s$^{-1}$. The same incorrect isotropic velocity model with $v_0 = 4$ km/s is used again. Figure 6a can be viewed as a low-pass filtered version of Figure 5a. A single reflection event is observed this time. It is important that the curve $\beta(t_0)$ shown in Figure 6c can also be thought as a low-resolution version of corresponding curve in Figure 5c. Thus, useful information about azimuthal anisotropy can be obtained even without isolating individual reflections.

The last example for model 1 is shown in Figure 7. This time, the Ricker wavelet with dominant frequency 20 Hz was convolved with the reservoir reflection response. Assuming that I am not aware of that, I, again, use an incorrect isotropic velocity model with $v_0 = 4$ km/s to obtain $\tilde{s}(t_0) \ast \tilde{r}(t_0, p)$ from equations (16). Comparing Figure 7a and 7b with Figure 5a and 5c shows that both AVO intercept and the principal values of AVO gradient are close to convolutions of signatures shown in Figure 5 with Ricker wavelet. The wavelet shape has its imprint on AVO azimuth as well: the curve $\beta(t_0)$ (Figure 7c) has a spurious lobe at time $t_0 = 55$ ms. Unfortunately, such false AVO azimuths cannot be avoided because the algorithm interprets wavelet lobes as real reflections. Therefore, Figure 7 emphasizes the necessity of wavelet estimation and deconvolution for AVO processing in fine layered reservoirs.

Model 2

For the second test I chose the model of carbonate fractured reservoir in the Barinas field, Venezuela (Table 2). This model was initially built by Micheletta et al. (1994) and Ata et al. (1994) to describe splitting of converted
waves and, then, refined by Perez and Gibson (1996) to explain P-wave AVO signatures recorded along different azimuths. I assumed HTI anisotropy in P member
(layer 5 in Table 2) and selected its parameters in such a way to produce a saddle-type AVO responses observed by Perez et al. (1998).

Figure 8 demonstrates the results of the first test for model 2. I again put reflection from the top (interface Paguey - Gobennador, see Table 2) at time \( t_0 = 0 \), convolve the reflection response with bell-shaped wavelet (19) \( [\nu = 100 \text{ s}^{-1}] \), and use incorrect isotropic constant-velocity \( (v_0 = 4.1 \text{ km/s}) \) model to obtain AVO intercept and gradient. Figure 8a shows three strong reflections: from interfaces Paguey - Gobennador \( (t_0 = 0) \), O member - P member \( (t_0 = 96 \text{ ms}) \), and Aquardiente - Basement \( (t_0 = 145 \text{ ms}) \). There are also several weaker reflections at \( t_0 \) from approximately 70 to 120 ms and some multiples observed at \( t_0 > 150 \text{ ms} \). Figure 8a clearly illustrates interference of all these events. However, even in the presence of interference, principal values of AVO gradient shown in Figure 8b indicate that there is no azimuthal anisotropy (i.e., \( b_1(t_0) = b_2(t_0) \)) in more shallow horizons than P member. At \( t_0 > 96 \text{ ms} \), after the reflection from interface O member - P member arrives, the difference between \( b_1 \) and \( b_2 \) becomes visible showing the presence of azimuthal anisotropy. The difference between \( b_1 \) and \( b_2 \) extends beyond P member, which can be attributed to combination of errors due to wrong velocity used for AVO analysis with some influence of ignored multiples and converted waves. Computing AVO azimuth \( \beta(t_0) \), I set \( \beta(t_0) \) to 0 when the difference \( b_1(t_0) - b_2(t_0) \) or AVO intercept \( A(t_0) \) is less than 25\% of their maximum values. Figure 8c shows that the azimuth of the symmetry-axis plane \((-\tau 0^\circ) \) within P member is correctly recovered.

P-wave seismic data recorded over Barinas field have much lower frequency than that in Figure 8 (Perez and Gibson, 1996; Perez et al., 1998). To simulate the expected data, I convolved reflection response from the reservoir with Ricker wavelet which has dominant frequency 25 Hz. Again, I used incorrect velocity model with \( v_0 = 4.1 \text{ km/s} \) to extract AVO attributes. The results of applying AVO analysis are shown in Figure 9. Despite obvious interference of various reflection events, which is seen at normal incidence (Figure 9a), a reasonable estimation of AVO azimuth \( \beta \) is obtained (Figure 9c). Although the curve \( \beta(t_0) \) contains small false

\[\frac{\text{The reservoir, filled with crude oil, includes O member, P member, S shale, and Aquardiente. Seismic measurements of fractures within O and P members are supported by borehole data (M. Perez, personal communication).}}{\text{The reservoir, filled with crude oil, includes O member, P member, S shale, and Aquardiente. Seismic measurements of fractures within O and P members are supported by borehole data (M. Perez, personal communication).}}\]
lobes at $t_0$ of about 70 and 130 ms (those correspond to the lobes of Ricker wavelet), it shows almost exact value $\beta = -72^\circ$ within P member. This result looks especially encouraging noting that time thickness of P member (16 ms) is less than half dominant period (20 ms) of the wavelet. Thus, the presented numerical example indicates the potential of AVO analysis in detecting azimuthal anisotropy in very thin layers.

The results shown in Figure 9 can be distorted by adding some random noise to the reflection response. The errors in AVO azimuths become visible when the variance of Gaussian noise reaches 30% of the maximum reflection amplitude. Figure 10 illustrates what happens in this case. Since AVO intercept is a better determined quantity than is AVO gradient, the values of $b_1(t_0)$ (Figure 10b) have greater distortions as compared to that in $A(t_0)$ (Figure 10a). The errors in AVO gradient produce spurious AVO azimuth $\beta$ at $t_0 = 0$ and corrupt AVO azimuth in the vicinity of $t_0 = 100$ ms (compare Figure 9c with Figure 10c). Even though this example combines all errors and inaccuracies I have examined before, the obtained curve $\beta(t_0)$ still shows where azimuthal anisotropy can be expected and thus provides useful information for interpretation.

It is of interest to compare estimations of AVO azimuth $\beta$ obtained from "conventional" azimuthal AVO analysis with the presented results. To simulate "conventional" AVO analysis, I transformed the reflection response $R(\omega, p)$ into time-offset domain and corrected obtained traces for normal moveout (NMO). Figure 11 shows some of those NMO-corrected traces and illustrates the change in reflection response as a function of incidence angle $\theta$ in upper half-space. Then, I picked amplitudes of peaks and troughs in time interval from 50 to 160 ms and used those amplitudes to find AVO azimuths [applying equation (9) with $N = 4$ and equations (13) and (17)] corresponding to each peak and trough. The results (dots) are shown in Figure 9c. Although AVO azimuth $\beta$ at $t_0 = 100$ ms was determined correctly, the overall result is difficult to interpret due to significant scatter in $\beta$ values. The scatter is caused by interference of various reflection events whose dependence on offset (or incidence angle) and azimuth cannot be properly accounted for in time-offset domain.

Discussion and conclusions
I have developed a method for AVO analysis in finely layered azimuthally anisotropic media. The method takes seismic reflection data in $\omega - p$ domain and reconstructs true AVO intercept $A$ and azimuthally varying AVO gradient $B$ which would be recorded in the case of isolated interfaces and in the absence of interference between closely spaced reflection events. The obtained AVO attributes, instantaneous AVO intercept $A(t_0)$ and gradient $B(t_0)$, are functions of the vertical time $t_0$ and represent the required input for various AVO inversion schemes because they can be interpreted in exactly the same way as AVO intercept and gradient at a single isolated reflector at time $t_0$. The technique I have proposed operates in slowness domain because the horizontal slowness $p = (p_1, p_2)$, which is preserved during wave propagation in horizontally layered media, is a natural parameter for reflection coefficients from interfaces located at different vertical times $t_0$. Otherwise, if I parameterize AVO attributes as functions of offset $X$ or the incidence angle $\theta$, I would have to recalculate $X$ or $\theta$ for each $t_0$ since they change in vertically varying media. Although this can be done for any given set of the model parameters, the whole AVO procedure becomes more complicated and probably less stable because the required model parameters are unknown.

The key idea of developed AVO procedure [sketched in Figure 1 and mathematically expressed by equation (4)] is to place each prospective primary plane-wave reflection, specified by the horizontal slowness $p$, at its proper vertical time $t_0$. The idea I applied here is fully equivalent to the concept initially proposed by Wapenaar et al. (1995) that one has to make the apparent vertical wavelength $\lambda_z$ independent of slowness $p$ to properly account for layering. In this regard the developed procedure can be viewed as an extension of technique of Wapenaar et al. (1995) to azimuthally anisotropic media.

To extract individual reflection coefficients $r(t_0, p)$, I used the "exploding reflector-type" reflectivity model, which contains primaries and approximately accounts for transmission effects. At first glance, the reflectivity model may seem too simplistic because it completely ignores multiples and converted waves. However, the numerical examples I have presented indicate that the simplifications made are adequate in the sense that they allowed reconstructing true AVO signatures in the absence of any other errors. I also have shown that the method produces good results even when correct anisotropic velocity within reservoir is replaced by some average isotropic constant-velocity model. This is a very important feature of the algorithm because it allows one to perform AVO analysis without knowing the fine-scale reservoir velocity model. This characteristic of the technique, in fact, could be expected because I used information

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5 The estimated AVO gradient $B(p)$ is also in the slowness domain. Although there is a relation between $B(p)$ and conventional AVO gradient $B(X)$ measured as a function of the offset $X = (x_1, x_2)$, generally $B(p) \neq B(X)$. 
about velocity just to calculate traveltimes of prospective reflection arrivals. It is known that traveltimes are influenced by only relatively long-period velocity variations and almost insensitive to velocity structure on a fine scale.

One of the quantities produced by the procedure is AVO azimuth $\beta(t_0)$ which corresponds to instantaneous direction of larger principal value of AVO gradient $\mathbf{B}(t_0)$. AVO azimuth is of great importance for seismic exploration in fractured reservoirs because it indicates the azimuth of vertical cracks at a given vertical time $t_0$. I have shown that $\beta(t_0)$ is a reasonable well determined quantity so one can expect to obtain it from real data. I have found that errors in $\beta$ are mostly associated with wrong estimation of seismic wavelet. Generally, the wavelet is supposed to be deconvolved before calculating AVO azimuth; otherwise spurious values of $\beta(t_0)$ are produced.

I compared estimated values of $\beta$ with those obtained by "conventional" azimuthal AVO analysis (i.e., when one looks at variation with offset and azimuth of reflection amplitude corresponding to a certain peak or trough on seismic traces). I found that "conventional" estimates of AVO azimuths are less robust and characterized by greater scatter. This scatter can be explained by offset- and azimuth-dependent interference, which is not properly accounted and compensated for in the time-offset domain.

I have tested the technique on seismic model developed for the Barinas field, Venezuela. This model contains several layers which are thinner than half dominant wavelength observed in seismic data. Although I have not examined the limits of spatial resolution of azimuthal AVO analysis in detail, the presented results suggest that it might be possible to detect the presence and principal directions of azimuthal anisotropy in layers with thickness less than half dominant wavelength.

Acknowledgments
I am grateful to Ilya Tsvankin and Petr Jilek (Center for Wave Phenomena, Colorado School of Mines) whose review improved the manuscript. I thank Andreas Rüger (Landmark) for helpful discussions. Special thanks to Reinaldo J. Michelsen and María A. Pérez (PDVSA-Intevep) for providing the seismic model of Barinas field and for sharing their knowledge of structure of the fractured reservoir.

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